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Existence Uniqueness and Stability of Nonlocal Neutral Stochastic Differential Equations with Random Impulses and Poisson Jumps

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Abstract

This manuscript aims to investigate the existence, uniqueness, and stability of non-local random impulsive neutral stochastic differential time delay equations (NRINSDEs) with Poisson jumps. First, we prove the existence of mild solutions to this equation using the Banach fixed point theorem. Next, we prove the stability via continuous dependence initial value. Our study extends the work of Wang and Wu [15] where the time delay is addressed by the prescribed phase space $\mathcal{B}$ (defined in Section 3). An example is given to illustrate the theory.

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1. Introduction

For the last few decades, interest in the study of integrodifferential and stochastic differential equations has grown among the scientific community. We know that the presence of noise and/or stochastic perturbations can be unavoidable when formulating a phenomenon. In such cases, stochastic models tend to offer

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better performance over their deterministic counterparts. The power of stochastic approaches is seen in the formulation and analysis of phenomena, such as population dynamics, stock prices, heat conduction in materials, etc.

Poisson jumps have also become a prevalent modeling component in economics, finance, physics, biology, medicine, and other sciences. It is natural and necessary to include a jump term in the stochastic differential equation. Moreover, many practical systems (such as sudden price variations/jumps due to stock-market crashes, earthquakes, epidemics, and so on) may undergo some jump-type stochastic perturbations. The sample paths of such systems are not continuous and it is more appropriate to consider stochastic processes with jumps to describe such models. In general, these jump models are derived from Poisson random measures. The sample paths of such systems are right continuous and have left limits (càdlàg in short). For more details, see the monographs [1, 3, 4], papers [5, 6], and references therein.

On the other hand, impulsive differential equations also attracted the attention of researchers (see [7, 8, 9]). Differential equations with fixed moments of impulses have become a natural framework for modeling processes in economics, physics, and population dynamics processes. The impulses in usual exist at deterministic or random points. The properties of fixed type impulses are established in many articles [8, 9, 10, 11]. Wu and Meng [12] was the first to consider a random impulsive ordinary differential system and established boundedness of solutions to the model by Liapunov's direct function. Moreover, Anguraj and Vinodkumar [16] investigated the existence and uniqueness of neutral functional differential equations with random impulses. Vinodkumar et al. [17] established the existence and stability results on random impulsive neutral partial differential equations. Recently, Li Zihan et al. [18] studied the existence of solutions for Sturm–Liouville differential equation with random impulses and boundary value problems using Green functions and Dhage's fixed point theorem. Li Zihan et al. [19] also discussed the existence of upper and lower solutions to second order random impulsive differential equation with boundary value problem. The later part of the manuscript involves to construct the sum of two monotonic iterative sequences and prove that they are convergent and thus they conclude that the system has upper and lower solutions. Very recently, Guo Yu [20] obtained the existence of solutions for first-order Hamiltonian random impulsive differential equations with Dirichlet boundary conditions.By using the variational method, they first obtained the corresponding energy functional and by using Legendre transformation, they obtained the conjugation of the functional. Then the existence of critical point was obtained by mountain pass lemma. Finally, authors asserted that the critical point of the energy functional is the mild solution of the first order Hamiltonian random impulsive differential equation. Existence and exponential stability of mild solutions for second-order neutral stochastic functional differential equation with random impulses in Hilbert space was studied by Shu Linxin [21] using Mönch fixed point theorem. Then the mean square exponential stability for the mild solution of the considered equations is obtained by establishing an integral inequality.

Various disturbance factors from random inputs influence stochastic differential equations (SDEs). By the interaction of stochastic processes and mathematical models, the real-world system can be interpreted. Several systems are modeled using stochastic functional differential equations with impulses. In general, impulses appear at random time points, i.e., impulse time and impulsive functions are random variables. SDEs with random impulses are widely used in medicine, biology, economy, finance, and so on. Wang and Wu [15] considered the random impulsive SDEs with stock prices model of the form:

$$\begin{aligned} d[\mathsf{S}(\mathsf{t})] &= \alpha \mathsf{S} dt + \beta \mathsf{S}(\mathsf{t}) dB(t), \ \mathsf{t} \geq 0, \ \mathsf{t} \neq \tau_k, \\ \mathsf{S}(\tau_k) &= a_k \mathsf{S}(\tau_k^-), \ k = 1, 2, ..., \\ \mathsf{S}(0) &= \mathsf{S}_0. \end{aligned}$$

Here B_t is a Brownian motion or Wiener process, $\mathsf{S}(\mathsf{t})$ represents the price of the stock at time t, $\{\tau_k\}$ represents the release time of the important information relating to the stock. $\mathsf{S}(\tau_k^-) = \lim_{\mathsf{t} \to \tau_k - 0} \mathsf{S}(\mathsf{t})$ and $\mathsf{S}(0) \in \mathbb{R}$. In reality, $\{\tau_k\}$ is a sequence of random variables, which satisfies $0 < \tau_1 < \tau_2 < \cdots$. Very recently, Anguraj et al. [14] investigated the stability of SDEs with random impulsive and Poisson jumps. However, to the best of our knowledge, so far, no work has been reported in the literature about NRINSDEs with

Poisson jumps. Inspired by the above-mentioned works, this paper aims to fill this gap by examining the existence, uniqueness, and stability of NRINSDEs with Poisson jumps.

The considered NRINSDEs with Poisson jumps is of the form:

$$d[\mathsf{x}(\mathsf{t}) + \mathfrak{h}(\mathsf{t}, \mathsf{x}_\mathsf{t})] = \mathfrak{f}(\mathsf{t}, \mathsf{x}_\mathsf{t})d\mathsf{t} + \mathfrak{g}(\mathsf{t}, \mathsf{x}_\mathsf{t})dw(\mathsf{t}) + \int_{\mathfrak{U}} \mathfrak{p}(\mathsf{t}, \mathsf{x}_\mathsf{t}, \mathsf{u})\widetilde{\mathsf{N}}(d\mathsf{t}, d\mathsf{u}),\ \mathsf{t} \neq \xi_k,\ \mathsf{t} \geq 0, \tag{1}$$

$$\mathsf{x}(\xi_k^-) = \mathsf{b}_k(\delta_k)\mathsf{x}(\xi_k^-), \qquad k = 1, 2, ..., \tag{2}$$

$$\mathsf{x}(0) + \mathsf{q}(\mathsf{x}) = \mathsf{x}_0 = \phi,\ -\delta \leq \theta \leq 0, \tag{3}$$

where δ_k is a random variable defined from Ω to $\mathfrak{D}_k =^{def} (0, d_k)$ with $0 < d_k < +\infty$ for $k = 1, 2, ...$ Suppose that δ_i and δ_j are independent of each other as $i \neq j$ for $i, j = 1, 2, ...$ Let us define $\mathfrak{C}([-\delta, 0], \mathcal{L}^2(\Omega, \mathbb{R}^d))$. Here, suppose $\mathfrak{T} \in (\mathsf{t}_0, +\infty)$, $\mathfrak{f} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \to \mathbb{R}^d$, $\mathfrak{g} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \to \mathbb{R}^{d\times m}$, $\mathfrak{h} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \to \mathbb{R}^d$, $\mathfrak{p} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \times \mathfrak{U} \to \mathbb{R}^d$, $\mathsf{q} : \mathfrak{C} \to \mathfrak{C}$ and $\mathsf{b}_k : \mathfrak{D}_k \to \mathbb{R}^{d\times d}$, and x_t is $\mathbb{R}^d$-valued stochastic process such that $\mathsf{x}_\mathsf{t} \in \mathbb{R}^d$, $\mathsf{x}_\mathsf{t} = \{\mathsf{x}(\mathsf{t} + \theta) : -\delta \leq \theta \leq 0\}$. The impulsive moments ξ_k from a strictly increasing sequence, i.e., $\xi_0 < \xi_1 < \cdots < \xi_k < \cdots < \lim_{k\to\infty}$, and $\mathsf{x}(\xi_k^-) = \lim_{t\to\xi_k-0} \mathsf{x}(\mathsf{t})$. We assume that $\xi_0 = \mathsf{t}_0$ and $\xi_k = \xi_{k-1} + \delta_k$ for $k = 1, 2, ...$ Obviously, $\{\xi_k\}$ is a process with independent increments. We suppose that $\{\mathsf{N}(\mathsf{t}), \mathsf{t} \geq 0\}$ is the simple counting process generated by $\{\xi_k\}$, and $\{\mathsf{w}(\mathsf{t}) : \mathsf{t} \geq 0\}$ is a given m-dimensional Wiener process. We denote $\Im_t^{(1)}$ the σ-algebra generated by $\{\mathsf{N}(\mathsf{t}), \mathsf{t} \geq 0\}$, and denote $\Im_\mathsf{t}^{(2)}$ the σ-algebra generated by $\{\mathsf{w}(s), s \leq \mathsf{t}\}$. We assume that $\Im_\infty^{(1)}, \Im_\infty^{(2)}$ and ξ are mutually independent. In (1)-(3), $\widetilde{\mathsf{N}}(d\mathsf{t}, d\mathsf{u}) = \mathsf{N}(d\mathsf{t}, d\mathsf{u}) - d\mathsf{t}\mathsf{v}(d\mathsf{u})$ denotes the compensated Poisson measure independent of $\mathsf{w}(\mathsf{t})$ and $\widetilde{\mathsf{N}}(d\mathsf{t}, d\mathsf{u})$ represents the Poisson counting measure associated with a characteristic measure v.

Highlights:

1. This work extends the work of Wang and Wu [15]
2. Time delay of NRINSDEs with Poisson jumps is taken care of by the prescribed phase space $\mathcal{B}$

The arrangement of the rest of the paper is as follows. In Section 2, some preliminaries and results applied in the latter part of the paper are presented. Section 3 is devoted to studying the existence and uniqueness of mild solutions of the system (1)-(3). In Section 4, the stability of the mild solution of the system (1)-(3) is studied.

2. Preliminaries

Let $(\Omega, \Im, \mathbb{P})$ is a probability space with filtration $\{\Im_\mathsf{t}\}$, $\mathsf{t} \geq 0$ satisfying $\Im_\mathsf{t} = \Im_\mathsf{t}^{(1)} \vee \Im_\mathsf{t}^{(2)}$. Let $\mathcal{L}^2(\Omega, \mathbb{R}^d)$ be the collection of all strongly measurable, $\Im_\mathsf{t}$ measurable, $\mathbb{R}^d$-valued random variables x with norm $\|\mathsf{x}\|_{\mathcal{L}^2} = \left(\mathbf{E}\,\|\mathsf{x}\|^2\right)^{\frac{1}{2}}$, where the expectation $\mathbf{E}$ is defined by $\mathbf{E}\mathsf{x} = \int_\Omega \mathsf{x} d\mathbb{P}$. Let $\delta > 0$ denote the Banach space of all piecewise continuous $\mathbb{R}^d$-valued stochastic process $\{\xi(\mathsf{t}), \mathsf{t} \in [-\delta, 0]\}$ by $\mathfrak{C}([-\delta, 0], \mathcal{L}^2(\Omega, \mathbb{R}^d))$ equipped with the norm

$$\|\psi\|_{\mathfrak{C}} = \sup_{\theta\in[-\delta,0]} \left(\mathbf{E}\,\|\psi(\theta)\|^2\right)^{\frac{1}{2}}.$$

The initial data

$$\mathsf{x}_0 = \phi = \mathsf{x}(0) + \mathsf{q}(\mathsf{x}) = \{\phi(\theta) : -\delta \leq \theta \leq 0\}, \tag{4}$$

is an $\Im_{\mathsf{t}_0}$ measurable, $[-\delta, 0]$ to $\mathbb{R}^d$-valued random variable such that $\mathbf{E}\,\|\zeta\|^2 < \infty$.

Poisson Jumps Process:

Let $\mathsf{p}(\mathsf{t}), \mathsf{t} \geq 0$, be an $\mathcal{H}$-valued, σ-finite stationary $\Im_\mathsf{t}$-adapted Poisson point process on $(\Omega, \Im, \Im_\mathsf{t}, \mathbb{P})$. The counting random measure N_p defined by $\mathsf{N}_\mathsf{p}((\mathsf{t}_1, \mathsf{t}_2] \times \Lambda)(\omega) = \sum_{\mathsf{t}_1<s<\mathsf{t}_2} I_\Lambda(\mathsf{p}(s))$ for any $\Lambda \in \mathfrak{B}_\sigma(\mathcal{H})$ is called the Poisson random measure associated with the Poisson point process p. Define the measure $\widetilde{\mathsf{N}}$ by

$$\widetilde{\mathsf{N}}(d\mathsf{t}, d\mathsf{u}) = \mathsf{N}_\mathsf{p}(d\mathsf{t}, d\mathsf{u}) - d\mathsf{t}\mathsf{v}(d\mathsf{u}),$$

where v is the characteristic measure on $\mathcal{H}$ called the compensated Poisson random measure associated with the Poisson point process p.

Definition 2.1. *For a given $\mathfrak{T} \in (\mathsf{t}_0, +\infty)$, a $\mathbb{R}^{-d}$-valued stochastic process $\mathsf{x}(\mathsf{t})$ on $\mathsf{t}_0 - \delta \le \mathsf{t} \le \mathfrak{T}$ is called a solution to (1)-(3) with the initial data (4) if for every $\mathsf{t}_0 \le \mathsf{t} \le \mathfrak{T}$, $\mathsf{x}(\mathsf{t}_0) = \phi$, $\{\mathsf{x}_\mathsf{t}\}_{\mathsf{t}_0 \le \mathsf{t} \le \mathfrak{T}}$ is $\Im_\mathsf{t}$-adapted and*

$$\begin{aligned}
\mathsf{x}(\mathsf{t}) &= \sum_{k=0}^{\infty} \Bigg[\prod_{i=1}^{k} \mathsf{b}_i(\delta_i)[\phi(0) - \mathfrak{q}(\mathsf{x}) + \mathfrak{h}(0,\phi)] - \prod_{i=1}^{k} \mathsf{b}_i(\delta_i)\mathfrak{h}(\mathsf{t}, \mathsf{x}_\mathsf{t}) \\
&+ \sum_{i=1}^{k} \prod_{j=i}^{k} \mathsf{b}_j(\delta_j) \int_{\xi_{i-1}}^{\xi_i} \mathfrak{f}(s, \mathsf{x}_s) ds + \int_{\xi_\mathsf{k}}^{\mathsf{t}} \mathfrak{f}(s, \mathsf{x}_s) ds \\
&+ \sum_{i=1}^{k} \prod_{j=i}^{k} \mathsf{b}_j(\delta_j) \int_{\xi_{i-1}}^{\xi_i} \mathfrak{g}(s, \mathsf{x}_s) dw(s) + \int_{\xi_k}^{t} \mathfrak{g}(s, \mathsf{x}_s) dw(s) \\
&+ \sum_{i=1}^{k} \prod_{j=i}^{k} \mathsf{b}_j(\delta_j) \int_{\mathfrak{U}} \int_{\xi_{i-1}}^{\xi_i} \mathfrak{p}(s, \mathsf{x}_s, \mathsf{u}) \widetilde{\mathsf{N}}(ds, d\mathsf{u}) + \int_{\mathfrak{U}} \int_{\xi_k}^{\mathsf{t}} \mathfrak{p}(s, \mathsf{x}_s, \mathsf{u}) \widetilde{\mathsf{N}}(ds, d\mathsf{u}) \Bigg] \mathsf{I}_{[\xi_k, \xi_{k+1})}(\mathsf{t}), \qquad (5)
\end{aligned}$$

where

$$\prod_{j=i}^{k} \mathsf{b}_j(\delta_j) = \mathsf{b}_k(\delta_k)\mathsf{b}_{k-1}(\delta_{k-1}) \cdots \mathsf{b}_i(\delta_i),$$

and $\mathsf{I}_{(A)}(.)$ is the index function, i.e.,

$$\mathsf{I}_{\mathrm{A}}(\mathsf{t}) = \begin{cases} 1, & if \quad \mathsf{t} \in \mathrm{A}, \\ 0, & if \quad \mathsf{t} \notin \mathrm{A}. \end{cases}$$

Lemma 2.2. *[2] For any $\mathsf{r} \ge 1$ and for arbitrary $\mathcal{L}_2^0$- valued predictable process $\Phi(.)$*

$$\sup_{s \in [0,\mathsf{t}]} \mathbb{E} \left\| \int_0^s \Phi(\mathsf{u}) dw(\mathsf{u}) \right\|_{\mathbb{X}}^{2\mathsf{r}} = (\mathsf{r}(2\mathsf{r}-1))^{\mathsf{r}} \left(\int_0^{\mathsf{t}} (\mathbb{E} \|\Phi(s)\|_{\mathcal{L}_2^0}^{2\mathsf{r}}) ds \right)^{\mathsf{r}}$$

3. Existence and Uniqueness

In order to derive the existence and uniqueness of the system (1)-(3), we shall impose the following assumptions:

(H1) The functions $\mathfrak{f} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \to \mathbb{R}^d$, $\mathfrak{g} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \to \mathbb{R}^{d \times m}$ and $\mathfrak{h} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \to \mathbb{R}^d$ satisfies the Lipschitz condition such that there exist constants $\mathcal{L}_\mathfrak{f} = \mathcal{L}_\mathfrak{f}(\mathfrak{T}) > 0$, $\mathcal{L}_\mathfrak{g} = \mathcal{L}_\mathfrak{g}(\mathfrak{T}) > 0$ and $\mathcal{L}_\mathfrak{h} = \mathcal{L}_\mathfrak{h}(\mathfrak{T}) > 0$ such that,

$$\begin{aligned}
\mathbb{E} \|\mathfrak{f}(\mathsf{t}, \mathsf{x}_\mathsf{t}) - \mathfrak{f}(\mathsf{t}, \mathsf{y}_\mathsf{t})\|^2 &\le \mathcal{L}_\mathfrak{f} \mathbb{E} \|x - y\|_\mathsf{t}^2, \\
\mathbb{E} \|\mathfrak{g}(\mathsf{t}, \mathsf{x}_\mathsf{t}) - \mathfrak{g}(\mathsf{t}, \mathsf{y}_\mathsf{t})\|^2 &\le \mathcal{L}_\mathfrak{g} \mathbb{E} \|x - y\|_\mathsf{t}^2, \\
\mathbb{E} \|\mathfrak{h}(\mathsf{t}, \mathsf{x}_\mathsf{t}) - \mathfrak{h}(\mathsf{t}, \mathsf{y}_\mathsf{t})\|^2 &\le \mathcal{L}_\mathfrak{h} \mathbb{E} \|x - y\|_\mathsf{t}^2,
\end{aligned}$$

for $\mathsf{x}, \mathsf{y} \in \mathfrak{C}$, $\mathsf{t} \in [\mathsf{t}_0, \mathfrak{T}]$.

(H2) The functions $\mathfrak{p} : [\mathsf{t}_0, \mathfrak{T}] \times \mathfrak{C} \times \mathfrak{U} \to \mathbb{R}^d$ satisfies the Lipschitz condition such that there exist constants $\mathcal{L}_\mathfrak{p} = \mathcal{L}_\mathfrak{p}(\mathfrak{T}) > 0$ such that,

$$(i) \int_{\mathfrak{U}} \mathbb{E} \|\mathfrak{p}(\mathsf{t}, \mathsf{x}_\mathsf{t}, \mathsf{u}) - \mathfrak{p}(\mathsf{t}, \mathsf{y}_\mathsf{t}, \mathsf{u})\|^2 \mathsf{v}(d\mathsf{u}) ds \vee$$

$$\left(\int_{\mathfrak{U}} \mathbb{E} \|\mathfrak{p}(\mathsf{t}, \mathsf{x}_\mathsf{t}, \mathsf{u}) - \mathfrak{p}(\mathsf{t}, \mathsf{y}_\mathsf{t}, \mathsf{u})\|^4 \mathsf{v}(d\mathsf{u}) ds \right)^{\frac{1}{2}} \le \mathcal{L}_\mathfrak{p} \mathbb{E} \|\mathsf{x} - \mathsf{y}\|_\mathsf{t}^2,$$

$$(ii) \left(\int_{\mathfrak{U}} \mathbb{E} \|\mathfrak{p}(\mathsf{t}, \mathsf{x}_\mathsf{t}, \mathsf{u})\|^4 \mathsf{v}(d\mathsf{u}) ds \right)^{\frac{1}{2}} \le \mathcal{L}_\mathfrak{p} \|\mathsf{x}\|_\mathsf{t}^2.$$

(H2) For all $\mathsf{t} \in [\mathsf{t}_0, \mathfrak{T}]$, it follows that $\mathfrak{f}(\mathsf{t}, 0), \mathfrak{g}(\mathsf{t}, 0)$, $\mathfrak{h}(\mathsf{t}, 0)$ and $\mathfrak{p}(\mathsf{t}, 0, \mathsf{u}) \in \mathcal{L}^2$, such that,

$$\mathbb{E}\,\|\mathfrak{f}(\mathsf{t},0)\|^2 \le \kappa_{\mathfrak{f}}, \quad \mathbb{E}\,\|\mathfrak{g}(\mathsf{t},0)\|^2 \le \kappa_{\mathfrak{g}}, \quad \mathbb{E}\,\|\mathfrak{h}(\mathsf{t},0)\|^2 \le \kappa_{\mathfrak{h}}, \quad \mathbb{E}\,\|\mathfrak{p}(\mathsf{t},0,\mathsf{u})\|^2 \le \kappa_{\mathfrak{p}},$$

where $\kappa_{\mathfrak{f}}, \kappa_{\mathfrak{g}}$ and $\kappa_{\mathfrak{h}}$ are constants.

(H3) The functions $\mathfrak{q} : \mathfrak{C} \to \mathfrak{C}$ is continuous, and there exists some constant $\mathcal{L}_{\mathfrak{q}} > 0$ such that

$$(i)\mathbb{E}\,\|\mathfrak{q}(\mathsf{t},\mathsf{x}_\mathsf{t}) - \mathfrak{q}(\mathsf{t},\mathsf{y}_\mathsf{t})\|^2 \le \mathcal{L}_{\mathfrak{q}}\mathbb{E}\,\|\mathsf{x}-\mathsf{y}\|_\mathsf{t}^2\,,$$
$$(ii)\mathbb{E}\,\|\mathfrak{q}(\mathsf{t},\mathsf{x}_\mathsf{t})\|^2 \le \mathcal{L}_{\mathfrak{q}}\,\|\mathsf{x}\|_\mathsf{t}^2\,.$$

for $\mathsf{x}, \mathsf{y} \in \mathfrak{C}$, $\mathsf{t} \in [\mathsf{t}_0, \mathfrak{T}]$.

(H4) The condition $\mathbb{E}\left\{\max_{i,k}\{\prod_{j=i}^{k}\|b_j(\delta_j)\|\}\right\}$ is uniformly bounded. That is, there exist constant $\mathcal{C} > 0$ such that,

$$\mathbb{E}\left\{\max_{i,k}\{\prod_{j=i}^{k}\|b_j(\delta_j)\|\}\right\} \;\le\; \mathcal{C}$$

for all $\delta_j \in \mathfrak{D}_j, j = 1, 2, 3...$

Theorem 3.1. *Let the hypotheses (H1)-(H3) be hold. Then there exists a unique continuous mild solution to the system (1)-(3) for any initial value (t_0, ϕ) with $t_0 \ge 0$ and $\phi \in \mathcal{B}$.*

Proof: Let $\mathcal{B}$ be the phase space $\mathcal{B} = \mathfrak{C}([\mathsf{t}_0 - \delta, \mathfrak{T}], \mathcal{L}^2(\Omega, \mathbb{R}^d))$ endowed with the norm

$$\|\mathsf{x}\|_{\mathcal{B}}^2 = \sup_{\mathsf{t}\in[\mathsf{t}_0,\mathsf{T}]} \|\mathsf{x}_\mathsf{t}\|_{\mathfrak{C}}^2\,,$$

where $\|\mathsf{x}_\mathsf{t}\|_{\mathfrak{C}} = \sup_{\mathsf{t}-\delta\le s\le \mathsf{t}} \mathbf{E}\,\|\mathsf{x}(s)\|^2$.
We define the operator $\Phi : \mathcal{B} \to \mathcal{B}$ by

$$(\Phi\mathsf{x})(\mathsf{t}) = \begin{cases} \phi(\mathsf{t}) - \mathfrak{q}(\mathsf{x}), & \mathsf{t} \in (+\infty, \mathsf{t}_0]; \\ \sum_{k=0}^{\infty}\Big[\prod_{i=1}^{k}\mathsf{b}_i(\delta_i)[\phi(0) - \mathfrak{q}(\mathsf{x}) + \mathfrak{h}(0,\phi)] - \prod_{i=1}^{k}\mathsf{b}_i(\delta_i)\mathfrak{h}(\mathsf{t},\mathsf{x}_\mathsf{t}) \\ +\displaystyle\sum_{i=1}^{k}\prod_{j=i}^{k}\mathsf{b}_j(\delta_j)\int_{\xi_{i-1}}^{\xi_i}\mathfrak{f}(s,\mathsf{x}_s)ds + \int_{\xi_k}^{\mathsf{t}}\mathfrak{f}(s,\mathsf{x}_s)ds \\ +\displaystyle\sum_{i=1}^{k}\prod_{j=i}^{k}\mathsf{b}_j(\delta_j)\int_{\xi_{i-1}}^{\xi_i}\mathfrak{g}(s,\mathsf{x}_s)dw(s) + \int_{\xi_k}^{t}\mathfrak{g}(s,\mathsf{x}_s)dw(s) \\ +\displaystyle\sum_{i=1}^{k}\prod_{j=i}^{k}\mathsf{b}_j(\delta_j)\int_{\mathfrak{U}}\int_{\xi_{i-1}}^{\xi_i}\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})\widetilde{\mathsf{N}}(ds,d\mathsf{u}) + \int_{\mathfrak{U}}\int_{\xi_k}^{\mathsf{t}}\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})\widetilde{\mathsf{N}}(ds,d\mathsf{u})\Big]\mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}), & \mathsf{t} \in [\mathsf{t}_0, \mathfrak{T}]. \end{cases}$$

Now we have to prove that Φ maps $\mathcal{B}$ into itself.

$$
\begin{aligned}
&\|(\Phi \mathsf{x})(\mathsf{t})\|^2 \\
&= \Bigg\| \sum_{k=0}^{+\infty} \Bigg[\prod_{i=1}^{k} \mathsf{b}_i(\delta_i)[\phi(0) - \mathfrak{q}(\mathsf{x}) + \mathfrak{h}(0,\phi)] - \prod_{i=1}^{k} \mathsf{b}_i(\delta_i)\mathfrak{h}(\mathsf{t},\mathsf{x}_\mathsf{t}) + \Bigg[\sum_{i=1}^{k} \prod_{j=i}^{k} \mathsf{b}_j(\delta_j) \int_{\xi_{i-1}}^{\xi_i} \mathfrak{f}(s,\mathsf{x}_s)ds \\
&\quad + \int_{\xi_k}^{\mathsf{t}} \mathfrak{f}(s,\mathsf{x}_s)ds \Bigg] + \Bigg[\sum_{i=1}^{k} \prod_{j=i}^{k} \mathsf{b}_j(\delta_j) \int_{\xi_{i-1}}^{\xi_i} \mathfrak{g}(s,\mathsf{x}_s)dw(s) + \int_{\xi_k}^{t} \mathfrak{g}(s,\mathsf{x}_s)dw(s) \Bigg] \\
&+ \Bigg[\sum_{i=1}^{k} \prod_{j=i}^{k} \mathsf{b}_j(\delta_j) \int_{\mathfrak{U}} \int_{\xi_{i-1}}^{\xi_i} \mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})\widetilde{\mathsf{N}}(ds,d\mathsf{u}) + \int_{\mathfrak{U}} \int_{\xi_k}^{\mathsf{t}} \mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})\widetilde{\mathsf{N}}(ds,d\mathsf{u}) \Bigg] \Bigg] \mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}) \Bigg\|^2 \\
&\le 5 \sum_{k=0}^{+\infty} \Bigg[\prod_{i=1}^{k} \|\mathsf{b}_i(\delta_i)\|^2 \, \|\phi(0) - \mathfrak{q}(\mathsf{x}) + \mathfrak{h}(0,\phi)\|^2 \, \mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}) \Bigg] + 5 \sum_{k=0}^{+\infty} \Bigg[\prod_{i=1}^{k} \|\mathfrak{h}(\mathsf{t},\mathsf{x}_\mathsf{t})\|^2 \, \mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}) \Bigg] \\
&\quad + 5 \Bigg[\max_{i,k} \Bigg\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\| \Bigg\} \Bigg]^2 \Bigg(\int_{\mathsf{t}_0}^{\mathsf{t}} \|\mathfrak{f}(s,\mathsf{x}_s)\| \, ds \mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}) \Bigg)^2 + 5 \Bigg[\max_{i,k} \Bigg\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\| \Bigg\} \Bigg]^2 \\
&\quad \times \Bigg(\int_{\mathsf{t}_0}^{\mathsf{t}} \|\mathfrak{g}(s,\mathsf{x}_s)\| \, dw(s) \mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}) \Bigg)^2 \Bigg] + 5 \Bigg[\max_{i,k} \Bigg\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\| \Bigg\} \Bigg]^2 \\
&\quad \times \Bigg(\int_{\mathsf{t}_0}^{\mathsf{t}} \|\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})\| \, \widetilde{\mathsf{N}}(ds,d\mathsf{u}) \mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}) \Bigg)^2 \\
&\le 10 \Bigg[\max_{k} \Bigg\{ \prod_{i=1}^{k} \|\mathsf{b}_i(\delta_i)\|^2 \Bigg\} \Bigg] \Big[\|\phi(0) - \mathfrak{q}(\mathsf{x})\|^2 + \|\mathfrak{h}(0,\phi)\|^2 \Big] + 10 \Bigg[\max_{k} \Bigg\{ \prod_{i=1}^{k} \|\mathsf{b}_i(\delta_i)\|^2 \Bigg\} \Bigg]
\end{aligned}
$$

$$
\begin{aligned}
&\times \Big[\|\mathfrak{h}(\mathsf{t},\mathsf{x}_\mathsf{t}) - \mathfrak{h}(\mathsf{t},0)\|^2 + \|\mathfrak{h}(\mathsf{t},0)\|^2 \Big] + 10 \Bigg[\max_{i,k} \Bigg\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\|^2 \Bigg\} \Bigg] \\
&\times (\mathsf{t} - \mathsf{t}_0) \int_{\mathsf{t}_0}^{\mathsf{t}} \Big[\|\mathfrak{f}(s,\mathsf{x}_s) - \mathfrak{f}(s,0)\|^2 + \|\mathfrak{f}(s,0)\|^2 \Big] ds + 10 \Bigg[\max_{i,k} \Bigg\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\|^2 \Bigg\} \Bigg] \\
&\times (\mathsf{t} - \mathsf{t}_0) \int_{\mathsf{t}_0}^{\mathsf{t}} \Big[\|\mathfrak{g}(s,\mathsf{x}_s) - \mathfrak{g}(s,0)\|^2 + \|\mathfrak{g}(s,0)\|^2 \Big] ds + 10 \Bigg[\max_{i,k} \Bigg\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\|^2 \Bigg\} \Bigg] \\
&\times (\mathsf{t} - \mathsf{t}_0) \Bigg[\int_{\mathsf{t}_0}^{\mathsf{t}} \int_{\mathfrak{U}} \Big[\|\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u}) - \mathfrak{p}(s,0,\mathsf{u})\|^2 + \|\mathfrak{p}(s,0,\mathsf{u})\|^2 \Big] \mathsf{v}(d\mathsf{u})ds \Bigg] \\
&+ 5 \Bigg[\max_{i,k} \Bigg\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\|^2 \Bigg\} \Bigg] \times (\mathsf{t} - \mathsf{t}_0) \Bigg[\int_{0}^{\mathsf{t}} \int_{\mathfrak{U}} \|\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})\|^4 \, \mathsf{v}(d\mathsf{u})ds \Bigg]^{\frac{1}{2}}.
\end{aligned}
$$

Then,

$$\begin{aligned}
&\mathbb{E}\left\|(\Phi \mathsf{x})(\mathsf{t})\right\|_{\mathsf{t}}^2 \\
&\leq 20\mathcal{C}^2\left[\mathbb{E}\left\|\phi(0)\right\|^2+\mathcal{L}_{\mathfrak{q}}\mathbb{E}\left\|\mathsf{x}\right\|^2\right]+10\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}\mathbb{E}\left\|\phi\right\|^2+10\mathcal{C}^2\left[\mathcal{L}_{\mathfrak{h}}\mathbb{E}\left\|\mathsf{x}\right\|_{\mathsf{t}}^2+\kappa_{\mathfrak{h}}\right] \\
&\quad+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\left[\mathcal{L}_{\mathfrak{f}}\mathbb{E}\left\|\mathsf{x}\right\|_s^2+\kappa_{\mathfrak{f}}\right]ds+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathbb{C}_2 \\
&\quad\times\int_{\mathsf{t}_0}^{\mathsf{t}}\left[\mathcal{L}_{\mathfrak{g}}\mathbb{E}\left\|\mathsf{x}\right\|_s^2+\kappa_{\mathfrak{g}}\right]ds+20\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\mathcal{L}_{\mathfrak{p}}\mathbb{E}\left\|\mathsf{x}\right\|_s^2ds \\
&\quad+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\kappa_{\mathfrak{p}} \\
&\leq 20\mathcal{C}^2\left[\mathbb{E}\left\|\phi(0)\right\|^2+\mathcal{L}_{\mathfrak{q}}\mathbb{E}\left\|\mathsf{x}\right\|^2\right]+10\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}\mathbb{E}\left\|\phi\right\|^2+10\mathcal{C}^2\kappa_{\mathfrak{h}}+10\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}\mathbb{E}\left\|\mathsf{x}\right\|_{\mathsf{t}}^2 \\
&\quad+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\mathcal{L}_{\mathfrak{f}}\mathbb{E}\left\|\mathsf{x}\right\|_s^2ds+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\kappa_{\mathfrak{f}} \\
&\quad+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathbb{C}_2\int_{\mathsf{t}_0}^{\mathsf{t}}\mathcal{L}_{\mathfrak{g}}\mathbb{E}\left\|\mathsf{x}\right\|_s^2ds+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\mathbb{C}_2\kappa_{\mathfrak{g}} \\
&\quad+20\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\mathcal{L}_{\mathfrak{p}}\mathbb{E}\left\|\mathsf{x}\right\|_s^2ds+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\kappa_{\mathfrak{p}}.
\end{aligned}$$

Taking supremum over t, we get

$$\begin{aligned}
&\sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]}\mathbb{E}\left\|(\Phi \mathsf{x})(\mathsf{t})\right\|_{\mathsf{t}}^2 \\
&\leq 20\mathcal{C}^2\left[\mathbb{E}\left\|\phi(0)\right\|^2+\mathcal{L}_{\mathfrak{q}}\sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]}\mathbb{E}\left\|\mathsf{x}\right\|^2\right]+10\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}\mathbb{E}\left\|\phi\right\|^2+10\mathcal{C}^2\kappa_{\mathfrak{h}}+10\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}\sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]}\mathbb{E}\left\|\mathsf{x}\right\|_{\mathsf{t}}^2
\end{aligned}$$

$$\begin{aligned}
&\quad+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\mathcal{L}_{\mathfrak{f}}\sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]}\mathbb{E}\left\|\mathsf{x}\right\|_s^2ds+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\kappa_{\mathfrak{f}} \\
&\quad+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathbb{C}_2\int_{\mathsf{t}_0}^{\mathsf{t}}\mathcal{L}_{\mathfrak{g}}\sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]}\mathbb{E}\left\|\mathsf{x}\right\|_s^2ds+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\mathbb{C}_2\kappa_{\mathfrak{g}} \\
&\quad+20\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\mathcal{L}_{\mathfrak{p}}\sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]}\mathbb{E}\left\|\mathsf{x}\right\|_s^2ds+10\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\kappa_{\mathfrak{p}} \\
&\leq 10\left[2\mathcal{C}^2\mathbb{E}\left\|\phi\right\|^2+\mathcal{C}^2\mathcal{L}^2\mathbb{E}\left\|\phi\right\|^2+\mathcal{C}^2\kappa_{\mathfrak{h}}+max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\left(\kappa_{\mathfrak{f}}+\mathbb{C}_2\kappa_{\mathfrak{g}}+\kappa_{\mathfrak{p}}\right)\right] \\
&\quad+10\left[2\mathcal{L}_{\mathfrak{q}}\mathcal{C}^2+\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}+max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\left(\mathcal{L}_{\mathfrak{f}}+\mathcal{L}_{\mathfrak{g}}\mathbb{C}_2+2\mathcal{L}_{\mathfrak{p}}\right)\right]\left\|\mathsf{x}\right\|_{\mathsf{t}}^2.
\end{aligned}$$

Thus we obtain,

$$\left\|\Phi \mathsf{x}\right\|_{\mathcal{B}}^2\leq m_1+m_2\left\|\mathsf{x}\right\|_{\mathcal{B}}^2,$$

where,

$$\begin{aligned}
m_1 &= 10\left[2\mathcal{C}^2\mathbb{E}\left\|\phi\right\|^2+\mathcal{C}^2\mathcal{L}^2\mathbb{E}\left\|\phi\right\|^2+\mathcal{C}^2\kappa_{\mathfrak{h}}+max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\left(\kappa_{\mathfrak{f}}+\mathbb{C}_2\kappa_{\mathfrak{g}}+\kappa_{\mathfrak{p}}\right)\right], \\
m_2 &= 10\left[2\mathcal{L}_{\mathfrak{q}}\mathcal{C}^2+\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}+max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\left(\mathcal{L}_{\mathfrak{f}}+\mathcal{L}_{\mathfrak{g}}\mathbb{C}_2+2\mathcal{L}_{\mathfrak{p}}\right)\right],
\end{aligned}$$

where m_1 and m_2 are constants. Hence Φ is bounded.
Now we have to prove that Φ is a contraction mapping. For any $\mathsf{x}, \mathsf{y} \in \mathcal{B}$, we have

$$
\begin{aligned}
&\|(\Phi \mathsf{x})(\mathsf{t}) - (\Phi \mathsf{y})(\mathsf{t})\|^2 \\
&\leq 5 \left[\max_k \left\{ \prod_{i=1}^{k} \|\mathsf{b}_i(\delta_i)\|^2 \right\} \|\mathfrak{q}(\mathsf{x}) - \mathfrak{q}(\mathsf{y})\| \, \mathsf{I}_{[\xi_k, \xi_{k+1})} \right]^2 \\
&\quad + 5 \left[\max_k \left\{ \prod_{i=1}^{k} \|\mathsf{b}_i(\delta_i)\|^2 \right\} \|\mathfrak{h}(\mathsf{t}, \mathsf{x}_\mathsf{t}) - \mathfrak{h}(\mathsf{t}, \mathsf{y}_\mathsf{t})\| \, \mathsf{I}_{[\xi_k, \xi_{k+1})} \right]^2 \\
&\quad + 5 \left[\max_{i,k} \left\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\|^2 \right\} \int_{\mathsf{t}_0}^{\mathsf{t}} \|\mathfrak{f}(s, \mathsf{x}_s) - \mathfrak{f}(s, \mathsf{y}_s)\| \, ds \mathsf{I}_{[\xi_k, \xi_{k+1})} \right]^2 \\
&\quad + 5 \left[\max_{i,k} \left\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\|^2 \right\} \int_{\mathsf{t}_0}^{\mathsf{t}} \|\mathfrak{g}(s, \mathsf{x}_s) - \mathfrak{g}(s, \mathsf{y}_s)\| \, dw(s) \mathsf{I}_{[\xi_k, \xi_{k+1})} \right]^2 \\
&\quad + 5 \left[\max_{i,k} \left\{ 1, \prod_{j=i}^{k} \|\mathsf{b}_j(\delta_j)\|^2 \right\} \int_{\mathsf{t}_0}^{\mathsf{t}} \int_{\mathfrak{U}} \|\mathfrak{p}(s, \mathsf{x}_s, \mathsf{u}) - \mathfrak{p}(s, \mathsf{y}_s, \mathsf{u})\| \, \widetilde{\mathsf{N}}(ds, d\mathsf{u}) \mathsf{I}_{[\xi_k, \xi_{k+1})} \right]^2
\end{aligned}
$$

$$
\begin{aligned}
&\mathbb{E} \|(\Phi \mathsf{x})(\mathsf{t}) - (\Phi \mathsf{y})(\mathsf{t})\|^2 \\
&\leq 5\mathcal{C}^2 \mathbb{E} \|\mathfrak{q}(\mathsf{x}) - \mathfrak{q}(\mathsf{y})\|^2 + 5\mathcal{C}^2 \mathbb{E} \|\mathfrak{h}(\mathsf{t}, \mathsf{x}_\mathsf{t}) - \mathfrak{h}(\mathsf{t}, \mathsf{y}_\mathsf{t})\|^2 + 5 \max\left\{1, \mathcal{C}^2\right\} (\mathfrak{T} - \mathsf{t}_0) \\
&\quad \times \int_{\mathsf{t}_0}^{\mathsf{t}} \mathbb{E} \|\mathfrak{f}(s, \mathsf{x}_s) - \mathfrak{f}(s, \mathsf{y}_s)\|^2 \, ds + 5 \max\left\{1, \mathcal{C}^2\right\} (\mathfrak{T} - \mathsf{t}_0) \\
&\quad \times \mathbb{C}_2 \int_{\mathsf{t}_0}^{\mathsf{t}} \mathbb{E} \|\mathfrak{g}(s, \mathsf{x}_s) - \mathfrak{g}(s, \mathsf{y}_s)\|^2 \, ds + 5 \max\left\{1, \mathcal{C}^2\right\} (\mathfrak{T} - \mathsf{t}_0) \\
&\quad \times \int_{\mathsf{t}_0}^{\mathsf{t}} \mathbb{E} \|\mathfrak{p}(s, \mathsf{x}_s, \mathsf{u}) - \mathfrak{p}(s, \mathsf{y}_s, \mathsf{u})\|^2 \, ds \\
&\leq 5\mathcal{C}^2 \mathcal{L}_\mathfrak{q} \|\mathsf{x} - \mathsf{y}\|_\mathsf{t}^2 + 5\mathcal{C}^2 \mathcal{L}_\mathfrak{h} \|\mathsf{x} - \mathsf{y}\|_\mathsf{t}^2 + 5 \max\left\{1, \mathcal{C}^2\right\} (\mathfrak{T} - \mathsf{t}_0) \int_{\mathsf{t}_0}^{\mathsf{t}} \mathcal{L}_\mathfrak{f} \mathbb{E} \|\mathsf{x} - \mathsf{y}\|_s^2 \, ds \\
&\quad + 5 \max\left\{1, \mathcal{C}^2\right\} (\mathfrak{T} - \mathsf{t}_0) \mathbb{C}^2 \int_{\mathsf{t}_0}^{\mathsf{t}} \mathcal{L}_\mathfrak{g} \mathbb{E} \|\mathsf{x} - \mathsf{y}\|_s^2 \, ds \\
&\quad + 5 \max\left\{1, \mathcal{C}^2\right\} (\mathfrak{T} - \mathsf{t}_0) \int_{\mathsf{t}_0}^{\mathsf{t}} \mathcal{L}_\mathfrak{p} \mathbb{E} \|\mathsf{x} - \mathsf{y}\|_s^2 \, ds,
\end{aligned}
$$

Moreover,

$$\begin{aligned}
&\sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]} \mathbb{E}\left\|(\Phi\mathsf{x})(\mathsf{t})-(\Phi\mathsf{y})(\mathsf{t})\right\|^2\\
&\leq 5\mathcal{C}^2\mathcal{L}_{\mathfrak{q}} \sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]} \|\mathsf{x}-\mathsf{y}\|_{\mathsf{t}}^2 + 5\mathcal{C}^2\mathcal{L}_{\mathfrak{h}} \sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]} \|\mathsf{x}-\mathsf{y}\|_{\mathsf{t}}^2\\
&\quad + 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\mathcal{L}_{\mathfrak{f}} \sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]} \mathbb{E}\|\mathsf{x}-\mathsf{y}\|_s^2\\
&\quad + 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\mathbb{C}^2\mathcal{L}_{\mathfrak{g}} \sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]} \mathbb{E}\|\mathsf{x}-\mathsf{y}\|_s^2\\
&\quad + 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2\mathcal{L}_{\mathfrak{p}} \sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]} \mathbb{E}\|\mathsf{x}-\mathsf{y}\|_s^2\\
&\leq \left\{5\mathcal{C}^2\mathcal{L}_{\mathfrak{q}} + 5\mathcal{C}^2\mathcal{L}_{\mathfrak{h}} + 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2[\mathcal{L}_{\mathfrak{f}}+\mathbb{C}_2\mathcal{L}_{\mathfrak{g}}+\mathcal{L}_{\mathfrak{p}}]\right\} \sup_{\mathsf{t}\in[\mathsf{t}_0,\mathfrak{T}]} \mathbb{E}\|\mathsf{x}-\mathsf{y}\|_{\mathsf{t}}^2.
\end{aligned}$$

Thus

$$\|(\Phi\mathsf{x})-(\Phi\mathsf{y})\|_{\mathcal{B}}^2 \leq \Upsilon(\mathfrak{T})\,\|\mathsf{x}-\mathsf{y}\|_{\mathcal{B}}^2,$$

with

$$\Upsilon(\mathfrak{T}) = 5\mathcal{C}^2\mathcal{L}_{\mathfrak{q}} + 5\mathcal{C}^2\mathcal{L}_{\mathfrak{h}} + 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)^2[\mathcal{L}_{\mathfrak{f}}+\mathbb{C}_2\mathcal{L}_{\mathfrak{g}}+\mathcal{L}_{\mathfrak{p}}]$$

By taking suitable $0 < \mathfrak{T}_1 < \mathfrak{T}$ sufficiently small such that, $\Upsilon(\mathfrak{T}_1) < 1$. Hence Φ is a contraction on $\mathcal{B}_{\mathfrak{T}_1}$ ($\mathcal{B}_{\mathfrak{T}_1}$ denotes $\mathcal{B}$ with $\mathfrak{T}$ substituted by $\mathfrak{T}_1$). By Banach Contraction Principle, a unique fixed point $\mathsf{x} \in \mathcal{B}_{\mathsf{T}_1}$ is obtained for the operator Φ and therefore $\Phi\mathsf{x} = \mathsf{x}$ is a mild solution of the system (1)-(3). The solution can be extended to the entire interval $(-\delta, \mathfrak{T}]$ in finitely many steps which completes the proof for the existence and uniqueness of mild solutions on the entire interval $(-\delta, \mathfrak{T}]$.

4. Stability

The stability through continuous dependence of solutions on initial condition are investigated.

Definition 4.1. *A mild solution* $\mathsf{x}(\mathsf{t})$ *of the system (1)-(3) with initial condition* ϕ *satisfies (4) is said to be stable in the mean square if for all* $\epsilon > 0$ *there exist,* $\eta > 0$ *such that,*

$$\begin{aligned}
\mathbb{E}\left\|\mathsf{x}(\mathsf{t})-\widehat{\mathsf{x}}(\mathsf{t})\right\|^2 &\leq \epsilon \ \ \textit{whenever},\\
\mathbb{E}\left\|\phi-\widehat{\phi}\right\|^2 &\leq \eta \ \textit{for all } t \in [\mathsf{t}_0,\mathfrak{T}],
\end{aligned}$$

where $\widehat{\mathsf{x}}(\mathsf{t})$ *is another mild solution of the system (1)-(3) with initial value* ϕ *defined in (4).*

Theorem 4.2. *Let* $\mathsf{x}(\mathsf{t})$ *and* $\mathsf{y}(\mathsf{t})$ *be mild solution of the system (1)-(3) with initial conditions* ϕ_1 *and* ϕ_2 *respectively. If the assumptions of theorem 3.1 gets satisfied, the mean solution of the system (1)-(3) is stable in the mean square.*

Proof: By assumptions, $\mathsf{x}(\mathsf{t})$ and $\mathsf{y}(\mathsf{t})$ be two mild solutions of the system (1)-(3) with initial values ϕ_1 and

ϕ_2 respectively.

$$\begin{aligned}
&\mathsf{x}(\mathsf{t})-\mathsf{y}(\mathsf{t})\\
&=\sum_{k=0}^{+\infty}\Bigg[\prod_{i=1}^{k}\mathsf{b}_i(\delta_i)\left[[\phi_1-\phi_2]+[\mathfrak{q}(\mathsf{x})-\mathfrak{q}(\mathsf{y})]+[\mathfrak{h}(0,\phi_1)-\mathfrak{h}(0,\phi_2)]\right]-\prod_{i=1}^{k}\mathsf{b}_i(\delta_i)\left[\mathfrak{h}(\mathsf{t},\mathsf{x}_\mathsf{t})-\mathfrak{h}(\mathsf{t},\mathsf{y}_\mathsf{t})\right]\\
&\quad+\sum_{i=1}^{k}\prod_{j=i}^{k}\mathsf{b}_j(\delta_j)\int_{\xi_{i-1}}^{\xi_i}[\mathfrak{f}(s,\mathsf{x}_s)-\mathfrak{f}(s,\mathsf{y}_s)]\,ds+\int_{\xi_k}^{\mathsf{t}}[\mathfrak{f}(s,\mathsf{x}_s)-\mathfrak{f}(s,\mathsf{y}_s)]\,ds\\
&\quad+\sum_{i=1}^{k}\prod_{j=i}^{k}\mathsf{b}_j(\delta_j)\int_{\xi_{i-1}}^{\xi_i}[\mathfrak{g}(s,\mathsf{x}_s)-\mathfrak{g}(s,\mathsf{y}_s)]\,dw(s)+\int_{\xi_k}^{\mathsf{t}}[\mathfrak{g}(s,\mathsf{x}_s)-\mathfrak{g}(s,\mathsf{y}_s)]\,dw(s)\\
&\quad+\sum_{i=1}^{k}\prod_{j=i}^{k}\mathsf{b}_j(\delta_j)\int_{\xi_{i-1}}^{\xi_i}\int_{\mathfrak{U}}[\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})-\mathfrak{p}(s,\mathsf{y}_s,\mathsf{u})]\,\widetilde{\mathsf{N}}(ds,d\mathsf{u})\\
&\quad+\int_{\xi_k}^{\mathsf{t}}\int_{\mathfrak{U}}[\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})-\mathfrak{p}(s,\mathsf{y}_s,\mathsf{u})]\,\widetilde{\mathsf{N}}(ds,d\mathsf{u})\Bigg]\mathsf{I}_{[\xi_k,\xi_{k+1})}(\mathsf{t}).
\end{aligned}$$

Then,

$$\begin{aligned}
&\mathbb{E}\left\|\mathsf{x}(\mathsf{t})-\mathsf{y}(\mathsf{t})\right\|^2\\
&\qquad\le 15\mathcal{C}^2\mathbb{E}\left\|\phi_1-\phi_2\right\|^2+15\mathcal{C}^2\mathbb{E}\left\|\mathfrak{q}(\mathsf{x})-\mathfrak{q}(\mathsf{y})\right\|^2\\
&\qquad\quad+15\mathcal{C}^2\mathbb{E}\left\|\mathfrak{h}(0,\phi_1)-\mathfrak{h}(0,\phi_2)\right\|^2+5\mathcal{C}^2\mathbb{E}\left\|\mathfrak{h}(\mathsf{t},\mathsf{x}_\mathsf{t})-\mathfrak{h}(\mathsf{t},\mathsf{y}_\mathsf{t})\right\|^2
\end{aligned}$$

$$\begin{aligned}
&\quad+5\max\left\{1,\mathcal{C}^2\right\}(\mathsf{t}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathfrak{f}(s,\mathsf{x}_s)-\mathfrak{f}(s,\mathsf{y}_s)\right\|^2ds\\
&\quad+5\max\left\{1,\mathcal{C}^2\right\}(\mathsf{t}-\mathsf{t}_0)\mathbb{C}_2\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathfrak{g}(s,\mathsf{x}_s)-\mathfrak{g}(s,\mathsf{y}_s)\right\|^2ds\\
&\quad+5\max\left\{1,\mathcal{C}^2\right\}(\mathsf{t}-\mathsf{t}_0)\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathfrak{p}(s,\mathsf{x}_s,\mathsf{u})-\mathfrak{p}(s,\mathsf{y}_s,\mathsf{u})\right\|^2ds\\
&\le 15\mathcal{C}^2\mathbb{E}\left\|\phi_1-\phi_2\right\|^2+15\mathcal{C}^2\mathcal{L}_\mathfrak{q}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_\mathsf{t}^2+15\mathcal{C}^2\mathcal{L}_\mathfrak{h}\mathbb{E}\left\|\phi_1-\phi_2\right\|^2\\
&\quad+5\mathcal{C}^2\mathcal{L}_\mathfrak{h}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_\mathsf{t}^2+5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathcal{L}_\mathfrak{f}\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_s^2ds\\
&\quad+5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathbb{C}_2\mathcal{L}_\mathfrak{g}\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_s^2ds\\
&\quad+5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathcal{L}_\mathfrak{p}\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_s^2ds\\
&\le 15\mathcal{C}^2\mathbb{E}\left\|\phi_1-\phi_2\right\|^2[1+\mathcal{L}_\mathfrak{h}]+5\left[3\mathcal{C}^2\mathcal{L}_\mathfrak{q}+\mathcal{C}^2\mathcal{L}_\mathfrak{h}\right]\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_\mathsf{t}^2\\
&\quad+5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathcal{L}_\mathfrak{f}\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_s^2ds+5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathbb{C}_2\mathcal{L}_\mathfrak{g}\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_s^2ds\\
&\quad+5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-\mathsf{t}_0)\mathcal{L}_\mathfrak{p}\int_{\mathsf{t}_0}^{\mathsf{t}}\mathbb{E}\left\|\mathsf{x}-\mathsf{y}\right\|_s^2ds
\end{aligned}$$

Furthermore,

$$\begin{aligned}\sup_{t\in[t_0,\mathfrak{T}]}\mathbb{E}\|x-y\|^2 &\leq 15\mathcal{C}^2\mathbb{E}\|\phi_1-\phi_2\|^2[1+\mathcal{L}_{\mathfrak{h}}]+5\left[3\mathcal{C}^2\mathcal{L}_{\mathfrak{q}}+\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}\right]\sup_{t\in[t_0,\mathfrak{T}]}\mathbb{E}\|x-y\|_t^2\\ &+ 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-t_0)\mathcal{L}_{\mathfrak{f}}\int_{t_0}^{t}\sup_{t\in[t_0,\mathfrak{T}]}\mathbb{E}\|x-y\|_s^2\,ds\\ &+ 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-t_0)\mathbb{C}_2\mathcal{L}_{\mathfrak{g}}\int_{t_0}^{t}\sup_{t\in[t_0,\mathfrak{T}]}\mathbb{E}\|x-y\|_s^2\,ds\\ &+ 5\max\left\{1,\mathcal{C}^2\right\}(\mathfrak{T}-t_0)\mathcal{L}_{\mathfrak{p}}\int_{t_0}^{t}\sup_{t\in[t_0,\mathfrak{T}]}\mathbb{E}\|x-y\|_s^2\,ds\end{aligned}$$

Thus,

$$\sup_{t\in[t_0,\mathfrak{T}]}\mathbb{E}\|x-y\|_t^2\leq\beta\mathbb{E}\|\phi_1-\phi_2\|^2$$

where,

$$\beta=\frac{15\mathcal{C}^2[1+\mathcal{L}_{\mathfrak{h}}]}{1-\left[5\left[3\mathcal{C}^2\mathcal{L}_{\mathfrak{q}}+\mathcal{C}^2\mathcal{L}_{\mathfrak{h}}\right]+5\max(1,\mathcal{C}^2)(\mathfrak{T}-t_0)^2\left[\mathcal{L}_{\mathfrak{f}}+\mathbb{C}_2\mathcal{L}_{\mathfrak{g}}+\mathcal{L}_{\mathfrak{p}}\right]\right]}$$

Given $\epsilon>0$ choose $\eta=\frac{\epsilon}{\beta}$ such that $\mathbb{E}\|\phi_1-\phi_2\|^2<\eta$. Then,

$$\|x-y\|_{\mathcal{B}}^2\leq\epsilon.$$

This completes the proof.

5. An application

The considered NRINSDEs with Poisson jumps is of the form:

$$\begin{aligned}d\left[x(t)+\int_{-\delta}^{0}v_1(\theta)x(t+\theta)\right] &= \left[\int_{-\delta}^{0}v_2(\theta)x(t+\theta)\right]dt+\left[\int_{-\delta}^{0}v_3(\theta)x(t+\theta)\right]dw(t)\\ &+ \left[\int_{-\delta}^{0}\int_{\mathfrak{U}}v_2(\theta)x(t+\theta)\right]\widetilde{N}(dt,du),\ t\geq 0,\ t\neq\xi_k, \quad (6)\\ x(\xi_k) &= b(k)\delta_k x(\xi_k^-),\ k=1,2,..., \quad (7)\\ x(0)+\sum_{i=1}^{n}c_i x(\mathfrak{q}_i,x) &= x_0,\ 0<\mathfrak{q}_1<\mathfrak{q}_2<\cdots\mathfrak{q}_{\mathfrak{p}}<\mathfrak{T}. \quad (8)\end{aligned}$$

Let $r>0$, u in $\mathbb{R}$-valued stochastic process, $\zeta\in\mathfrak{C}([-\delta,0],\mathcal{L}^2(\Omega,\mathbb{R}))$. δ_k is defined from Ω to $D_k\overset{def.}{=}(0,d_k)$ for all $k=1,2,...$, Suppose that δ_k following Erlang distribution and δ_i and δ_j are independent of each other as $i\neq j$ for $i,j=1,2,...t_0=\xi_0<\xi_1<\xi_2<\cdots<\xi_k<\cdots$, and $\xi_k=\xi_{k-1}+\delta_k$ for $k=1,2,...$ Let $w(t)\in\mathbb{R}$ is a one-dimensional Brownian motions, where b is a function of k. $v_1,v_2,v_3,v_4:[-\delta,0]\to\mathbb{R}$ are continuous functions. Define $\mathfrak{f}:[t_0,\mathfrak{T}]\times\mathfrak{C}\to\mathbb{R}^d$, $\mathfrak{g}:[t_0,\mathfrak{T}]\times\mathfrak{C}\to\mathbb{R}^{d\times m}$, $\mathfrak{h}:[t_0,\mathfrak{T}]\times\mathfrak{C}\to\mathbb{R}^d$, $\mathfrak{p}:[t_0,\mathfrak{T}]\times\mathfrak{C}\times\mathfrak{U}\to\mathbb{R}^d$, $\mathfrak{q}:\mathfrak{C}\to\mathfrak{C}$ and $b_k:\mathfrak{D}_k\to\mathbb{R}^{d\times d}$ by

$$\mathfrak{h}(t,x(t))(\cdot)=\int_{-r}^{0}v_1u(t+\theta)d\theta(\cdot),\qquad\mathfrak{f}(t,x(t))(\cdot)=\int_{-r}^{0}v_2u(t+\theta)d\theta(\cdot),$$
$$\mathfrak{g}(t,x(t))(\cdot)=\int_{-r}^{0}v_3u(t+\theta)d\theta(\cdot),\qquad\mathfrak{p}(t,x(t))(\cdot)=\int_{-r}^{0}v_4u(t+\theta)d\theta(\cdot),$$

For $\mathsf{x}(\mathsf{t}+\theta) \in \mathfrak{C}$, we suppose that the following conditions hold:

$$(i). \max_{i,k} \left\{ \prod_{j=i}^{k} \mathbf{E} \|\mathsf{b}(j)(\tau_j)\|^2 \right\} < \infty,$$

$$(ii). \int_{-r}^{0} v_1(\theta)^2 d\theta, \int_{-r}^{0} v_2(\theta)^2 d\theta, \int_{-r}^{0} v_3(\theta)^2 d\theta, \int_{-r}^{0} v_3(\theta)^4 d\theta < \infty.$$

Suppose the state (i) and (ii) gets satisfied from which we can prove that the assumptions (H1)-(H4) holds. Thus system (1)-(3) has a unique mild solution x and is stable.

Remark 5.1. *If $\mathfrak{p} = 0$ in (1)-(3), then the system behaves as NRINSDEs of the form:*

$$d[\mathsf{x}(\mathsf{t}) + \mathfrak{h}(\mathsf{t}, \mathsf{x}_\mathsf{t})] = \mathfrak{f}(\mathsf{t}, \mathsf{x}_\mathsf{t}) d\mathsf{t} + \mathfrak{g}(\mathsf{t}, \mathsf{x}_\mathsf{t}) dw(\mathsf{t}), \quad t \neq \xi_k, \ \mathsf{t} \geq 0, \tag{9}$$

$$\mathsf{x}(\xi_k^-) = \mathsf{b}_k(\delta_k)\mathsf{x}(\xi_k^-), \quad k = 1, 2, ..., \tag{10}$$

$$\mathsf{x}(0) + \mathfrak{q}(\mathsf{x}) = \mathsf{x}_0 = \phi, \ -\delta \leq \theta \leq 0, \tag{11}$$

By applying Theorem 3.1 under the assumptions (H1)-(H4), then the above guarantees the existence of the mild solution.

6. Conclusion

This manuscript is devoted to studying the existence, uniqueness, and stability of NRINSDEs with Poisson jumps. We proved the existence of mild solutions to the equation using the Banach fixed point theorem. Then, we proved the stability via continuous dependence initial value. Further, this result could be extended to investigate the controllability of random impulsive neutral stochastic differential equations finite/infinite state-dependent delay in the future. The fractional-order of NRINSDEs with Poisson jumps would be quite interesting. The controllability of these systems can be studied obviously. Numerical approximation of the given system will lead us to a new direction and be considered future work.

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