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Spin 3-Body Problem with Radiation Terms (II) – Existence of Periodic Solutions of Spin Equations

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Abstract

The present paper is devoted to the existence of a periodic solution of the spin equations for three-body problem of classical electrodynamics. These equations are derived in a previous paper. We present in an operator form the system in consideration and by fixed point method prove an existence of a periodic solution.

Keywords: classical electrodynamics spin equations spin three-body problem.
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1. Introduction

The present paper is an immediate continuation of our investigations on the spin equations for three-body problem of classical electrodynamics from [8]. In [8] we have derived a general system of equations of motion describing three-body problem with radiation terms and spin. The results obtained rely on the previous papers [23], [2] – [7], [16] – [22], [24].

The paper consists of seven sections, Appendix and References. In Section 2 preliminary results for three-body problem equations of motion with radiation terms and their relations to spin equations are considered. In Section 3 the spin equations are derived and the elements of the spin tensor are defined. In Section 4 a vector form of the Lorentz part and radiation parts of the first three spin equations is presented. The Lorentz parts we call the summands in the right-hand sides of the spin equations which we take into account for the influence of a given particle to the rest ones. The terms which take into account the self-interaction of every particle we call radiation terms. In Section 5 we transform the radiation parts of the right-hand sides by similar calculations. In fact, to every moving and spinning particle six functions are assigned, that is, the system for spin functions is overdetermined. We, however, transform every equation and show that the last three equations are consequences of the first three ones [8]. In this way, we obtain nine equations for

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nine unknown spin functions, or we get as many equations as the spin functions are. In Section 6 we derive the radiation terms and present some their estimates. In Section 7 an existence theorem for spin equations is proved.

2. Preliminary Results for Three-Body Problem Equations of Motion with Radiation Terms and Spin Equations

First, we recall some denotations from [8]. The investigations are in the Minkowski space. The Roman subscribes run over 1,2,3,4, while the Greek – over 1,2,3 with Einstein summation convention. By $\langle .,. \rangle_4$ we denote the dot product in the Minkowski space, while by $\langle .,. \rangle$ – the dot product in 3-dimensional Euclidean subspace.

The general system describing the motion of three mass charged particles with radiation terms and spins in the frame of classical electrodynamics derived in [8] is:

$$\frac{d\lambda_r^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left(\sum_{n=1,n\neq k}^{3} F_{rs}^{(kn)}\lambda_s^{(k)} + F_{rs}^{(k)rad}\lambda_s^{(k)}\right),$$

$$\frac{d\sigma_{ij}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left(\sum_{n=1,n\neq k}^{3} F_{im}^{(kn)}\sigma_{mj}^{(k)} - \sum_{n=1,n\neq k}^{3}\sigma_{im}^{(k)}F_{mj}^{(kn)} + F_{im}^{(k)rad}\sigma_{mj}^{(k)} - \sigma_{im}^{(k)}F_{mj}^{(k)rad}\right) \qquad (k=1,2,3),$$

or in details

$$\begin{aligned}
\frac{d\lambda_r^{(1)}}{ds_1} &= \frac{e_1}{m_1 c^2}\left(\left(F_{rs}^{(12)} + F_{rs}^{(13)}\right)\lambda_s^{(1)} + F_{rs}^{(1)rad}\lambda_s^{(1)}\right);\\
\frac{d\lambda_r^{(2)}}{ds_2} &= \frac{e_2}{m_2 c^2}\left(\left(F_{rs}^{(21)} + F_{rs}^{(23)}\right)\lambda_s^{(2)} + F_{rs}^{(2)rad}\lambda_s^{(2)}\right);\\
\frac{d\lambda_r^{(3)}}{ds_3} &= \frac{e_3}{m_3 c^2}\left(\left(F_{rs}^{(31)} + F_{rs}^{(32)}\right)\lambda_s^{(3)} + F_{rs}^{(3)rad}\lambda_s^{(3)}\right);
\end{aligned} \qquad (1)$$

$$\begin{aligned}
\frac{d\sigma_{ij}^{(1)}}{ds_1} &= \frac{e_1}{m_1c^2}\left(\left(F_{im}^{(12)} + F_{im}^{(13)} + F_{im}^{(1)rad}\right)\sigma_{mj}^{(1)} - \sigma_{im}^{(1)}\left(F_{mj}^{(12)} + F_{mj}^{(13)} + F_{mj}^{(1)rad}\right)\right);\\
\frac{d\sigma_{ij}^{(2)}}{ds_2} &= \frac{e_2}{m_2c^2}\left(\left(F_{im}^{(21)} + F_{im}^{(23)} + F_{im}^{(2)rad}\right)\sigma_{mj}^{(2)} - \sigma_{im}^{(2)}\left(F_{mj}^{(21)} + F_{mj}^{(23)} + F_{mj}^{(2)rad}\right)\right);\\
\frac{d\sigma_{ij}^{(3)}}{ds_3} &= \frac{e_3}{m_3c^2}\left(\left(F_{im}^{(31)} + F_{im}^{(32)} + F_{im}^{(3)rad}\right)\sigma_{mj}^{(3)} - \sigma_{im}^{(3)}\left(F_{mj}^{(31)} + F_{mj}^{(32)} + F_{mj}^{(3)rad}\right)\right),
\end{aligned} \qquad (2)$$

where there is a summation in repeated Roman subscribes.

The electromagnetic tensors $F_{rs}^{(kn)}$ in the right-hand sides we call Lorentz parts. For each particle, they take into account the influence of the other particles [2]. The quantities relating to the particles are: $(x_1^{(k)}(t), x_2^{(k)}(t), x_3^{(k)}(t), x_4^{(k)}(t) = ict) \equiv (\vec{x}^{(k)}, ict)$ – space-time coordinates of the moving particles P_k; c – the vacuum speed of light; m_k – proper masses; e_k – charges $(k = 1,2,3)$.

The elements of the electromagnetic tensors $F_{rl}^{(kn)} = \frac{\partial \Phi_l^{(n)}}{\partial x_r^{(k)}} - \frac{\partial \Phi_r^{(n)}}{\partial x_l^{(k)}}$ can be calculated by the Lienard-Wiechert retarded potentials (cf. [23], [20]) $\Phi_r^{(n)} = -\frac{e_n\,\lambda_r^{(n)}}{\left\langle \lambda^{(n)},\, \xi^{(kn)}\right\rangle_4}$, where

$$\begin{aligned}
\xi^{(kn)} &= (\xi_1^{(kn)}, \xi_2^{(kn)}, \xi_3^{(kn)}, \xi_4^{(kn)}) =\\
&= (x_1^{(k)}(t) - x_1^{(n)}(t-\tau_{kn}), x_2^{(k)}(t) - x_2^{(n)}(t-\tau_{kn}), x_3^{(k)}(t) - x_3^{(n)}(t-\tau_{kn}), ic\tau_{kn}(t))
\end{aligned}$$

are isotropic vectors and

$$\lambda^{(k)}=\left(\lambda_1^{(k)},\lambda_2^{(k)},\lambda_3^{(k)},\lambda_4^{(k)}\right)=\left(\vec{\lambda}^{(k)},\lambda_4^{(k)}\right)=\left(\frac{u_1^{(k)}}{\Delta_k},\frac{u_2^{(k)}}{\Delta_k},\frac{u_3^{(k)}}{\Delta_k},\frac{ic}{\Delta_k}\right)=\left(\frac{\vec{u}^{(k)}}{\Delta_k},\frac{ic}{\Delta_k}\right);$$

$$\lambda^{(n)}=\left(\lambda_1^{(n)},\lambda_2^{(n)},\lambda_3^{(n)},\lambda_4^{(n)}\right)=\left(\vec{\lambda}^{(n)},\lambda_4^{(k}\right)=\left(\frac{u_1^{(n)}}{\Delta_{kn}},\frac{u_2^{(n)}}{\Delta_{kn}},\frac{u_3^{(n)}}{\Delta_{kn}},\frac{ic}{\Delta_{kn}}\right)=\left(\frac{\vec{u}^{(n)}}{\Delta_{kn}},\frac{ic}{\Delta_{kn}}\right)$$

are unit tangent vectors to the world lines, where

$$\Delta_k=\sqrt{c^2-\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle};\ \Delta_{kn}=\sqrt{c^2-\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\vec{u}^{(n)}(t-\tau_{kn})\right\rangle}.$$

For non-relativistic case when $\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle\ll c^2$ we have $\lambda^{(k)}=\left(\vec{\lambda}^{(k)},\lambda_4^{(k)}\right)=\left(\frac{u_1^{(k)}}{c},\frac{u_2^{(k)}}{c},\frac{u_3^{(k)}}{c},i\right)$.

Since $\xi^{(kn)}$ are isotropic 4-vectors, i.e. $\left\langle\xi^{(kn)},\xi^{(kn)}\right\rangle_4=0$, the retarded functions $\tau_{kn}(t)$ can be defined as solutions of the functional equations

$$\tau_{kn}(t)=\frac{1}{c}\sqrt{\left\langle\vec{\xi}^{(kn)},\vec{\xi}^{(kn)}\right\rangle}\equiv\frac{1}{c}\sqrt{\sum_{\alpha=1}^{3}\left[x_\alpha^{(k)}(t)-x_\alpha^{(n)}(t-\tau_{kn}(t)\right]^2},$$

where $(kn)=(12),(13),(21),(23),(31),(32)$.

Following [7] we obtain

$$\frac{d\lambda_r^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}F_{rm}^{(kn)}\lambda_m^{(k)};\ \ F_{rm}^{(kn)}=\frac{\partial\Phi_m^{(n)}}{\partial x_r^{(k)}}-\frac{\partial\Phi_r^{(n)}}{\partial x_m^{(k)}}=e_n\left(P_r^{(kn)}\xi_m^{(kn)}-P_m^{(kn)}\xi_r^{(kn)}\right);$$

$$\frac{d\lambda_r^{(k)}}{ds_k}=\frac{e_k}{m_kc^2}\sum_{n=1,n\neq k}^{3}e_n\left(P_r^{(kn)}\xi_m^{(kn)}-P_m^{(kn)}\xi_r^{(kn)}\right)\lambda_m^{(k)}\quad(r=1,2,3,4);$$

$$\frac{d\vec{\lambda}^{(k)}}{ds_k}=-\frac{e_k^2}{m_kc^2}\sum_{n=1,n\neq k}^{3}e_n\left[\vec{P}^{(kn)}\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4-\vec{\xi}^{(kn)}\left\langle\lambda^{(k)},P^{(kn)}\right\rangle_4\right];$$

$$\begin{aligned}\xi^{(kn)}&=(\xi_1^{(kn)},\xi_2^{(kn)},\xi_3^{(kn)},\xi_4^{(kn)})=\\&=(x_1^{(k)}(t)-x_1^{(n)}(t-\tau_{kn}),x_2^{(k)}(t)-x_2^{(n)}(t-\tau_{kn}),x_3^{(k)}(t)-x_3^{(n)}(t-\tau_{kn}),ic\tau_{kn}(t));\end{aligned}$$

$$P_r^{(kn)}=-\frac{\lambda_r^{(n)}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}\left(1+\left\langle\xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4\right)+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4{}^2}\frac{d\lambda_r^{(n)}}{ds_n};$$

$$\vec{P}^{(kn)}=-\frac{\vec{\lambda}^{(n)}M_{kn}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4{}^2}\frac{d\vec{\lambda}^{(n)}}{ds_n};$$

$$M_{kn}=1+\left\langle\xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4=1+\frac{D_{kn}}{\Delta_{kn}^2}\left(\left\langle\vec{\xi}^{(kn)},\dot{\vec{u}}^{(n)}\right\rangle+\frac{\left(\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle-\tau_{kn}c^2\right)\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^2}\right);$$

$$L_{kn}=\frac{M_{kn}}{\Delta_{kn}\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}-\frac{D_{kn}\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^2\Delta_{kn}^4};\quad D_{kn}=\frac{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle}{c^2\tau_{kn}-\left\langle\vec{\xi}^{(kn)},\vec{u}^{(k)}\right\rangle};$$

$$P_4^{(kn)}=-\frac{\lambda_4^{(n)}M_{kn}}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4^3}+\frac{1}{\left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4{}^2}\frac{d\lambda_4^{(n)}}{ds_n}=-icL_{kn};\quad\frac{i}{c}P_4^{(kn)}=L_{kn};$$

$$\frac{d\vec{\lambda}^{(k)}}{ds_k}=\frac{\dot{\vec{u}}^{(k)}(t)}{\Delta_k^2}+\frac{\vec{u}^{(k)}(t)\left\langle\vec{u}^{(k)}(t),\,\dot{\vec{u}}^{(k)}(t)\right\rangle}{\Delta_k^4};\qquad \frac{d\lambda_4^{(k)}}{ds_k}=\frac{ic\left\langle\vec{u}^{(k)}(t),\,\dot{\vec{u}}^{(k)}(t)\right\rangle}{\Delta_k^4};$$

$$\left\langle\vec{\lambda}^{(k)},\vec{\lambda}^{(k)}\right\rangle=\frac{\left\langle\vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}{\Delta_k{}^2};\qquad \frac{d\vec{\lambda}^{(n)}}{ds_n}=\frac{D_{kn}}{\Delta_{kn}^2}\left(\dot{\vec{u}}^{(n)}+\frac{\vec{u}^{(n)}}{\Delta_{kn}^2}\left\langle\vec{u}^{(n)},\,\dot{\vec{u}}^{(n)}\right\rangle\right);$$

$$\frac{d\lambda_4^{(n)}}{ds_n}=\frac{icD_{kn}}{\Delta_{kn}^4}\left\langle\vec{u}^{(n)},\dot{\vec{u}}^{(n)}\right\rangle;\qquad \left\langle\lambda^{(k)},\lambda^{(n)}\right\rangle_4=\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(n)}(t-\tau_{kn})\right\rangle-c^2}{\Delta_k\Delta_{kn}};$$

$$\left\langle\lambda^{(k)},\xi^{(kn)}\right\rangle_4=\frac{\left\langle\vec{u}^{(k)}(t),\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_k};\ \left\langle\lambda^{(n)},\xi^{(kn)}\right\rangle_4=\frac{\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\vec{\xi}^{(kn)}\right\rangle-c^2\tau_{kn}}{\Delta_{kn}}\quad(k,n=1,2,3);$$

$$\left\langle\xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4=D_{kn}\left(\frac{\left\langle\vec{\xi}^{(kn)},\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle}{\Delta_{kn}^2}+\frac{\left\langle\vec{\xi}^{(kn)},\vec{u}^{(n)}(t-\tau_{kn})\right\rangle-c^2\tau_{kn}}{\Delta_{kn}^4}\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle\right);$$

$$\left\langle\lambda^{(k)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4=\frac{D_{kn}}{\Delta_k}\left(\frac{\left\langle\vec{u}^{(k)}(t),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle}{\Delta_{kn}^2}+\frac{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(n)}(t-\tau_{kn})\right\rangle-c^2\tau_{kn}}{\Delta_{kn}^4}\left\langle\vec{u}^{(n)}(t-\tau_{kn}),\dot{\vec{u}}^{(n)}(t-\tau_{kn})\right\rangle\right).$$

The radiation terms are defined as a half of a difference of retarded and advanced potentials

$$\Phi_r^{(k)ret}=-\frac{e_k\lambda_r^{(k)ret}}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4},\ \Phi_r^{(k)adv}=-\frac{e_k\lambda_r^{(k)adv}}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4}$$

in accordance with [7]

$$F_{ml}^{(k)rad}=\frac{1}{2}\left[\left(\frac{\partial\Phi_l^{(k)ret}}{\partial x_m^{(k)ret}}-\frac{\partial\Phi_m^{(n)ret}}{\partial x_l^{(k)ret}}\right)-\left(\frac{\partial\Phi_l^{(k)adv}}{\partial x_m^{(k)adv}}-\frac{\partial\Phi_m^{(n)adv}}{\partial x_l^{(k)adv}}\right)\right],$$

$$F_{rs}^{(k)ret}=\frac{\partial\Phi_s^{(k)ret}}{\partial x_r^{(k)}}-\frac{\partial\Phi_r^{(k)ret}}{\partial x_s^{(k)}}=e_k\left(P_r^{(k)ret}\xi_s^{(k)ret}-P_s^{(k)ret}\xi_r^{(k)ret}\right),$$

$$F_{rs}^{(k)adv}=\frac{\partial\Phi_s^{(k)adv}}{\partial x_r^{(k)}}-\frac{\partial\Phi_r^{(k)adv}}{\partial x_s^{(k)}}=e_k\left(P_r^{(k)adv}\xi_s^{(k)adv}-P_s^{(k)adv}\xi_r^{(k)adv}\right),$$

where

$$P_r^{(k)ret}=-\frac{\lambda_r^{(k)ret}}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4^3}\left[1+\left\langle\xi^{(k)ret},\frac{d\lambda^{(k)ret}}{ds_k}\right\rangle_4\right]+\frac{1}{\left\langle\lambda^{(k)ret},\xi^{(k)ret}\right\rangle_4{}^2}\frac{d\lambda_r^{(k)ret}}{ds_k},$$

$$P_r^{(k)adv}=-\frac{\lambda_r^{(k)adv}}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4^3}\left[1+\left\langle\xi^{(k)adv},\frac{d\lambda^{(k)adv}}{ds_k}\right\rangle_4\right]+\frac{1}{\left\langle\lambda^{(k)adv},\xi^{(k)adv}\right\rangle_4{}^2}\frac{d\lambda_r^{(k)adv}}{ds_k},$$

$$F_{rs}^{(k)rad}=\frac{F_{rs}^{(k)ret}-F_{rs}^{(k)adv}}{2}=$$

$$=e_k\frac{P_r^{(k)ret}\xi_s^{(k)ret}-P_s^{(k)ret}\xi_r^{(k)ret}-\left(P_r^{(k)adv}\xi_s^{(k)adv}-P_s^{(k)adv}\xi_r^{(k)adv}\right)}{2}.$$

In [7] it is proved an existence of periodic solution of (1) under assumption

(C): $\sqrt{\left\langle\vec{u}^{(k)}(t),\vec{u}^{(k)}(t)\right\rangle}\le\bar{c}<c\ \ (k=1,2,3).$

3. Transformation of the Spin Equations in a Vector Form

In view of [8], $\vec{\theta}^{(k)} = \left(\theta_1^{(k)}(t), \theta_2^{(k)}(t), \theta_3^{(k)}(t)\right)$, $\vec{\sigma}^{(k)} = \left(\sigma_1^{(k)}(t), \sigma_2^{(k)}(t), \sigma_3^{(k)}(t)\right)$,

$$\vec{\theta}^{(k)} = \frac{1}{c}\left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}\right) = \frac{1}{c}\begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \frac{1}{c}\begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix}\vec{e}_1 - \frac{1}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix}\vec{e}_2 + \frac{1}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix}\vec{e}_3$$

and then the spin tensors $\sigma_{\mu\nu}^{(k)} (k = 1, 2, 3)$ are

$$\sigma_{\mu\nu}^{(k)} = \begin{pmatrix} 0 & \sigma_3^{(k)} & -\sigma_2^{(k)} & i\,\theta_1^{(k)} \\ -\sigma_3^{(k)} & 0 & \sigma_1^{(k)} & i\,\theta_2^{(k)} \\ \sigma_2^{(k)} & -\sigma_1^{(k)} & 0 & i\,\theta_3^{(k)} \\ -i\,\theta_1^{(k)} & -i\,\theta_2^{(k)} & -i\,\theta_3^{(k)} & 0 \end{pmatrix} =$$

$$= \begin{pmatrix} 0 & \sigma_3^{(k)} & -\sigma_2^{(k)} & \frac{i}{c}\begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \\ -\sigma_3^{(k)} & 0 & \sigma_1^{(k)} & -\frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \\ \sigma_2^{(k)} & -\sigma_1^{(k)} & 0 & \frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \\ -\frac{i}{c}\begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} & \frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} & -\frac{i}{c}\begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} & 0 \end{pmatrix}.$$

Recalling the summation in repeated Roman indices we present an explicit form of the spin equations obtained in [8]:

$$\frac{d\sigma_{12}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left[\left(\sum_{n=1,n\neq k}^{3} F_{1\,m}^{(kn)} + F_{1m}^{(k)rad}\right)\sigma_{m2}^{(k)} - \sigma_{1\,m}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{m2}^{(kn)} + F_{m2}^{(k)rad}\right)\right];$$

$$\frac{d\sigma_{13}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left[\left(\sum_{n=1,n\neq k}^{3} F_{1\,m}^{(kn)} + F_{1\,m}^{(k)rad}\right)\sigma_{m3}^{(k)} - \sigma_{1m}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{m3}^{(kn)} + F_{m3}^{(k)rad}\right)\right];$$

$$\frac{d\sigma_{23}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left[\left(\sum_{n=1,n\neq k}^{3} F_{2\,m}^{(kn)} + F_{2\,m}^{(k)rad}\right)\sigma_{m3}^{(k)} - \sigma_{2m}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{m3}^{(kn)} + F_{m3}^{(k)rad}\right)\right];$$

$$\frac{d\sigma_{14}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left[\left(\sum_{n=1,n\neq k}^{3} F_{1\,m}^{(kn)} + F_{1\,m}^{(k)rad}\right)\sigma_{m4}^{(k)} - \sigma_{1m}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{m4}^{(kn)} + F_{m4}^{(k)rad}\right)\right];$$

$$\frac{d\sigma_{24}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left[\left(\sum_{n=1,n\neq k}^{3} F_{2\,m}^{(kn)} + F_{2\,m}^{(k)rad}\right)\sigma_{m4}^{(k)} - \sigma_{2m}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{m4}^{(kn)} + F_{m4}^{(k)rad}\right)\right];$$

$$\frac{d\sigma_{34}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2}\left[\left(\sum_{n=1,n\neq k}^{3} F_{3\,m}^{(kn)} + F_{3\,m}^{(k)rad}\right)\sigma_{m4}^{(k)} - \sigma_{3m}^{(k)}\left(\sum_{n=1,n\neq k}^{3} F_{m4}^{(kn)} + F_{m4}^{(k)rad}\right)\right].$$

We have proved in [8] that the last three equations are consequence of the first three ones in a weak sense so that we consider just the first three equations for every particle ($k = 1, 2, 3$):

$$\frac{d\sigma_{12}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n\neq k}^{3} \left(F_{1\,m}^{(kn)} \sigma_{m2}^{(k)} - F_{m2}^{(kn)} \sigma_{1\,m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m2}^{(k)} - F_{m2}^{(k)rad} \sigma_{1\,m}^{(k)} \right); \tag{3}$$

$$\frac{d\sigma_{13}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n\neq k}^{3} \left(F_{1\,m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{1m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{1\,m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{1m}^{(k)} \right); \tag{4}$$

$$\frac{d\sigma_{23}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n\neq k}^{3} \left(F_{2\,m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{2m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{2\,m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{2m}^{(k)} \right). \tag{5}$$

In [8] we have presented (3) – (5) in a vector form:

$$\begin{aligned}
\frac{d\vec{\sigma}^{(k)}}{ds_k} = \sum_{n=1, n\neq k}^{3} \frac{e_k e_n}{m_k c^2} \Big[& \vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) + \tau_{kn} \vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \\
& - \vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + L_{kn} \vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \Big] + \\
+ \frac{{e_k}^2}{2m_k c^2} \Big[& \vec{P}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) + {\tau_k}^{ret} \vec{P}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \\
& - \vec{\xi}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) + L_{k,ret} \vec{\xi}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \Big] - \\
- \frac{{e_k}^2}{2m_k c^2} \Big[& \vec{P}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) + {\tau_k}^{adv} \vec{P}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \\
& - \vec{\xi}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) + L_{k,adv} \vec{\xi}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \Big].
\end{aligned}$$

Using the known relations from vector calculus we transform the last equations in the form:

$$\begin{aligned}
\frac{d\vec{\sigma}^{(k)}}{ds_k} = & \sum_{n=1, n\neq k}^{3} \frac{e_k e_n}{m_k c^2} \Big[\left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \vec{P}^{(kn)} - \left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \vec{\xi}^{(kn)} + \\
& + \left(\tau_{kn} \left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle + L_{kn} \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \right) \vec{\lambda}^{(k)} - \left(\tau_{kn} \left\langle \vec{P}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle + L_{kn} \left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle \right) \vec{\sigma}^{(k)} \Big] + \\
& + \frac{{e_k}^2}{2m_k c^2} \Big[\left\langle \vec{\xi}^{(k)ret}, \vec{\sigma}^{(k)} \right\rangle \vec{P}^{(k)ret} - \left\langle \vec{P}^{(k)ret}, \vec{\sigma}^{(k)} \right\rangle \vec{\xi}^{(k)ret} + \\
& \qquad + \left(\tau_k^{ret} \left\langle \vec{P}^{(k)ret}, \vec{\sigma}^{(k)} \right\rangle + L_{k,ret} \left\langle \vec{\xi}^{(k)ret}, \vec{\sigma}^{(k)} \right\rangle \right) \vec{\lambda}^{(k)} - \\
& \qquad - \left(\tau_k^{ret} \left\langle \vec{P}^{(k)ret}, \vec{\lambda}^{(k)} \right\rangle + L_{k,ret} \left\langle \vec{\xi}^{(k)ret}, \vec{\lambda}^{(k)} \right\rangle \right) \vec{\sigma}^{(k)} \Big] - \\
& - \frac{{e_k}^2}{2m_k c^2} \Big[\left\langle \vec{\xi}^{(k)adv}, \vec{\sigma}^{(k)} \right\rangle \vec{P}^{(k)adv} - \left\langle \vec{P}^{(k)adv}, \vec{\sigma}^{(k)} \right\rangle \vec{\xi}^{(k)adv} + \\
& \qquad + \left(\tau_k^{adv} \left\langle \vec{P}^{(k)adv}, \vec{\sigma}^{(k)} \right\rangle + L_{k,adv} \left\langle \vec{\xi}^{(k)adv}, \vec{\sigma}^{(k)} \right\rangle \right) \vec{\lambda}^{(k)} - \\
& \qquad - \left(\tau_k^{adv} \left\langle \vec{P}^{(k)adv}, \vec{\lambda}^{(k)} \right\rangle + L_{k,adv} \left\langle \vec{\xi}^{(k)adv}, \vec{\lambda}^{(k)} \right\rangle \right) \vec{\sigma}^{(k)} \Big].
\end{aligned} \tag{6}$$

We notice that (6) is a linear system for 9 unknown spin functions

$$\left(\sigma_1^{(1)}, \sigma_2^{(1)}, \sigma_3^{(1)}, \sigma_1^{(2)}, \sigma_2^{(2)}, \sigma_3^{(2)}, \sigma_1^{(3)}, \sigma_2^{(3)}, \sigma_3^{(3)} \right).$$

The coefficient before unknown functions are functions of velocities and trajectories of the moving particles. We have proved however an existence of a periodic solution of (1), namely $\left(u_1^{(1)}, u_2^{(1)}, u_3^{(1)}, u_1^{(2)}, u_2^{(2)}, u_3^{(2)}, u_1^{(3)}, u_2^{(3)}, u_3^{(3)}\right)$ (cf. [7]). Therefore, we can substitute these solutions into coefficient of (6). So we obtain a system with known coefficient. It remains to prove an existence of periodic solution of (6).

4. Operator Presentation of the Periodic Problem

We present the above vector equations in coordinate form under assumption $\tau_k^{ret} = \tau_k^{adv} = \tau$ and obtain a system of 9 equations for 9 unknown spin functions ${\sigma_\alpha}^{(k)}$ ($k = 1, 2, 3;\ \alpha = 1, 2, 3$):

$$\frac{d\sigma_\alpha^{(k)}}{\Delta_k dt} = \sum_{n=1, n\neq k}^{3} \frac{e_k e_n}{m_k c^2}\left[P_\alpha^{(kn)}\left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle - \xi_\alpha^{(kn)}\left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle + \right.$$
$$\left. + \left(\tau_{kn}\left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle + L_{kn}\left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle\right)\lambda_\alpha^{(k)} - \left(\tau_{kn}\left\langle \vec{P}^{(kn)}, \vec{\lambda}^{(k)}\right\rangle + L_{kn}\left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)}\right\rangle\right)\sigma_\alpha^{(k)}\right] +$$
$$+ \frac{{e_k}^2}{2m_k c^2}\left[{P_\alpha}^{(k)ret}\left\langle \vec{\xi}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle - {\xi_\alpha}^{(k)ret}\left\langle \vec{P}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle + \left(\tau\left\langle \vec{P}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle + L_{k,ret}\left\langle \vec{\xi}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle\right)\lambda_\alpha^{(k)} - \right.$$
$$- \left(\tau\left\langle \vec{P}^{(k)ret}, \vec{\lambda}^{(k)}\right\rangle + L_{k,ret}\left\langle \vec{\xi}^{(k)ret}, \vec{\lambda}^{(k)}\right\rangle\right)\sigma_\alpha^{(k)} - {P_\alpha}^{(k)adv}\left\langle \vec{\xi}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle + {\xi_\alpha}^{(k)adv}\left\langle \vec{P}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle -$$
$$\left. - \left(\tau\left\langle \vec{P}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle + L_{k,adv}\left\langle \vec{\xi}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle\right)\lambda_\alpha^{(k)} + \left(\tau\left\langle \vec{P}^{(k)adv}, \vec{\lambda}^{(k)}\right\rangle + L_{k,adv}\left\langle \vec{\xi}^{(k)adv}, \vec{\lambda}^{(k)}\right\rangle\right)\sigma_\alpha^{(k)}\right].$$

Introduce denotations for the above system

$$\frac{d{\sigma_\alpha}^{(k)}(t)}{dt} = F_\alpha^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) \tag{7}$$

and separate the right-hand sides into Lorentz part $F_{\alpha,L}^{(k)}$ and radiation part $F_{\alpha,rad}^{(k)}$:

$$F_\alpha^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) = F_{\alpha,L}^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) + F_{\alpha,rad}^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}),$$

where

$$F_{\alpha,L}^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) \equiv \sum_{n=1, n\neq k}^{3} \frac{e_k e_n \Delta_k}{m_k c^2}\left[P_\alpha^{(kn)}\left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle - \xi_\alpha^{(kn)}\left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle + \right.$$
$$\left. + \left(\tau_{kn}\left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle + L_{kn}\left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)}\right\rangle\right)\lambda_\alpha^{(k)} - \left(\tau_{kn}\left\langle \vec{P}^{(kn)}, \vec{\lambda}^{(k)}\right\rangle + L_{kn}\left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)}\right\rangle\right)\sigma_\alpha^{(k)}\right]; \tag{8}$$

$$F_{\alpha,rad}^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) \equiv \frac{e_k^2 \Delta_k}{2m_k c^2}\left[P_\alpha^{(k)ret}\left\langle \vec{\xi}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle - \xi_\alpha^{(k)ret}\left\langle \vec{P}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle + \right.$$
$$+ \left(\tau\left\langle \vec{P}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle + L_{k,ret}\left\langle \vec{\xi}^{(k)ret}, \vec{\sigma}^{(k)}\right\rangle\right)\lambda_\alpha^{(k)} - \left(\tau\left\langle \vec{P}^{(k)ret}, \vec{\lambda}^{(k)}\right\rangle + L_{k,ret}\left\langle \vec{\xi}^{(k)ret}, \vec{\lambda}^{(k)}\right\rangle\right)\sigma_\alpha^{(k)} -$$
$$- {P_\alpha}^{(k)adv}\left\langle \vec{\xi}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle + {\xi_\alpha}^{(k)adv}\left\langle \vec{P}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle -$$
$$- \left(\tau\left\langle \vec{P}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle + L_{k,adv}\left\langle \vec{\xi}^{(k)adv}, \vec{\sigma}^{(k)}\right\rangle\right)\lambda_\alpha^{(k)} + \left(\tau\left\langle \vec{P}^{(k)adv}, \vec{\lambda}^{(k)}\right\rangle + L_{k,adv}\left\langle \vec{\xi}^{(k)adv}, \vec{\lambda}^{(k)}\right\rangle\right)\sigma_\alpha^{(k)}. \tag{9}$$

Remark 4.1. It is easy to see that the above functions are linear ones with respect to the spin functions.

The velocity functions $u_\alpha^{(k)}(t), (k = 1, 2, 3;\ \alpha = 1, 2, 3)$ appearing in $\vec{F}_L^{(k)}(t, \vec{\sigma}^{(k)})$ and $\vec{F}_{rad}^{(k)}(t, \vec{\sigma}^{(k)})$ are solution of (1) belonging to the set:

$$M = \left\{ u \in C_T^\infty[0, \infty) : \left|u^{(n)}(t)\right| \le U_0 \omega^n e^{\mu(t-pT)},\ \int_{pT}^{(p+1)T} u(t)dt = 0\ \ (n = 0, 1, 2, 3, ...);\ t \in [pT, (p+1)T] \right\},$$

where $p = 0,1,2,...,\mu,\omega,U_0,T > 0, U_0 e^{\mu T} \le \bar{c} < c; \omega = \dfrac{2\pi}{T} < \mu;$ $C_T^{\infty}[0,\infty)$ is the space of all infinite differentiable T -periodic functions with a saturated family of pseudo-metrics

$$\rho_{(p,m)}\left(u_\alpha^{(k)},\bar{u}_\alpha^{(k)}\right) = \left\{e^{-\mu(t-pT)}\omega^{-m}\left|\frac{d^m u_\alpha^{(k)}(t)}{dt^m} - \frac{d^m \bar{u}_\alpha^{(k)}(t)}{dt^m}\right| : t \in [pT,(p+1)T]\right\},$$

$(p = 0,1,2,...\ ;\ m = 0,1,2,...)$. It follows that trajectories $x_\alpha^{(k)}(t), (k = 1,2,3;\ \alpha = 1,2,3)$ are T-periodic functions, too.

Introduce the set of spin continuous T-periodic functions

$$SP = \left\{\sigma \in C_T[0,\infty) : |\sigma(t)| \le S_0 e^{\mu(t-pT)}, t \in [pT,(p+1)T\,]\right\}$$

and the family of pseudo-metrics

$$\rho_p(\sigma,\bar{\sigma}) = \text{ess sup}\left\{e^{-\mu(t-pT)}\left|\sigma(t)-\bar{\sigma}(t)\right| : t \in [pT,(p+1)T]\right\}, (p = 0,1,2,...).$$

We consider SP with a family of pseudo-metrics and in this way obtain a uniform space with a countable family of pseudo-metrics.

The problem of an existence of T-periodic solution of (7) is equivalent to the existence of T-periodic solution of the integral equations (cf. [18]):

$$\sigma_\alpha^{(k)}(t) = \sigma_\alpha^{(k)}((p+1)T) + \int\limits_{pT}^{t} F_\alpha^{(k)}(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})(s)ds\,,\ t \in [pT,(p+1)T]\ (p = 0,1,2,...),\ t \in [pT,(p+1)T],$$

$(k = 1,2,3;\ \alpha = 1,2,3)$, where $F_\alpha^{(k)}(s,\sigma_1,\sigma_2,\sigma_3)$ is T-periodic with respect to the first variable.

5. Explicit Form and Estimate of the Lorentz Part

First we give an explicit form of the Lorentz part of the spin equations. Since

$$\xi^{(kn)} = \left(\xi_1^{(kn)},\xi_2^{(kn)},\xi_3^{(kn)},\xi_4^{(kn)}\right) =$$
$$= \left(x_1^{(k)}(t)-x_1^{(n)}(t-\tau_{kn}), x_2^{(k)}(t)-x_2^{(n)}(t-\tau_{kn}), x_3^{(k)}(t)-x_3^{(n)}(t-\tau_{kn}), ic\tau_{kn}(t)\right);$$

$$\vec{u}^{(k)} = \vec{u}^{(k)}(t)\,;\ \vec{u}^{(n)} = \vec{u}^{(n)}(t-\tau_{kn});$$

$$\left(\vec{\lambda}^{(k)},\vec{\lambda}^{(k)}\right) = \frac{\left\langle \vec{u}^{(k)},\vec{u}^{(k)}\right\rangle}{\Delta_k{}^2};\ \frac{d\vec{\lambda}^{(n)}}{ds_n} = \frac{D_{kn}}{\Delta_n^2}\left(\dot{\vec{u}}^{(n)} + \frac{\vec{u}^{(n)}}{\Delta_{kn}^2}\left\langle \vec{u}^{(n)},\ \dot{\vec{u}}^{(n)}\right\rangle\right);\ \frac{d\lambda_4^{(n)}}{ds_n} = \frac{icD_{kn}}{\Delta_{kn}^4}\left\langle \vec{u}^{(n)},\ \dot{\vec{u}}^{(n)}\right\rangle;$$

$$M_{kn} = 1 + \left\langle \xi^{(kn)},\frac{d\lambda^{(n)}}{ds_n}\right\rangle_4 = 1 + \frac{D_{kn}}{\Delta_{kn}^2}\left(\left\langle \vec{\xi}^{(kn)},\dot{\vec{u}}^{(n)}\right\rangle + \frac{\left(\left\langle \vec{\xi}^{(kn)},\vec{u}^{(n)}\right\rangle - \tau_{kn}c^2\right)\left\langle \vec{u}^{(n)},\ \dot{\vec{u}}^{(n)}\right\rangle}{\Delta_{kn}^2}\right);$$

$$\left\langle \lambda^{(k)},\lambda^{(n)}\right\rangle_4 = \frac{\left\langle \vec{u}^{(k)},\vec{u}^{(n)}\right\rangle - c^2}{\Delta_k\Delta_{kn}};\ \left\langle \lambda^{(k)},\xi^{(kn)}\right\rangle_4 = \frac{\left\langle \vec{u}^{(k)},\vec{\xi}^{(kn)}\right\rangle - c^2\tau_{kn}}{\Delta_k};$$

$$\left\langle \lambda^{(n)},\xi^{(kn)}\right\rangle_4 = \frac{\left\langle \vec{u}^{(n)},\vec{\xi}^{(kn)}\right\rangle - c^2\tau_{kn}}{\Delta_{kn}};\ L_{kn} = \frac{M_{kn}\Delta_{kn}{}^2}{\left(\left\langle \vec{u}^{(n)},\vec{\xi}^{(kn)}\right\rangle - c^2\tau_{kn}\right)^3} - \frac{D_{kn}\left\langle \vec{u}^{(n)},\ \dot{\vec{u}}^{(n)}\right\rangle}{\left(\left\langle \vec{u}^{(n)},\vec{\xi}^{(kn)}\right\rangle - c^2\tau_{kn}\right)^2\Delta_{kn}^2};$$

$$\left\langle \xi^{(kn)}, \frac{d\lambda^{(n)}}{ds_n} \right\rangle_4 = D_{kn} \left(\frac{\left\langle \vec{\xi}^{(kn)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}^2} + \frac{\left\langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \right\rangle - c^2\tau_{kn}}{\Delta_{kn}^4} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \right);$$

$$\left\langle \lambda^{(k)}, \frac{d\lambda^{(n)}}{ds_n} \right\rangle_4 = \frac{D_{kn}}{\Delta_k} \left(\frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}^2} + \frac{\left\langle \vec{u}^{(k)}, \vec{u}^{(n)} \right\rangle - c^2\tau_{kn}}{\Delta_{kn}^4} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \right);$$

$$P_\gamma^{(kn)} = -\frac{{\Delta_{kn}}^2 M_{kn} u_\gamma^{(n)}}{\left(\left\langle \vec{u}^{(n)}(t-\tau_{kn}), \vec{\xi}^{(kn)} \right\rangle - c^2\tau_{kn} \right)^3} + D_{kn} \frac{\Delta_{kn}^2 \dot{u}_\gamma{}^{(n)} + u_\gamma{}^{(n)} \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}^2 \left(\left\langle \vec{u}^{(n)}(t-\tau_{kn}), \vec{\xi}^{(kn)} \right\rangle - c^2\tau_{kn} \right)^2};$$

$$\left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle = \frac{{\Delta_{kn}}^2 M_{kn} \left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle}{\left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^3} + D_{kn} \frac{\Delta_{kn}^2 \left\langle \dot{\vec{u}}^{(n)}, \vec{\sigma}^{(k)} \right\rangle + \left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}^2 \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^2};$$

$$\left\langle \vec{P}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle = \frac{{\Delta_{kn}}^2 M_{kn} \left\langle \vec{u}^{(n)}, \vec{u}^{(k)} \right\rangle}{\Delta_k \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^3} + D_{kn} \frac{\Delta_{kn}^2 \left\langle \dot{\vec{u}}^{(n)}, \vec{u}^{(k)} \right\rangle + \left\langle \vec{u}^{(n)}, \vec{u}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_k \Delta_{kn}^2 \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^2}$$

we obtain

$$F_{\alpha,L}^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) =$$

$$= \sum_{n=1, n\neq k}^{3} \frac{e_k e_n \Delta_k}{m_k c^2} \Big[P_\alpha^{(kn)} \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle - \xi_\alpha^{(kn)} \left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle + \left(\tau_{kn} \left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle + L_{kn} \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \right) \lambda_\alpha^{(k)} -$$

$$- \left(\tau_{kn} \left\langle \vec{P}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle + L_{kn} \left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle \right) \sigma_\alpha^{(k)} \Big] =$$

$$= \sum_{n=1, n\neq k}^{3} \frac{e_k e_n \Delta_k}{m_k c^2} \left[\frac{\left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle u_\alpha^{(n)}}{\left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^3} {\Delta_{kn}}^2 M_{kn} + \right.$$

$$+ D_{kn} \frac{\Delta_{kn}^2 \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \dot{u}_\alpha{}^{(n)} + \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle u_\alpha{}^{(n)}}{\Delta_{kn}^2 \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^2} - \frac{\left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \left(\xi_\alpha^{(kn)} - \tau_{kn} \lambda_\alpha^{(k)} \right)}{\left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^3} {\Delta_{kn}}^2 M_{kn} -$$

$$- D_{kn} \frac{\Delta_{kn}^2 \left\langle \dot{\vec{u}}^{(n)}, \vec{\sigma}^{(k)} \right\rangle + \left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle}{\Delta_{kn}^2 \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^2} \left(\xi_\alpha^{(kn)} - \tau_{kn} \lambda_\alpha^{(k)} \right) +$$

$$+ \frac{\left(- \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \lambda_\alpha^{(k)} + \left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle \sigma_\alpha^{(k)} \right)}{\left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^3} {\Delta_{kn}}^2 M_{kn} -$$

$$- D_{kn} \frac{\left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \left(\left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \lambda_\alpha^{(k)} - \left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle \sigma_\alpha^{(k)} \right)}{\Delta_{kn}^2 \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^2} - \frac{\tau_{kn} \left\langle \vec{u}^{(n)}, \vec{u}^{(k)} \right\rangle \sigma_\alpha^{(k)}}{\Delta_k \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^3} {\Delta_{kn}}^2 M_{kn} -$$

$$\left. - D_{kn} \frac{\Delta_{kn}^2 \left\langle \dot{\vec{u}}^{(n)}, \vec{u}^{(k)} \right\rangle \tau_{kn} \sigma_\alpha^{(k)} + \left\langle \vec{u}^{(n)}, \vec{u}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \tau_{kn} \sigma_\alpha^{(k)}}{\Delta_k \Delta_{kn}^2 \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle \right)^2} \right] =$$

$$= \sum_{n=1,n\neq k}^{3} \frac{e_k e_n \Delta_k}{m_k c^2} \Bigg\{ \frac{{\Delta_{kn}}^2 M_{kn}}{\Delta_k \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle\right)^3} \Big[- \Delta_k \left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \xi_\alpha^{(kn)} + \Delta_k \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle u_\alpha^{(n)} +$$

$$+ \left(\left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \tau_{kn} - \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \right) u_\alpha^{(k)} + \left(\Delta_k \left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle - \tau_{kn} \left\langle \vec{u}^{(n)}, \vec{u}^{(k)} \right\rangle \right) \sigma_\alpha^{(k)} \Big] +$$

$$+ \frac{D_{kn}}{\Delta_{kn}^2 \left(c^2\tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\xi}^{(kn)} \right\rangle\right)^2} \Big[\Delta_{kn}^2 \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \dot{u}_\alpha{}^{(n)} +$$

$$+ \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle u_\alpha{}^{(n)} + \left(\Delta_{kn}^2 \left\langle \dot{\vec{u}}^{(n)}, \vec{\sigma}^{(k)} \right\rangle + \left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \right) \xi_\alpha^{(kn)} +$$

$$+ \Big(-\Delta_{kn}^2 \left\langle \dot{\vec{u}}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \tau_{kn} - \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \left\langle \vec{\xi}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \Big) \lambda_\alpha^{(k)} +$$

$$+ \left(\left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \left\langle \vec{\xi}^{(kn)}, \vec{\lambda}^{(k)} \right\rangle - \Delta_{kn}^2 \left\langle \dot{\vec{u}}^{(n)}, \vec{u}^{(k)} \right\rangle \tau_{kn} - \left\langle \vec{u}^{(n)}, \vec{u}^{(k)} \right\rangle \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \tau_{kn} \right) \sigma_\alpha^{(k)} \Big] \Bigg\}.$$

Now we are able to obtain an upper bound of the Lorentz part of the right-hand sides of (7) for $t \in [pT,(p+1)T]$. Indeed, in view of $\beta = \bar{c}/c < 1$,

$$|D_{kn}| = \left| \frac{c^2\tau_{kn} - \left\langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \right\rangle}{c^2\tau_{kn} - \left\langle \vec{\xi}^{(kn)}, \vec{u}^{(k)} \right\rangle} \right| \le \frac{c^2\tau_{kn} + c\bar{c}\tau_{kn}}{c^2\tau_{kn} - c\bar{c}\tau_{kn}} = \frac{1+\beta}{1-\beta}; \quad \left|\lambda_\alpha^{(k)}\right| = \frac{\left|u_\alpha^{(k)}\right|}{\Delta} \le \frac{\bar{c}}{c(1-\beta^2)^{1/2}} = \frac{\beta}{(1-\beta^2)^{1/2}};$$

$$|M_{kn}| \le 1 + |D_{kn}| \, \frac{c^2 \left| \left\langle \vec{\xi}^{(kn)}, \dot{\vec{u}}^{(n)} \right\rangle \right| + \left| c^2\tau_{kn} - \left\langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \right\rangle \right| \left| \left\langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \right\rangle \right|}{\Delta_{kn}^4} \le 1 + \frac{6\tau_{kn}\omega U_0 e^{\mu T}}{c(1-\beta)^3};$$

$$\left| F_{\alpha,L}^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) \right| \le \sum_{n=1,n\neq k}^{3} \frac{|e_k e_n|}{m_k c^2} \Bigg[\frac{3c^3 + 3\,c^2}{c^6\,(1-\beta)^3\,{\tau_{kn}}^3} \tau_{kn} c^2 \left(1 + \frac{6\tau_{kn}\omega U_0 e^{\mu T}}{c(1-\beta)^3} \right) \left| \vec{\sigma}^{(k)} \right| +$$

$$+ \frac{2D_{kn} e^{\mu T} \left| \vec{\sigma}^{(k)} \right| \tau_{kn}}{(1-\beta^2) c^6 \,(1-\beta)^2\, {\tau_{kn}}^2} \Big(c^3\omega U_0 + c^3\omega U_0 + c^3\omega U_0 + c^3\omega U_0 + c^3\omega U_0 + c^3\omega U_0 +$$

$$+ c^3\omega U_0 + c^2\omega U_0 c + c^3\omega U_0 + c^3\omega U_0 \Big) \Bigg] \le$$

$$\le \sum_{n=1,n\neq k}^{3} \frac{|e_k e_n|}{m_k} e^{\mu T} \left[\frac{6}{c^3\,(1-\beta)^3\,\tau_{kn}^2} + \frac{36\omega U_0}{c^4(1-\beta)^6\tau_{kn}} + \frac{20\omega U_0}{c^5(1-\beta)^6\tau_{kn}} \right] \left| \vec{\sigma}^{(k)} \right| \approx$$

$$\approx \sum_{n=1,n\neq k}^{3} \frac{|e_k e_n|}{m_k} e^{\mu T} \left[\frac{6}{c^3\,(1-\beta)^3\,{\tau_{kn}}^2} + \frac{36\omega U_0}{c^4(1-\beta)^6\tau_{kn}} \right] \left| \vec{\sigma}^{(k)} \right|.$$

6. Explicit Form and Estimate of the Radiation Part

Here we transform the radiation part of the spin equation using some reasoning from [7]. Indeed, we recall assumption $\tau_k^{ret} = {\tau_k}^{adv} = \tau \approx 10^{-24}$ and then:

$$\xi_\gamma^{(k)adv} = x_\gamma^{(k)}(t+\tau) - x_\gamma^{(k)}(t) \approx \tau\, u_\gamma^{(k)}(t); \; \xi_\gamma^{(k)ret} = x_\gamma^{(k)}(t) - x_\gamma^{(k)}(t-\tau) \approx \tau\, u_\gamma^{(k)}(t);$$

$$u_\gamma^{(k)}(t+\tau) \approx u_\gamma^{(k)}(t); \; u_\gamma^{(k)}(t-\tau) \approx u_\gamma^{(k)}(t); \; u_\gamma^{(k)}(t) u_\gamma^{(k)}(t+\tau) \approx \left(u_\gamma^{(k)}(t)\right)^2; \; u_\gamma^{(k)}(t) u_\gamma^{(k)}(t-\tau) \approx \left(u_\gamma^{(k)}(t)\right)^2;$$

$$\left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv} \right\rangle = \left\langle \vec{u}^{(k)}, \vec{u}^{(k)}(t+\tau) \right\rangle \approx \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \right\rangle; \; \left\langle u^{(k)}, u^{(k)ret} \right\rangle \approx \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \right\rangle;$$

$$\left\langle \vec{u}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle \approx \left\langle \vec{u}^{(k)}, \vec{u}^{(k)adv} \right\rangle; \; \left\langle \vec{u}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle \approx \left\langle \vec{u}^{(k)}, \vec{u}^{(k)ret} \right\rangle;$$

$$c^2\tau_k^{ret} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle \approx \tau \left(c^2 - \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \right\rangle \right) = \tau\, {\Delta_k}^2;$$

$$c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle \approx \tau \left(c^2 - \left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \right\rangle \right) = \tau\, {\Delta_k}^2;$$

$$D_{(k),ret} = \frac{c^2\tau_k^{ret} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)ret} \right\rangle}{c^2\tau_k^{ret} - \left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)} \right\rangle} \approx \frac{c^2\tau - \tau \left\langle \vec{u}^{(k)}, \vec{u}^{(k)} \right\rangle}{c^2\tau - \tau \left\langle \vec{u}^{(k)}, \vec{u}^{(k)} \right\rangle} = 1;\; D_{(k),adv} = \frac{c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)adv} \right\rangle}{c^2\tau_k^{adv} - \left\langle \vec{\xi}^{(k)adv}, \vec{u}^{(k)} \right\rangle} \approx 1;$$

$$M_{(k)ret} \approx 1 + \frac{\tau\left(\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)} \right\rangle - \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)} \right\rangle \right)}{\Delta_k^2} = 1;\; M_{(k)adv} \approx 1 + \frac{\tau\left(\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)} \right\rangle - \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)} \right\rangle \right)}{\Delta_k^2} = 1;$$

$$\left\langle \xi^{(k)ret}, \lambda^{(k)ret} \right\rangle_4 = \frac{\left\langle \vec{u}^{(k)}(t-\tau_k^{ret}), \vec{\xi}^{(k)ret} \right\rangle - c^2\tau_k^{ret}}{\Delta_{(k)ret}} \approx \tau \frac{\left\langle \vec{u}^{(k)}(t), \vec{u}^{(k)} \right\rangle - c^2}{\Delta_k} = -\tau\Delta_k;$$

$$\left\langle \xi^{(k)ret}, \frac{d\lambda^{(k)ret}}{ds_{ret}} \right\rangle_4 =$$

$$= D_k^{ret} \left(\frac{\left\langle \vec{\xi}^{(k)ret}, \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle}{\Delta_{(k)ret}^2} + \frac{\left\langle \vec{\xi}^{(k)ret}, \vec{u}^{(k)}(t-\tau_k^{ret}) \right\rangle - c^2\tau_k^{ret}}{\Delta_{(k)ret}^4} \left\langle \vec{u}^{(k)}(t-\tau_k^{ret}), \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle \right) \approx$$

$$\approx \tau \frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle - \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle}{\Delta_k^2} = 0;$$

$$\frac{d\lambda_\alpha^{(k)ret}}{ds_k} = D_k^{ret} \left(\frac{\dot{u}_\alpha^{(k)}(t-\tau_k^{ret})}{\Delta_{(k)ret}^2} + \frac{u_\alpha^{(k)}(t-\tau_k^{ret}) \left\langle \vec{u}^{(k)}(t-\tau_k^{ret}), \dot{\vec{u}}^{(k)}(t-\tau_k^{ret}) \right\rangle}{\Delta_{(k)ret}^4} \right) \approx$$

$$\approx \frac{\dot{u}_\alpha^{(k)}(t-\tau)}{\Delta_k^2} + \frac{u_\alpha^{(k)} \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle}{\Delta_k^4};$$

$$P_\alpha^{(k)ret} = u_\alpha^{(k)} \frac{{\Delta_k}^2}{\tau^3 \left(c^2 - \left\langle \vec{u}^{(k)}, \vec{u}^{(k)} \right\rangle \right)^3} \left[1 + \tau \left(\frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle}{\Delta_k^2} - \frac{\left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle}{\Delta_k^2} \right) \right] +$$

$$+ \frac{{\Delta_k}^2}{\tau^2 \left(c^2 - \left\langle \vec{u}^{(k)}, \vec{u}^{(k)} \right\rangle \right)^2} \left(\frac{\dot{u}_\alpha^{(k)}(t-\tau)}{\Delta_k^2} + \frac{u_\alpha^{(k)}(t) \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle}{\Delta_k^4} \right) \approx$$

$$\approx \frac{u_\alpha^{(k)}}{\tau^3 {\Delta_k}^4} + \frac{1}{\tau^2 {\Delta_k}^2} \left(\frac{\dot{u}_\alpha^{(k)}(t-\tau)}{\Delta_k^2} + \frac{u_\alpha^{(k)} \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle}{\Delta_k^4} \right);$$

$$P_\alpha^{(k)adv} \approx \frac{u_\alpha^{(k)}}{\tau^3 \Delta_k^4} + \frac{1}{\tau^2 \Delta_k^2} \left(\frac{\dot{u}_\alpha^{(k)}(t+\tau)}{\Delta_k^2} + \frac{u_\alpha^{(k)} \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t+\tau) \right\rangle}{\Delta_k^4} \right);$$

$$\frac{P_\alpha^{(k)adv} - P_\alpha^{(k)ret}}{2} = \frac{u_\alpha^{(k)}}{2\tau^3 \Delta_k^4} + \frac{1}{2\tau^2 \Delta_k^2} \left(\frac{\dot{u}_\alpha^{(k)}(t+\tau)}{\Delta_k^2} + \frac{u_\alpha^{(k)} \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t+\tau) \right\rangle}{\Delta_k^4} \right) -$$

$$- \frac{u_\alpha^{(k)}}{2\tau^3 \Delta_k^4} - \frac{1}{2\tau^2 \Delta_k^2} \left(\frac{\dot{u}_\alpha^{(k)}(t-\tau)}{\Delta_k^2} + \frac{u_\alpha^{(k)} \left\langle \vec{u}^{(k)}, \dot{\vec{u}}^{(k)}(t-\tau) \right\rangle}{\Delta_k^4} \right) \approx \frac{1}{\tau} \left(\frac{\ddot{u}_\alpha^{(k)}}{{\Delta_k}^4} + \frac{u_\alpha^{(k)}}{\Delta_k^6} \left\langle \vec{u}^{(k)}, \ddot{\vec{u}}^{(k)} \right\rangle \right);$$

$$L_{(k)ret} = \frac{M_{(k)ret}}{\Delta_{(k)ret}\left\langle \lambda^{(k)ret}, \xi^{(k)ret}\right\rangle_4^3} - \frac{D_{(k)ret}\left\langle \vec{u}^{(k)ret},\, \dot{\vec{u}}^{(k)ret}\right\rangle}{\left\langle \lambda^{(k)ret}, \xi^{(k)ret}\right\rangle_4{}^2\Delta^4_{(k)ret}} \approx \frac{1}{-\tau^3\,\Delta_k^4} - \frac{\left\langle \vec{u}^{(k)},\, \dot{\vec{u}}^{(k)}(t-\tau)\right\rangle}{\tau^2\Delta_k^6};$$

$$L_{(k)adv} = \frac{M_{(k)adv}}{\Delta_{(k)adv}\left\langle \lambda^{(k)adv}, \xi^{(k)adv}\right\rangle_4^3} - \frac{D_{(k)adv}\left\langle \vec{u}^{(k)adv},\, \dot{\vec{u}}^{(k)adv}\right\rangle}{\left\langle \lambda^{(k)adv}, \xi^{(k)asdv}\right\rangle_4{}^2\Delta^4_{(k)adv}} \approx \frac{1}{-\tau^3\,\Delta_k^4} - \frac{\left\langle \vec{u}^{(k)},\, \dot{\vec{u}}^{(k)}(t+\tau)\right\rangle}{\tau^2\Delta_k^6};$$

$$\frac{L_{(k)adv}-L_{(k)ret}}{2} = -\frac{\left\langle \vec{u}^{(k)},\, \dot{\vec{u}}^{(k)}(t+\tau)\right\rangle}{2\tau^2\Delta_k^6} + \frac{\left\langle \vec{u}^{(k)},\, \dot{\vec{u}}^{(k)}(t-\tau)\right\rangle}{2\tau^2\Delta_k^6} \approx -\frac{\left\langle \vec{u}^{(k)},\, \ddot{\vec{u}}^{(k)}\right\rangle}{\tau\Delta_k^6}.$$

Let us present the radiation part (9) in the form

$$F^{(k)}_{\alpha,rad}(t,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)}) =$$

$$= \frac{{e_k}^2\Delta_k}{m_k c^2}\Bigg[-\frac{1}{2}\sum_{\gamma=1}^{3}\left({P_\alpha}^{(k)adv}\xi_\gamma^{(k)adv} - {P_\alpha}^{(k)ret}\xi_\gamma^{(k)ret}\right)\sigma_\gamma^{(k)} + \frac{1}{2}\sum_{\gamma=1}^{3}\left({\xi_\alpha}^{(k)adv}P_\gamma^{(k)adv} - {\xi_\alpha}^{(k)ret}P_\gamma^{(k)ret}\right)\sigma_\gamma^{(k)} -$$

$$-\frac{1}{2}\lambda_\alpha^{(k)}\tau\sum_{\gamma=1}^{3}\left(P_\gamma^{(k)adv} - P_\gamma^{(k)ret}\right)\sigma_\gamma^{(k)} - \frac{1}{2}\lambda_\alpha^{(k)}\sum_{\gamma=1}^{3}\left(L_{(k)adv}\xi_\gamma^{(k)adv} - L_{(k)ret}\xi_\gamma^{(k)ret}\right)\sigma_\gamma^{(k)} +$$

$$+\frac{1}{2}\sigma_\alpha^{(k)}\tau\sum_{\gamma=1}^{3}\left({P_\gamma}^{(k)adv} - P_\gamma^{(k)ret}\right)\lambda_\gamma^{(k)} + \frac{1}{2}\sigma_\alpha^{(k)}\sum_{\gamma=1}^{3}\left(L_{(k)adv}\xi_\gamma^{(k)adv} - L_{(k)ret}\xi_\gamma^{(k)ret}\right)\lambda_\gamma^{(k)}\Bigg] \approx$$

$$\approx \frac{{e_k}^2\Delta_k}{m_k c^2}\Bigg[-\tau\frac{{P_\alpha}^{(k)adv} - {P_\alpha}^{(k)ret}}{2}\left\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\right\rangle + \tau{u_\alpha}^{(k)}\sum_{\gamma=1}^{3}\frac{P_\gamma^{(k)adv} - P_\gamma^{(k)ret}}{2}\sigma_\gamma^{(k)} -$$

$$-\frac{u_\alpha^{(k)}}{\Delta_k}\tau\sum_{\gamma=1}^{3}\frac{P_\gamma^{(k)adv} - P_\gamma^{(k)ret}}{2}\sigma_\gamma^{(k)} - \tau\frac{L_{(k)adv}-L_{(k)ret}}{2}\frac{u_\alpha^{(k)}}{\Delta_k}\left\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\right\rangle +$$

$$+\sigma_\alpha^{(k)}\tau\sum_{\gamma=1}^{3}\frac{{P_\gamma}^{(k)adv} - P_\gamma^{(k)ret}}{2}\frac{u_\gamma^{(k)}}{\Delta_k} + \tau\frac{L_{(k)adv}-L_{(k)ret}}{2}\sigma_\alpha^{(k)}\frac{\langle \vec{u}^{(k)}, \vec{u}^{(k)}\rangle}{\Delta_k}\Bigg] =$$

$$= \frac{{e_k}^2}{m_k c^2}\Bigg[-\frac{\Delta_k\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\rangle}{{\Delta_k}^4}\ddot{u}_\alpha^{(k)} - \frac{\Delta_k\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\rangle\left\langle \vec{u}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{\Delta_k^6}u_\alpha^{(k)} +$$

$$+\frac{\Delta_k\left\langle \vec{\sigma}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{{\Delta_k}^4}{u_\alpha}^{(k)} + \frac{\Delta_k\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\rangle\left\langle \vec{u}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{\Delta_k^6}{u_\alpha}^{(k)} -$$

$$-\left(\frac{\left\langle \vec{\sigma}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{{\Delta_k}^4} + \frac{\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\rangle\left\langle \vec{u}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{\Delta_k^6}\right)u_\alpha^{(k)} + \frac{\left\langle \vec{u}^{(k)},\, \ddot{\vec{u}}^{(k)}\right\rangle\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\rangle}{{\Delta_k}^6}u_\alpha^{(k)} +$$

$$+\left(\frac{\left\langle \vec{u}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{{\Delta_k}^4} + \frac{\langle \vec{u}^{(k)}, \vec{u}^{(k)}\rangle\left\langle \vec{u}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{\Delta_k^6} - \frac{\left\langle \vec{u}^{(k)},\, \ddot{\vec{u}}^{(k)}\right\rangle\langle \vec{u}^{(k)}, \vec{u}^{(k)}\rangle}{{\Delta_k}^6}\right)\sigma_\alpha^{(k)}\Bigg] =$$

$$= \frac{{e_k}^2}{m_k c^2}\left(-\frac{\Delta_k\langle \vec{u}^{(k)}, \vec{\sigma}^{(k)}\rangle}{\Delta_k^4}\ddot{u}_\alpha^{(k)} + \frac{\Delta_k\left\langle \vec{\sigma}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle - \left\langle \vec{\sigma}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{\Delta_k^4}{u_\alpha}^{(k)} + \frac{\left\langle \vec{u}^{(k)}, \ddot{\vec{u}}^{(k)}\right\rangle}{{\Delta_k}^4}\sigma_\alpha^{(k)}\right).$$

To obtain an upper bound we notice that

$$\left|F_{\alpha,rad}^{(k)}(t,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})\right| \leq$$

$$\leq \frac{{e_k}^2}{m_k c^2}\left(\frac{\left|\left\langle \vec{u}^{(k)},\vec{\sigma}^{(k)}\right\rangle\right|}{{\Delta_k}^3}\left|\ddot{u}_\alpha^{(k)}\right| + \frac{\Delta_k+1}{{\Delta_k}^4}\left|\left\langle \vec{\sigma}^{(k)},\ddot{\vec{u}}^{(k)}\right\rangle\right|\left|{u_\alpha}^{(k)}\right| + \frac{\left|\left\langle \vec{u}^{(k)},\ddot{\vec{u}}^{(k)}\right\rangle\right|}{{\Delta_k}^4}\left|\sigma_\alpha^{(k)}\right|\right) \leq$$

$$\leq \frac{{e_k}^2}{m_k c^2}\omega^2 U_0 e^{\mu T}\frac{c\Delta_k+2c^2}{{\Delta_k}^4}\left|\vec{\sigma}^{(k)}\right| \leq \frac{{e_k}^2}{m_k c^2}\omega^2 U_0 e^{\mu T}\frac{3c^2}{c^4(1-\beta^2)^2}\left|\vec{\sigma}^{(k)}\right| \leq$$

$$\leq \frac{{e_k}^2}{m_k}\frac{3\omega^2 U_0 e^{\mu T}}{c^4(1-\beta^2)^2}\sqrt{\left(\sigma_1^{(k)}\right)^2+\left(\sigma_2^{(k)}\right)^2+\left(\sigma_3^{(k)}\right)^2}.$$

7. Existence Theorem for Spin Equations

In what follows we prove an existence theorem for T-periodic solution of the spin equations. **Main Theorem.** Let the following conditions be fulfilled:

$$\frac{\left|\sigma_{\alpha 0}^{(k)}\right|}{S_0} + \frac{\sqrt{3}}{\mu}\sum_{n=1,n\neq k}^{3}\frac{|e_k e_n|}{m_k}e^{\mu T}\left(\frac{24c^2}{c^3(1-\beta)^3{r_{kn}}^2} + \frac{36\omega U_0 2c}{c^4(1-\beta)^6 r_{kn}}\right) + \frac{\sqrt{3}}{\mu}\frac{{e_k}^2}{m_k}\frac{3\omega^2 U_0 e^{\mu T}}{c^4(1-\beta^2)^2} \leq 1 \ (k=1,2,3).$$

Then the system (7) has a unique continuous T-periodic solution.
Proof: Define an operator by the formulas

$$H = \left(H_1^{(1)},H_2^{(1)},H_3^{(1)},H_1^{(2)},H_2^{(2)},H_3^{(2)},H_1^{(3)},H_2^{(3)},H_3^{(3)}\right) : (SP)^9 \to (SP)^9,$$

where

$$H_\alpha^{(k)}(t) = \sigma_\alpha^{(k)}((p+1)T) + \int_{pT}^{t} F_\alpha^{(k)}(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})(s)ds,\ t\in[pT,(p+1)T]; p=0,1,2,...; k=1,2,3; \alpha=1,2,3.$$

It is easy to see that every T-periodic solution of (7) is a fixed point of H and vice versa. It is not difficulty to check that H maps $(SP)^9$ into itself. Indeed, $F_\alpha^{(k)}(s,\sigma_1,\sigma_2,\sigma_3)$ is T-periodic with respect to the first variable and therefore, $F_\alpha^{(k)}(s,\sigma_1(s),\sigma_2(s),\sigma_3(s))$ is T-periodic function for every $\sigma_1(s),\sigma_2(s),\sigma_3(s)$. Besides in view of Appendix we obtain

$$\left|H_\alpha^{(k)}(t)\right| \leq \left|\int_{pT}^{t} F_\alpha^{(k)}(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})(s)ds\right| \leq$$

$$\leq \left|\sigma_\alpha^{(k)}((p+1)T)\right| + \left|\int_{pT}^{t} F_{\alpha,L}^{(k)}(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})(s)ds\right| + \left|\int_{pT}^{t} F_{\alpha,rad}^{(k)}(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})(s)ds\right| \leq \left|\sigma_\alpha^{(k)}(0)\right| +$$

$$+\left|\int_{pT}^{t}\sum_{n=1,n\neq k}^{3}\frac{|e_k e_n|}{m_k}e^{\mu T}\left(\frac{6}{c^3(1-\beta)^3{\tau_{kn}}^2} + \frac{36\omega U_0}{c^4(1-\beta)^6\tau_{kn}}\right)\left|\vec{\sigma}^{(k)}\right|ds\right| + \left|\int_{pT}^{t}\frac{{e_k}^2}{m_k}\frac{3\omega^2 U_0 e^{\mu T}}{c^4(1-\beta^2)^2}\left|\vec{\sigma}^{(k)}\right|ds\right| \leq$$

$$\leq \left[\left|\sigma_{\alpha 0}^{(k)}\right| + \frac{S_0\sqrt{3}}{\mu}\sum_{n=1,n\neq k}^{3}\frac{|e_k e_n|}{m_k}e^{\mu T}\left(\frac{24c^2}{c^3(1-\beta)^3{r_{kn}}^2} + \frac{36\omega U_0 2c}{c^4(1-\beta)^6 r_{kn}}\right) + \frac{{e_k}^2}{m_k}\frac{3\omega^2 U_0 e^{\mu T}}{c^4(1-\beta^2)^2}\right]e^{\mu(t-pT)} \leq$$

$$\leq S_0 e^{\mu(t-pT)},$$

that is, H maps $(SP)^9$ into itself.

The set $(SP)^9$ turns out into a complete uniform space with respect to the family of pseudo-metrics

$$\rho_p(\sigma,\bar\sigma)=ess\sup\left\{e^{-\mu(t-pT)}\ |\sigma(t)-\bar\sigma(t)|:\ t\in[pT,(p+1)T]\right\},$$

$$\rho_p\left(\left(\sigma_1^{(1)},...,\sigma_3^{(3)}\right),\left(\bar\sigma_1^{(1)},...,\bar\sigma_3^{(3)}\right)\right)=\sum_{k=1}^{3}\sum_{\gamma=1}^{3}\rho_p(\sigma_\gamma^{(k)},\bar\sigma_\gamma^{(k)}),(p=0,1,2,...).$$

It remains to show that H is a contractive operator in the sense of [1]. Indeed, in view of Appendix for $t\in[pT,(p+1)T];(p=0,1,2,...)$ and $(k=1,2,3;\alpha=1,2,3)$ we have:

$$\left|H_\alpha^{(k)}(\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})(t)-H_\alpha^{(k)}(\bar\sigma_1^{(k)},\bar\sigma_2^{(k)},\bar\sigma_3^{(k)})(t)\right|\le$$
$$\le\int_{pT}^{t}\left|F_\alpha^{(k)}(s,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})(s)-F_\alpha^{(k)}(s,\bar\sigma_1^{(k)},\bar\sigma_2^{(k)},\bar\sigma_3^{(k)})(s)\right|ds\ \le$$
$$\le\sum_{n=1,n\ne k}^{3}\frac{|e_ke_n|}{m_k}\left[\frac{33}{c^4(1-\beta)^4}\frac{1}{r_{kn}^2}+\ \frac{26\sqrt3}{c^4(1-\beta)^8}\frac{\omega}{r_{kn}}\right]\sum_{\gamma=1}^{3}\int_{pT}^{t}\left|\sigma_\gamma^{(k)}(s)-\bar\sigma_\gamma^{(k)}(s)\right|ds+$$
$$+\frac{6{e_k}^2\omega^2}{m_kc^3(1-\beta^2)^2}\sum_{\gamma=1}^{3}\int_{pT}^{t}\left|\sigma_\gamma^{(k)}(s)-\bar\sigma_\gamma^{(k)}(s)\right|ds\le$$
$$\le\sum_{n=1,n\ne k}^{3}\frac{|e_ke_n|}{m_k}\left[\frac{33}{c^4(1-\beta)^4}\frac{1}{r_{kn}^2}+\ \frac{26\sqrt3}{c^4(1-\beta)^8}\frac{\omega}{r_{kn}}\right]\frac{e^{\mu(t-pT)}-1}{\mu}\rho_p\left(\left(\sigma_1^{(1)},...,\sigma_3^{(3)}\right),\left(\bar\sigma_1^{(1)},...,\bar\sigma_3^{(3)}\right)\right)+$$
$$+\frac{6{e_k}^2\omega^2}{m_kc^3(1-\beta^2)^2}\frac{e^{\mu(t-pT)}-1}{\mu}\rho_p\left(\left(\sigma_1^{(1)},...,\sigma_3^{(3)}\right),\left(\bar\sigma_1^{(1)},...,\bar\sigma_3^{(3)}\right)\right)\le$$
$$\le\frac{e^{\mu(t-pT)}}{\mu}\left[\sum_{n=1,n\ne k}^{3}\frac{|e_ke_n|}{m_k}\left(\frac{33}{c^4(1-\beta)^4}\frac{1}{r_{kn}^2}+\frac{26\sqrt3}{c^4(1-\beta)^8}\frac{\omega}{r_{kn}}\right)+\right.$$
$$\left.+\frac{6{e_k}^2\omega^2}{m_kc^3(1-\beta^2)^2}\right]\rho_p\left(\left(\sigma_1^{(1)},...,\sigma_3^{(3)}\right),\left(\bar\sigma_1^{(1)},...,\bar\sigma_3^{(3)}\right)\right).$$

It follows

$$\rho_p\left((H_1^{(1)}(\sigma_1^{(k)},...,\sigma_3^{(3)}),...,H_3^{(3)}(\sigma_1^{(1)},...,\sigma_3^{(3)})),(H_1^{(1)}(\bar\sigma_1^{(1)},...,\bar\sigma_3^{(k)}),...,H_3^{(3)}(\bar\sigma_1^{(1)},...,\bar\sigma_3^{(k)}))\right)\le$$
$$\le\frac{1}{\mu}\left[9\sum_{n=1,n\ne k}^{3}\frac{|e_ke_n|}{m_k}\left(\frac{33}{c^4(1-\beta)^4}\frac{1}{r_{kn}^2}+\frac{26\sqrt3}{c^4(1-\beta)^8}\frac{\omega}{r_{kn}}\right)+\right.$$
$$\left.+\frac{6{e_k}^2\omega^2}{m_kc^3(1-\beta^2)^2}\right]\rho_p\left(\left(\sigma_1^{(1)},...,\sigma_3^{(3)}\right),\left(\bar\sigma_1^{(1)},...,\bar\sigma_3^{(3)}\right)\right).$$

Consequently, the operator H has a fixed point which is a T-periodic solution of (7).

The Main Theorem is thus proved.

Appendix. Lipschitz estimates of the right-hand sides of the spin equations

$$\left|F_{\alpha,L}^{(k)}(t,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})-F_{\alpha,L}^{(k)}(t,\sigma_1^{(k)},\sigma_2^{(k)},\sigma_3^{(k)})\right|\le$$
$$\le\sum_{n=1,n\ne k}^{3}\frac{|e_ke_n|\,\Delta_k}{m_kc^2}\left[\left|P_\alpha^{(kn)}\right|\sum_{\gamma=1}^{3}\xi_\gamma^{(kn)}\left|\sigma_\gamma^{(k)}-\bar\sigma_\gamma^{(k)}\right|+\left|\xi_\alpha^{(kn)}\right|\sum_{\gamma=1}^{3}\left|P_\gamma^{(kn)}\right|\left|\sigma_\gamma^{(k)}-\bar\sigma_\gamma^{(k)}\right|+\right.$$
$$+\left(\tau_{kn}\sum_{\gamma=1}^{3}\left|P_\gamma^{(kn)}\right|\left|\sigma_\gamma^{(k)}-\bar\sigma_\gamma^{(k)}\right|+|L_{kn}|\sum_{\gamma=1}^{3}\left|\xi_\gamma^{(kn)}\right|\left|\sigma_\gamma^{(k)}-\bar\sigma_\gamma^{(k)}\right|\right)\left|\lambda_\alpha^{(k)}\right|+$$
$$\left.+\left(\tau_{kn}\sum_{\gamma=1}^{3}\left|P_\gamma^{(kn)}\right|\left|\lambda_\gamma^{(k)}\right|+|L_{kn}|\sum_{\gamma=1}^{3}\left|\xi_\gamma^{(kn)}\right|\left|\lambda_\gamma^{(k)}\right|\right)\left|\sigma_\alpha^{(k)}-\bar\sigma_\alpha^{(k)}\right|\right]\le$$

$$\leq \sum_{n=1,n\neq k}^{3} \frac{|e_k e_n|}{m_k c} \left[\left(\frac{1}{c^6 {\tau_{kn}}^3 (1-\beta)^{7/2}} + \left(\frac{3\sqrt{3}}{c^8 (1-\beta)^{15/2}} + \frac{2\sqrt{3}}{c^5 (1-\beta)^5} \right) \frac{\omega}{{\tau_{kn}}^2} \right) c\ \tau_{kn} \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| + \right.$$

$$+ c\tau_{kn} \left(\frac{1}{c^6 {\tau_{kn}}^3 (1-\beta)^{7/2}} + \left(\frac{3\sqrt{3}}{c^8 (1-\beta)^{15/2}} + \frac{2\sqrt{3}}{c^5 (1-\beta)^5} \right) \frac{\omega}{{\tau_{kn}}^2} \right) \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| +$$

$$+ \left(\frac{1}{c^7 (1-\beta)^{7/2} \tau_{kn}^3} + \frac{3\sqrt{3}\omega}{c^7 \tau_{kn}^2 (1-\beta)^{13/2}} + \frac{\omega\sqrt{3}}{c^6 (1-\beta)^5 \tau_{kn}^2} \right) c\tau_{kn} \frac{c}{\Delta_k} \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| +$$

$$+ \tau_{kn} \left(\frac{1}{{\tau_{kn}}^3(t) (1-\beta)^{7/2}} + \left(\frac{3\sqrt{3}}{c^6 (1-\beta)^{15/2}} + \frac{2\sqrt{3}}{c^3 (1-\beta)^5} \right) \frac{\omega}{c^2 {\tau_{kn}}^2} \right) \left| \sigma_\alpha^{(k)} - \bar{\sigma}_\alpha^{(k)} \right| \sum_{\gamma=1}^{3} \left| \lambda_\gamma^{(k)} \right| +$$

$$\left. + \left| \sigma_\alpha^{(k)} - \bar{\sigma}_\alpha^{(k)} \right| \left(\frac{1}{c^7 (1-\beta)^{7/2} \tau_{kn}^3} + \frac{3\sqrt{3}\omega}{c^7 \tau_{kn}^2 (1-\beta)^{13/2}} + \frac{\omega\sqrt{3}}{c^6 (1-\beta)^5 \tau_{kn}^2} \right) c\tau_{kn} \sum_{\gamma=1}^{3} \left| \lambda_\gamma^{(k)} \right| \right] \leq$$

$$\leq \sum_{n=1,n\neq k}^{3} \frac{|e_k e_n|}{m_k c} \left[\left(\frac{1}{c^5 (1-\beta)^{7/2} {\tau_{kn}}^2} + \frac{3\sqrt{3}}{c^7 (1-\beta)^{15/2}} \frac{\omega}{\tau_{kn}} + \frac{2\sqrt{3}}{c^4 (1-\beta)^5} \frac{\omega}{\tau_{kn}} \right) \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| + \right.$$

$$+ \left(\frac{1}{c^5 (1-\beta)^{7/2} {\tau_{kn}}^2} + \frac{3\sqrt{3}}{c^7 (1-\beta)^{15/2}} \frac{\omega}{\tau_{kn}} + \frac{2\sqrt{3}}{c^4 (1-\beta)^5} \frac{\omega}{\tau_{kn}} \right) \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| +$$

$$+ \left(\frac{1}{c^6 (1-\beta)^4 \tau_{kn}^2} + \frac{3\sqrt{3}\omega}{c^6 (1-\beta)^7 \tau_{kn}} + \frac{\omega\sqrt{3}}{c^5 (1-\beta)^{11/2} \tau_{kn}} \right) \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| +$$

$$+ 3 \left(\frac{1}{c^5 (1-\beta)^{7/2} {\tau_{kn}}^2} + \frac{3\sqrt{3}}{c^7 (1-\beta)^{15/2}} \frac{\omega}{\tau_{kn}} + \frac{2\sqrt{3}}{c^4 (1-\beta)^5} \frac{\omega}{\tau_{kn}} \right) \left| \sigma_\alpha^{(k)} - \bar{\sigma}_\alpha^{(k)} \right| +$$

$$\left. +3 \left(\frac{1}{c^5 (1-\beta)^{7/2} \tau_{kn}^2} + \frac{3\sqrt{3}\omega}{c^5 \tau_{kn} (1-\beta)^{13/2}} + \frac{\omega\sqrt{3}}{c^4 (1-\beta)^5 \tau_{kn}} \right) \left| \sigma_\alpha^{(k)} - \bar{\sigma}_\alpha^{(k)} \right| \right] \leq$$

$$\leq \sum_{n=1,n\neq k}^{3} \frac{|e_k e_n|}{m_k c} \left[\frac{33}{c^3 (1-\beta)^4} \frac{1}{r_{kn}^2} + \frac{\omega}{r_{kn}} \left(\frac{30\sqrt{3}}{c^6 (1-\beta)^8} + \frac{26\sqrt{3}}{c^3 (1-\beta)^5} + \frac{26\sqrt{3}}{c^4 (1-\beta)^7} \right) \right] \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| \leq$$

$$\leq \sum_{n=1,n\neq k}^{3} \frac{|e_k e_n|}{m_k} \left[\frac{33}{c^4 (1-\beta)^4} \frac{1}{r_{kn}^2} + \frac{26\sqrt{3}}{c^4 (1-\beta)^8} \frac{\omega}{r_{kn}} \right] \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right|;$$

$$\left| F_{\alpha,rad}^{(k)}(t, \sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)}) - F_{\alpha,rad}^{(k)}(t, \bar{\sigma}_1^{(k)}, \bar{\sigma}_2^{(k)}, \bar{\sigma}_3^{(k)}) \right| \leq \frac{{e_k}^2 \omega^2 U_0 e^{\mu(t-pT)} c}{m_k c^2 {\Delta_k}^3} \left(2 + \frac{1+\sqrt{3}}{\Delta_k} \right) \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| \leq$$

$$\leq \frac{e_k^2 \omega^2}{m_k \Delta_k^3} \frac{2c+4}{c(1-\beta^2)^{1/2}} \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| \leq \frac{e_k^2 \omega^2}{m_k} \frac{6c}{c^4 (1-\beta^2)^2} \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right| = \frac{6{e_k}^2 \omega^2}{m_k c^3 (1-\beta^2)^2} \sum_{\gamma=1}^{3} \left| \sigma_\gamma^{(k)} - \bar{\sigma}_\gamma^{(k)} \right|.$$

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