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Certain ways of constructing new spaces using Δ-approach

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Abstract

Many authors have given various structures to Cesàro spaces as cited in the text. The aim of this work is to define new spaces $[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \lambda, \coprod]_0, [C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \lambda, \coprod]_1$ and $[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \lambda, \coprod]_\infty$. We will study their basic topological properties. In our final section, we will compute some inclusion relations.

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1. Introduction and Background

By Ξ we mean all real valued sequences and any subspace of Ξ is termed as sequence space [9, 27, 28, 29, 30, 31] and references therein. Here $\mathbb{R}$ and $\mathbb{C}$, we abbreviate as set of all real and complex numbers, respectively.

In [10], the author propounded his work in the direction of differential geometry and was practicing methods of average of the divergent series called as Cesaro mean and defined it for order 1, $C_1 = (c_ij)$ as

$$c_{mn} = \begin{cases} \frac{1}{m} & if \quad 1 \le n \le m, \\ 0 & if \quad n \le m. \end{cases}$$

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Further, it was considered in [30], and defined the following:

$$cep_p = \left\{ v \in \Xi : \sum_{j=1}^{\infty} \left(\frac{1}{j} \sum_{k=1}^{j} \left| v_j \right| \right)^p < \infty \right\}, \text{ for } 1 < p < \infty,$$

and

$$cep_\infty = \left\{ v \in \Xi : \sup_{j \geq 1} \left| \frac{1}{j} \sum_{k=1}^{j} \left| v_j \right| \right| < \infty \right\}.$$

It was later studied by many authors as can be seen in [4, 8, 12] and many others therein.

A function $\mathcal{G} : \beth \to \mathbb{R}$, where $\beth$ is a linear space, is known as a paranorm, if for all $\varkappa, \varsigma \in \beth$:

(i) $\mathfrak{G}(\varkappa) \geq 0$ for all $\varkappa \geq 0$,

(ii) $\mathfrak{G}(\varkappa + \varsigma) \leq \mathfrak{G}(\varkappa) + \mathfrak{G}(\varsigma)$, and

(iii) if $(\varkappa_j)$ is a sequence of vectors with $\lim_{j \to \infty} \mathfrak{G}(\varkappa_j - \varkappa) = 0$ and (a_j) is a scalar sequence with $\lim_{j \to \infty} a_j = a$, then $\lim_{j \to \infty} \mathfrak{G}(a_j \varkappa_j - a\varkappa) = 0$.

A paranormed space $(\beth, \mathfrak{G})$ is a linear space equipped with a paranorm $\mathfrak{G}$ [1, 3, 5, 24, 32, 36].

A seminorm $\coprod$ is a function on vector space $\mathcal{X}$ if there exists a constant $a > 0$ such that $\coprod(u + v) \leq \coprod(u) + \coprod(v)$ and $\coprod(au) = |a| \coprod(u)$. Note that every semi norm is a paranorm but not conversely.

The function $\mathfrak{T}$ to called as modulus function $\mathfrak{T} : [0, \infty) \to [0, \infty)$ satisfying (i) $\mathfrak{T}(u) = 0$ if and only if $u = 0$, (ii) $\mathfrak{T}(u + v) = \mathfrak{T}(u) + \mathfrak{T}(v)$ for all $u = 0$, $v = 0$, (iii) $\mathfrak{T}$ is increasing, and (iv) $\mathfrak{T}$ is continuous from the right at zero. Consequently, $\mathfrak{T}$ should be continuous everywhere on $[0, \infty)$ and were further studied in [2, 6, 7, 34], and many others.

The statistical convergence concept was first given in [11], and its various structures can be seen in [21, 22, 32]. Let $\mathfrak{K} \subseteq \mathbb{N}$, then natural density $\delta(\mathfrak{K})$ is given as

$$\delta(\mathfrak{K}) = \lim_{i \to \infty} \frac{1}{i} card\left(\{ j \leq i : j \in \mathfrak{K} \} \right),$$

where card (.) represents the cardinality of the enclosed set with $\delta(\mathbb{N} - \mathfrak{K}) = 1 - \delta(\mathfrak{K})$.

A number sequence $\varkappa = (\varkappa_k)$ is said to be statistically convergent to the number $L \in \mathbb{C}$ if for each $\in > 0$, we have

$$\lim_{i \to \infty} \frac{1}{i} card\left(\{ j \leq i : j \in \mathfrak{K} : \left| \varkappa_j - L \right| > \in \} \right) = 0.$$

Throughout the text, S will represent set of all statistically convergent sequences.

The notion of Δ-operator to sequence spaces was given in [25, ?] and is defined as:

$$\beth(\Delta) = \{ (u_i) : (\Delta u_i) = (u_i - u_{i+1}) \in \beth,$$

where $\beth = l_\infty$, c and c_0 and are Banach spaces, normed by

$$\left\| u \right\|_\Delta = |u_1| + \sup_i |\Delta u_i|.$$

The notion of statistical convergence, modulus function and difference sequences were studied by many researchers as can be seen in [2, 17, 19, 20, 32], may others Throughout the text, $\mathfrak{X}$ will represent a locally convex T_2 and linear topological space inducing the topology by

continuous seminorm $\mathfrak{q}$; also the space of $\mathfrak{X}$-valued sequences will be represented by $S(\mathfrak{X})$. Denote $\mathfrak{h} = \inf_{m \geq 1} \mathfrak{s}_m, \mathcal{H} = \sup_{m \geq 1} \mathfrak{s}_m and\ C = max\{1, 2^{\mathcal{H}-1}\}$. Also, if a_m, $b_m \in \mathbb{C}$, then

$$| a_m + b_m |^{\mathfrak{s}_m} \leq \mathfrak{L}\left[| a_m |^{\mathfrak{s}_m} + | b_m |^{\mathfrak{s}_m} \right],$$

Also, for any complex β_j, we have

$$| \beta_j |^{\mathfrak{s}i} \leq max\left(1, \beta^{\mathcal{H}}\right), \tag{1}$$

where $\mathcal{H} = \sup_i \mathfrak{s}_i$ and $\mathfrak{s}$ is bounded sequence of strictly positive real numbers [26].

2. New Type of Spaces

In this part oftext, we introduce the space $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$, $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_1$ and $\left[{}_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_\infty$, where $g = (g_j)$ is a sequence such that $g_j \neq 0\ \forall_j \in \mathrm{N}$.

Following the authors cited in [8, 14, 15, 16, 18, 33, 35], we now define new spaces of the following form:

$$\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0 = \left\{ v \in S(\mathfrak{X}) : \lim_{j \to \infty} \frac{1}{j} \sum_{l=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{l} \sum_{i=1}^{l} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_l} = 0 \right\},$$

$$\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_1 = \left\{ v \in S(\mathfrak{X}) : \lim_{j \to \infty} \frac{1}{j} \sum_{l=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{l} \sum_{i=1}^{l} \Delta(g) v_i - a \right) \right) \right]^{\mathfrak{s}_l} = 0 \right\},$$

where $\mathfrak{a} \in \mathfrak{X}$ and

$$\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_\infty = \left\{ v \in S(\mathcal{X}) : \sup_j \frac{1}{j} \sum_{l=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{l} \sum_{i=1}^{l} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_l} < \infty \right\},$$

for $\Delta(g)v_i = g_i(v_i - v_i{+}1)$.

Theorem 2.1. *The sets* $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$, $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_1$ *and* $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_\infty$ *are linear spaces.*

Proof. We only consider $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$ and rest will follow on similar lines. Suppose $u, v \in \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$, hence for scalars a, b, we can find $\mathcal{Z}_a$ and $\mathcal{Z}_b$ such that $|a| \leq \mathcal{Z}_a$ and $|b| \leq \mathcal{Z}_b$ and thus we see by sub-additivity of $\mathfrak{T}$ that

$$\left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g)(a u_i + b v_i) \right) \right) \right]^{\mathfrak{s}_k} \leq \left[\mathfrak{T}\left(|a| \amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) u_i \right) \right) + \mathfrak{T}\left(|b| \amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_k}$$

$$\leq \mathcal{L}\left[\mathcal{Z}_a \mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) u_i \right) \right) \right]^{\mathfrak{s}_k} + \mathcal{L}\left[\mathcal{Z}_b \mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_k}$$

$$\leq \mathcal{L}\mathcal{Z}_a^{\mathcal{H}}\left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) u_i \right) \right) \right]^{\mathfrak{s}k} + \mathcal{L}\mathcal{Z}_b^{\mathcal{H}}\left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_k}$$

$$\to 0 \quad as \quad n \to \infty.$$

This shows that $(au + bv) \in \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$ and the result follows.

Theorem 2.2. *The sets* $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$, $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_1$ *and* $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_\infty$ *are paranormed spaces with paranorm* $\mathfrak{H}\Delta$ *as*

$$\mathfrak{H}_\Delta(v) = \sup_j \left(\frac{1}{j} \sum_{k=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_k} \right)^{\frac{1}{\mathfrak{P}}},$$

where $\mathfrak{P} = max\left\{1, \sup_j \mathfrak{s}_j\right\}$.

Proof. : We prove the result for $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$ and rest will be proved on similar grounds. It is obvious that

(i) $\mathfrak{H}_\Delta(-v) = \mathfrak{H}_\Delta(v)$.

(ii) $\mathfrak{H}_\Delta(\theta) = 0$ *for* $v = \theta$.

(iii) By Minkowski inequality, we have

$$\begin{aligned}
&\mathfrak{H}_\Delta(u+v) \\
&= \left(\frac{1}{j} \sum_{k=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g)(u_i + v_i) \right) \right) \right]^{\mathfrak{s}_k} \right)^{\frac{1}{\mathfrak{P}}} \leq \left(\frac{1}{j} \sum_{k=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) u_i \right) \right) + \mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_k} \right)^{\frac{1}{\mathfrak{P}}} \\
&\leq \left(\frac{1}{j} \sum_{k=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) u_i \right) \right) \right]^{\mathfrak{s}_k} \right)^{\frac{1}{\mathfrak{P}}} + \left(\frac{1}{j} \sum_{k=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} \Delta(g) v_i \right) \right) \right]^{\mathfrak{s}_k} \right)^{\frac{1}{\mathfrak{P}}} \leq \mathfrak{H}_\Delta(u) + \mathfrak{H}_\Delta(v).
\end{aligned}$$

Finally, we prove that the scalar multiplication is continuous, let $a \in \mathbb{C}$. By using definition of $\mathfrak{H}\Delta$, we have $\mathfrak{H}_\Delta(av) \leq \sigma^{\frac{H}{P}} \mathfrak{H}_\Delta(v)$, where σ is a positive integer such that $|a| \leq \sigma$. Consequently, $\mathfrak{H}\Delta(av) \to 0$ *for* $a \to 0$ and $v \to \theta$.

Also, for fixed a, $\mathfrak{H}\Delta(av) \to 0$ *for* $v \to 0$. So, we only need to prove that $a \to 0$ yields $\mathfrak{H}_\Delta(av) \to 0$ for fixed v.

For a fixed $v \in \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}, \mathfrak{s}, \amalg\right]_0$, we have

$$\lim_j \frac{1}{j} \sum_{k=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} a\Delta(g) v_i \right) \right) \right]^{\mathfrak{s}k} = 0.$$

Thus, for $\in > 0$ and small, we can find $j_0 \in \mathbb{N}$ such that

$$\frac{1}{j} \sum_{k=1}^{j} \left[\mathfrak{T}\left(\amalg\left(\frac{1}{k} \sum_{i=1}^{k} a\Delta(g) v_i \right) \right) \right]^{\mathfrak{s}k} = 0, \forall j \geq j_0. \tag{2}$$

Also, by continuity of $\mathfrak{T}$ with $n \le n_0$, we see

$$\frac{1}{j}\sum_{k=1}^{j}\left[\mathfrak{T}\left(\amalg\left(\frac{1}{k}\sum_{i=1}^{k} a\Delta(g)v_i\right)\right)\right]^{\mathfrak{s}k} = 0, \forall j \ge j_0. \tag{3}$$

Hence, from (2) and (3), we conclude that $\mathfrak{H}\Delta(av) \to 0$ *for* $a \to 0$.

3. Inclusion Relations

In this section, we investigate some inclusion relations in given spaces.

Theorem 3.1. *Consider tùe modulus functions* $\mathfrak{T}_1$ *and* $\mathfrak{T}_2$. *Then, for* $0 < \mathfrak{h} < \inf_m \mathfrak{s}_m \le \mathfrak{s} \le \sup_m \mathfrak{s}_m = \mathcal{H} < \infty$, *we have*

(i) $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_0 \cap \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_0 \subseteq \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1 + \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_0$,

(ii) $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_1 \cap \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_1 \subseteq \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1 + \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_1$,

(iii) $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_\infty \cap \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_\infty \subseteq \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1 + \mathfrak{T}_2, \mathfrak{s}, q\right]_\infty$.

Proof. To prove (i), let $\varkappa \in \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_0 \cap \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_0$. We have

$$\frac{1}{n}\sum_{i=1}^{n}\left[(\mathfrak{T}_1 + \mathfrak{T}_2)\left(\amalg\left(\frac{1}{i}\sum_{j=1}^{i}\Delta(g)\varkappa_j\right)\right)\right]^{\mathfrak{s}_i} = \frac{1}{n}\sum_{i=1}^{n}\left[\mathfrak{T}_1\left(\amalg\left(\frac{1}{i}\sum_{j=1}^{i}\Delta(g)\varkappa_j\right)\right) + \frac{1}{n}\sum_{i=1}^{n}\left[\mathfrak{T}_2\left(\amalg\left(\frac{1}{i}\sum_{j=1}^{i}\Delta(g)\varkappa\right)\right)\right]^{\mathfrak{s}_i}\right.$$

$$\le C.\frac{1}{n}\sum_{i=1}^{n}\left[\mathfrak{T}_1\left(\amalg\left(\frac{1}{i}\sum_{j=1}^{i}\Delta(g)\varkappa_j\right)\right)\right]^{\mathfrak{s}_i} + C.\frac{1}{n}\sum_{i=1}^{n}\left[\mathfrak{T}_2\left(\amalg\left(\frac{1}{i}\sum_{j=1}^{i}\Delta(g)\varkappa_j\right)\right)\right]^{\mathfrak{s}_i} \to 0 \text{ as } n \to \infty.$$

This shows that $\varkappa \in \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_0$, there by proving (i). In a similar fashion, (ii) and (iii) can be proved.

Theorem 3.2. *Consider tùe modulus functions* $\mathfrak{T}_1$ *and* $\mathfrak{T}_2$. *Then, for* $0 < h < \inf_m \mathfrak{s}_m \le \mathfrak{s} \le \sup_m \mathfrak{s}_m = H < \infty$, *we have*

(i) $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_0 \subseteq \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1 \circ \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_0$,

(ii) $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_1 \subseteq \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1 \circ \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_1$,

(iii) $\left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_\infty \subseteq \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1 \circ \mathfrak{T}_2, \mathfrak{s}, \amalg\right]_\infty$.

Proof. To prove (i), let $\varkappa \in \left[C_1^g, \Delta, \mathfrak{X}, \mathfrak{T}_1, \mathfrak{s}, \amalg\right]_0$.

Now for $\in > 0$, we can nd δ $(0 < \delta < 1)$, such that $\mathfrak{T}_1(x) < \in$ for $0 \le x < \delta$. Setting $\pi_1 = \mathfrak{T}_i\left(\amalg\left(\frac{1}{i}\sum_{j=1}^{i}\Delta(g)x_j\right)\right)$, we then have

$$\sum_{i=1}^{n}\left[\mathfrak{T}_1(\pi_i)\right]^{\mathfrak{s}_i}=\sum_{\pi_i\le\delta}\left[\mathfrak{T}_1(\pi_i)\right]^{\mathfrak{s}_i}+\sum_{\pi_i>\delta}\left[\mathfrak{T}_1(\pi_i)\right]^{\mathfrak{s}_i}.$$

Also, the continuity of $\mathfrak{T}_1$ gives

$$\sum_{\pi_i\le\delta}\left[\mathfrak{T}_1(\pi_i)\right]^{\mathfrak{s}i}<n\quad\max(\epsilon^h,\epsilon^H). \tag{4}$$

Since $\pi_i>\delta$ and $0<\delta<1$, therefore, we have

$$\pi_i<\frac{\pi_i}{\delta}\le 1+\left[\frac{\pi_i}{\delta}\right].$$

We thus have

$$\mathfrak{T}_1(\pi_i)<\mathfrak{T}_1\left(\left[1+\frac{\pi_i}{\delta}\right]\right)$$

$$\le\left(1+\left[\frac{\pi_i}{\delta}\right]\mathfrak{T}_1(1)\right.$$

$$\le 2.\frac{\pi_i}{\delta}\mathfrak{T}_1(1).$$

Hence, we can see that

$$\frac{1}{n}\sum_{\pi_i>\delta}\left[\mathfrak{T}_1(\pi_i)\right]^{\mathfrak{s}i}\le\max\left\{1,\left(\frac{2\mathfrak{T}_1(1)}{\delta}\right)^H\right\}.\frac{1}{n}\sum_{i=1}^{n}\left[\mathfrak{T}_1(\pi_i)\right]^{\mathfrak{s}i}. \tag{5}$$

Consequently, using (4) and (5), we have $\left[C_1^g,\Delta,\mathfrak{X},\mathfrak{T}_1,\mathfrak{s},\amalg\right]_0\subseteq\left[C_1^g,\Delta,\mathfrak{X},\mathfrak{T}_1\circ\mathfrak{T}_2,\mathfrak{s},\amalg\right]_0$.

In a similar fashion, (ii) and (iii) can be proved.

Conclusion: In this paper, the authors have studied the spaces of the form $\left[C_1^g,\Delta,\mathfrak{X},\mathfrak{T},\lambda,\amalg\right]_0$, $\left[C_1^g,\Delta,\mathfrak{X},\mathfrak{T},\lambda,\amalg\right]_1$ *and* $\left[C_1^g,\Delta,\mathfrak{X},\mathfrak{T},\lambda,\amalg\right]_\infty$. The various properties like paranormed nature, inclusion relations have been given. The researchers may work it for further generalized nature and structures on Δ-operator.

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