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S-Pseudo Quasi-2-Absorbing Submodules

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Abstract

Let R be a commutative ring with identity and let Y be a unitary left R-module. We introduce the concept of an S-pseudo quasi-2-absorbing submodule as a generalization of an endomorphism 2-absorbing submodule and as a stronger form of a pseudo quasi-2-absorbing submodule. Several examples, elementary properties, and characterizations are established. In the classes of scalar and multiplication modules, the notions of S-pseudo quasi-2-absorbing and pseudo quasi-2-absorbing submodules coincide. We also give conditions under which the notions of S-pseudo quasi-2-absorbing and endomorphism 2-absorbing submodules are equivalent.

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1. Introduction

Several generalizations of prime and 2-absorbing submodules have been studied in the literature. Semi-2-absorbing submodules were considered in [1]; pseudo 2-absorbing and related submodules were studied in [2, 3]; and extended nearly pseudo 2-absorbing submodules were introduced in [4]. Motivated by these developments and by the notion of a pseudo quasi-2-absorbing submodule [5], we introduce and investigate a stronger endomorphism-based version.

Throughout this paper, R denotes a commutative ring with identity, and all modules are unitary left R-modules. For an R-module Y, let $S = End_R(Y)$.

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A proper submodule A of Y is called *endomorphism* 2-*absorbing* (briefly, *End*-2-*absorbing*) if, for all $g_1, g_2 \in End_R(Y)$ and $y \in Y$,

$$(g_1 \circ g_2)(y) \in A$$

implies that $g_1(y) \in A$, or $g_2(y) \in A$, or $(g_1 \circ g_2)(Y) \subseteq A$.

A proper submodule A of Y is called *pseudo quasi*-2-*absorbing* if, whenever $g_1, g_2, g_3 \in End_R(Y)$ and $y \in Y$ satisfy

$$(g_1 \circ g_2 \circ g_3)(y) \in A,$$

then

$$(g_1 \circ g_2)(y) \in A + Soc(Y), \quad (g_1 \circ g_3)(y) \in A + Soc(Y),$$

or

$$(g_2 \circ g_3)(Y) \subseteq A + Soc(Y) \text{ [5].}$$

A nonzero submodule A of Y is *essential* if $A \cap B \neq (0)$ for every nonzero submodule B of Y [6]. The socle $Soc(Y)$ is the sum of all simple submodules of Y [7]. A module Y is called *scalar* if, for every $g \in End_R(Y)$, there exists $r \in R$ such that $g(y) = ry$ for all $y \in Y$ [8]. It is called a *multiplication module* if every submodule A of Y is of the form $A = IY$ for some ideal I of R; equivalently, $A = [A :_R Y]Y$ [9]. A module is *semisimple* if every submodule is a direct summand, equivalently, $Soc(Y) = Y$ [10]. A submodule A is *fully invariant* if $g(A) \subseteq A$ for every $g \in End_R(Y)$ [11]. A module Y is projective if every homomorphism from Y to a quotient module lifts through the corresponding epimorphism. If Y is projective, then $Soc(Y) = Soc(R)Y$ under the hypotheses used below [12]. Finally, Y is called Z-regular if, for every $y \in Y$, there exists $f \in Hom_R(Y, R)$ such that $y = f(y)y$ [11].

2. S-Pseudo Quasi-2-Absorbing Submodules

In this section, $S = End_R(Y)$. We introduce the principal definition and establish elementary properties and characterizations.

Definition 2.1 *A proper submodule A of an R-module Y is called an S-pseudo quasi*-2-*absorbing submodule if, whenever* $g_1, g_2, g_3 \in S$ *and* $y \in Y$ *satisfy*

$$(g_1 \circ g_2 \circ g_3)(y) \in A,$$

then at least one of the following holds:

$$(g_1 \circ g_2)(y) \in A + Soc(Y), \qquad (g_1 \circ g_3)(y) \in A + Soc(Y),$$

$$(g_2 \circ g_3)(y) \in A + Soc(Y).$$

A proper ideal I of S is called an S-pseudo quasi-2-*absorbing ideal if it is an S-pseudo quasi*-2-*absorbing submodule of the regular S-module S.*

Remark 2.2. *The following observations and examples are immediate from the definition.*

1. *The submodule* $(0) \oplus (\overline{4})$ *is an S-pseudo quasi-2-absorbing submodule of the* $\mathbb{Z}$*-module* $\mathbb{Z} \oplus \mathbb{Z}_8$.
2. *Every End-2-absorbing submodule is S-pseudo quasi-2-absorbing. The converse does not hold in general. For example,* $(\overline{12})$ *is an S-pseudo quasi-2-absorbing submodule of the -module* $\mathbb{Z}_{48}$, *but it is not End-2-absorbing.*

3. *If A is a maximal essential submodule of Y, then A is End-2-absorbing if and only if it is S-pseudo quasi-2-absorbing. Indeed, one implication follows from item (2), whereas the reverse implication follows from* $Soc(Y) \subseteq A$.
4. *The S-pseudo property implies the pseudo quasi-2-absorbing property.*
   The converse does not hold in general. In particular, the submodule $(0) \oplus 3\mathbb{Z}_{12}$ *of* $\mathbb{Z} \oplus \mathbb{Z}_{12}$ *is pseudo quasi-2-absorbing, but it is not S-pseudo quasi-2-absorbing.*
5. *The intersection of a chain of S-pseudo quasi-2-absorbing submodules is S-pseudo quasi-2-absorbing.*

For $h \in S$ and a submodule C of Y, we write

$$[C : h] = \{y \in Y : h(y) \in C\}.$$

The colon notation with subscript S is used for the corresponding set of endomorphisms.

Proposition 2.3. *A proper submodule A of Y is S-pseudo quasi-2-absorbing if and only if, for all* $g_1, g_2, g_3 \in S$,

$$[A : g_1 \circ g_2 \circ g_3] \subseteq [A + Soc(Y) :_S g_1 \circ g_2] \cup [A + Soc(Y) :_S g_1 \circ g_3] \cup [A + Soc(Y) :_S g_2 \circ g_3].$$

Proof. Suppose that A is S-pseudo quasi-2-absorbing and let $y \in [A : g_1 \circ g_2 \circ g_3]$. Then $(g_1 \circ g_2 \circ g_3)(y) \in A$, so one of $(g_1 \circ g_2)(y)$, $(g_1 \circ g_3)(y)$, and $(g_2 \circ g_3)(y)$ belongs to $A + Soc(Y)$. Hence the stated inclusion holds. Conversely, if the inclusion holds and $(g_1 \circ g_2 \circ g_3)(y) \in A$, then y belongs to the left-hand side and therefore to one of the three sets on the right-hand side. The defining condition follows. □

Proposition 2.4 *A proper submodule A of Y is S-pseudo quasi-2-absorbing if and only if, for all* $g_1, g_2 \in S$ *and* $y \in Y$ *such that* $(g_1 \circ g_2)(y) \notin A + Soc(Y)$,

$$[A :_S (g_1 \circ g_2)(y)] \subseteq [A + Soc(Y) :_S g_1(y)] \cup [A + Soc(Y) :_S g_2(y)].$$

Proof. Assume that A is S-pseudo quasi-2-absorbing. If g_3 belongs to the set on the left, then $(g_1 \circ g_2 \circ g_3)(y) \in A$. Since $(g_1 \circ g_2)(y) \notin A + Soc(Y)$, the defining condition yields either $(g_1 \circ g_3)(y) \in A + Soc(Y)$ or $(g_2 \circ g_3)(y) \in A + Soc(Y)$. Thus the inclusion follows. The converse is obtained by applying the assumed inclusion whenever the first alternative in the definition fails. □

Corollary 2.5 *For a proper submodule A of Y, the following statements are equivalent:*

1. *A is S-pseudo quasi-2-absorbing;*
2. *the inclusion in Proposition 2.3 holds for every* $g_1, g_2, g_3 \in S$;
3. *the inclusion in Proposition 2.4 holds for every* $g_1, g_2 \in S$ *and every* $y \in Y$ *with* $(g_1 \circ g_2)(y) \notin A + Soc(Y)$.

Proposition 2.6 *A proper submodule A of Y is S-pseudo quasi-2-absorbing if and only if, whenever* $g_1, g_2, g_3 \in S$ *and B is a submodule of Y satisfying*

$$(g_1 \circ g_2 \circ g_3)(B) \subseteq A,$$

then

$$(g_1 \circ g_2)(B) \subseteq A + Soc(Y),$$

or

$$(g_1 \circ g_3)(B) \subseteq A + Soc(Y),$$

or

$$(g_2 \circ g_3)(B) \subseteq A + Soc(Y).$$

Proof. The forward implication follows by applying the defining condition to elements of B. The reverse implication is obtained by taking $B = Ry$ for an arbitrary $y \in Y$. □

3. S-Pseudo Quasi-2-Absorbing Submodules in Special Classes of Modules

In this section, we compare S-pseudo quasi-2-absorbing and pseudo quasi-2-absorbing submodules in scalar, cyclic, finitely generated, and multiplication modules.

Proposition 3.1 *Let Y be a scalar module and A a proper submodule of Y. Then A is S-pseudo quasi-2-absorbing if and only if A is pseudo quasi-2-absorbing.*

Proof. The forward implication follows from Remark 2.2(4). Conversely, suppose that A is pseudo quasi-2-absorbing and $(g_1 \circ g_2 \circ g_3)(y) \in A$. Since Y is scalar, there exist $r_1, r_2, r_3 \in R$ such that $g_i(y) = r_i y$ for $i = 1,2,3$. Thus $r_1 r_2 r_3 y \in A$. The pseudo quasi-2-absorbing property gives

$$r_1 r_2 y \in A + Soc(Y), \quad r_1 r_3 y \in A + Soc(Y),$$

or $r_2 r_3 y \in A + Soc(Y)$. These are precisely the required alternatives. □

Corollary 3.2 *Let Y be a cyclic module and A a proper submodule of Y. Then A is A-pseudo quasi-2-absorbing if and only if it is pseudo quasi-2-absorbing.*

Corollary 3.3 *Let Y be a finitely generated module and A a proper submodule of Y. Then A is A-pseudo quasi-2-absorbing if and only if it is pseudo quasi-2-absorbing.*

Recall that a monomorphism $\varphi : X \to Y$ is called *essential* if $\varphi(X)$ is an essential submodule of Y.

Proposition 3.4 *Let $\varphi : X \to Y$ be an essential monomorphism, and let A be a proper fully invariant submodule of Y. If A is S-pseudo quasi-2-absorbing, then $\varphi^{-1}(A)$ is an S-pseudo quasi-2-absorbing submodule of X.*

Proof. Since A is proper, $\varphi^{-1}(A)$ is proper. Suppose that $(g_1 \circ g_2 \circ g_3)(x) \in \varphi^{-1}(A)$ and that $\varphi((g_1 \circ g_2)(x)) \notin A + Soc(Y)$. Then

$$\varphi((g_1 \circ g_2 \circ g_3)(x)) \in A.$$

Because A is S-pseudo quasi-2-absorbing, either

$$\varphi((g_1 \circ g_3)(x)) \in A + Soc(Y)$$

or

$$\varphi((g_2 \circ g_3)(x)) \in A + Soc(Y).$$

For an essential monomorphism, $\varphi^{-1}(Soc(Y)) = Soc(X)$ under the cited hypotheses. Consequently, either $(g_1 \circ g_3)(x)$ or $(g_2 \circ g_3)(x)$ belongs to $\varphi^{-1}(A) + Soc(X)$, proving the assertion. □

Proposition 3.5 *Let A be a multiplication module and A a proper submodule of Y. Then A is S-pseudo quasi-2-absorbing if and only if A is pseudo quasi-2-absorbing.*

Proof. The forward implication follows from Remark 2.2(4). Conversely, let $(g_1 \circ g_2 \circ g_3)(y) \in A$. In the class considered here, write $g_i(y) = r_i y$ for suitable $r_i \in R$. Hence $r_1 r_2 r_3 y \in A$. The pseudo quasi-2-absorbing property gives one of

$$r_1 r_2 y, \quad r_1 r_3 y, \quad r_2 r_3 y$$

in $A + Soc(Y)$, which is equivalent to the defining S-pseudo quasi-2-absorbing alternatives. □

Lemma 3.6 *Let A and B be submodules of Y with $B \subseteq A$. If A is pseudo quasi-2-absorbing in Y, then A / B is pseudo quasi-2-absorbing in Y / B.*

Proof. Let $r_1, r_2, r_3 \in R$ and $y \in Y$ satisfy

$$r_1 r_2 r_3 (y + B) \in A / B.$$

Then $r_1 r_2 r_3 y \in A$. Hence at least one of $r_1 r_2 y$, $r_1 r_3 y$, and $r_2 r_3 y$ belongs to $A + Soc(Y)$. Passing to the quotient gives the corresponding element in

$$\frac{A + Soc(Y)}{B} = \frac{A}{B} + \frac{Soc(Y) + B}{B} \subseteq \frac{A}{B} + Soc(Y / B).$$

Thus A / B is pseudo quasi-2-absorbing in Y / B. □

Lemma 3.7 *Let Y be semisimple, and let $B \subseteq A$ be submodules of Y. If B is pseudo quasi-2-absorbing in Y and A / B is pseudo quasi-2-absorbing in Y / B, then A is pseudo quasi-2-absorbing in Y.*

Proof. Suppose that $r_1 r_2 r_3 y \in A$. Then

$$r_1 r_2 r_3 (y + B) \in A / B.$$

Since A / B is pseudo quasi-2-absorbing, one of $r_1 r_2 (y + B)$, $r_1 r_3 (y + B)$, and $r_2 r_3 (y + B)$ lies in $A / B + Soc(Y / B)$. Because Y is semisimple, $Soc(Y) = Y$, and the corresponding element lies in $A + Soc(Y)$. Hence A is pseudo quasi-2-absorbing. □

Proposition 3.8 *Let A and B be submodules of a multiplication module Y, with $B \subseteq A$. If A is S-pseudo quasi-2-absorbing in Y, then A / B is S-pseudo quasi-2-absorbing in Y / B.*

Proof. By Remark 2.2(4), A is pseudo quasi-2-absorbing. Lemma 3.6 yields that A / B is pseudo quasi-2-absorbing in Y / B. Since quotients of multiplication modules are multiplication modules, Proposition 3.5 implies that A / B is S-pseudo quasi-2-absorbing. □

Proposition 3.9 *Let Y be a semisimple multiplication module and let $B \subseteq A$ be submodules. If B is S-pseudo quasi-2-absorbing in Y and A / B is S-pseudo quasi-2-absorbing in Y / B, then A is S-pseudo quasi-2-absorbing in Y.*

Proof. By Remark 2.2(4), both B and A / B are pseudo quasi-2-absorbing in their respective modules. Lemma 3.7 shows that A is pseudo quasi-2-absorbing in Y. Proposition 3.5 then gives the required S-pseudo quasi-2-absorbing property. □

4. Characterizations in Projective and Z-Regular Cyclic Modules

We now give characterizations of S-pseudo quasi-2-absorbing submodules in cyclic projective and cyclic Z-regular modules.

Proposition 4.1 *Let Y be a cyclic projective module and A a proper submodule of Y. Then A is S-pseudo quasi-2-absorbing if and only if* $[A :_R Y]$ *is an S-pseudo quasi-2-absorbing ideal of R.*

Proof. Assume first that A is S-pseudo quasi-2-absorbing. If

$$(g_1 \circ g_2 \circ g_3)(r) \in [A :_R Y],$$

then $(g_1 \circ g_2 \circ g_3)(rY) \subseteq A$. Proposition 2.6 gives one of

$$(g_1 \circ g_2)(rY), \quad (g_1 \circ g_3)(rY), \quad (g_2 \circ g_3)(rY)$$

contained in $A + Soc(Y)$. Since Y is cyclic, it is a multiplication module and $A = [A :_R Y]Y$. Since Y is projective, $Soc(Y) = Soc(R)Y$. It follows that one of

$$(g_1 \circ g_2)(r), \quad (g_1 \circ g_3)(r), \quad (g_2 \circ g_3)(r)$$

belongs to $[A :_R Y] + Soc(R)$. Therefore $[A :_R Y]$ is S-pseudo quasi-2-absorbing.

Conversely, assume that $[A :_R Y]$ is S-pseudo quasi-2-absorbing. Let B be a submodule of Y with $(g_1 \circ g_2 \circ g_3)(B) \subseteq A$. Since Y is cyclic and hence multiplication, $B = IY$ for some ideal I of R. Then

$$(g_1 \circ g_2 \circ g_3)(I) \subseteq [A :_R Y].$$

The ideal property gives one of $(g_1 \circ g_2)(I), (g_1 \circ g_3)(I)$, and $(g_2 \circ g_3)(I)$ contained in $[A :_R Y] + Soc(R)$. Multiplication by Y and the equality $Soc(R)Y = Soc(Y)$ yield the alternatives in Proposition 2.6. Hence A is S-pseudo quasi-2-absorbing. □

Proposition 4.2 *Let Y be a cyclic projective module and let I be a proper ideal of R with* $ann_R(Y) \subseteq I$. *Then I is an S-pseudo quasi-2-absorbing ideal of R if and only if IY is an S-pseudo quasi-2-absorbing submodule of Y.*

Proof. Suppose that I is S-pseudo quasi-2-absorbing and let $(g_1 \circ g_2 \circ g_3)(B) \subseteq IY$. Since Y is cyclic, write $B = JY$. Then

$$(g_1 \circ g_2 \circ g_3)(J) \subseteq I.$$

Consequently, one of $(g_1 \circ g_2)(J)$, $(g_1 \circ g_3)(J)$, and $(g_2 \circ g_3)(J)$ is contained in $I + Soc(R)$. Multiplying by Y and using $Soc(R)Y = Soc(Y)$ gives the required alternatives for IY.

Conversely, suppose that IY is S-pseudo quasi-2-absorbing. If $(g_1 \circ g_2 \circ g_3)(r) \in I$, then $(g_1 \circ g_2 \circ g_3)(rY) \subseteq IY$. Applying the submodule property and using that Y is cyclic projective gives one of $(g_1 \circ g_2)(r)$, $(g_1 \circ g_3)(r)$, and $(g_2 \circ g_3)(r)$ in $I + Soc(R)$. Hence I is S-pseudo quasi-2-absorbing. □

Corollary 4.3 *If Y is a cyclic projective module and A is a proper submodule of Y, then the following statements are equivalent:*

1. *A is S-pseudo quasi-2-absorbing;*
2. $[A :_R Y]$ *is an S-pseudo quasi-2-absorbing ideal of R;*
3. $A = IY$ *for some S-pseudo quasi-2-absorbing ideal I of R satisfying* $ann_R(Y) \subseteq I$.

Lemma 4.4 *Let Y be a cyclic Z-regular module and A a proper submodule of Y. Then A is pseudo quasi-2-absorbing if and only if* $[A :_R Y]$ *is a pseudo quasi-2-absorbing ideal of R.*

Proof. Assume that A is pseudo quasi-2-absorbing and that $(r_1r_2r_3)J \subseteq [A :_R Y]$ for an ideal J of R. Then $(r_1r_2r_3)(JY) \subseteq A$. Therefore one of $(r_1r_2)(JY)$, $(r_1r_3)(JY)$, and $(r_2r_3)(JY)$ is contained in $A + Soc(Y)$. Since Y is cyclic, $A = [A :_R Y]Y$, and the Z-regular hypothesis gives the corresponding socle relation. Hence one of $(r_1r_2)J$, $(r_1r_3)J$, and $(r_2r_3)J$ is contained in $[A :_R Y] + Soc(R)$.

Conversely, let $(r_1r_2r_3)B \subseteq A$. Write $B = JY$. Then $(r_1r_2r_3)J \subseteq [A :_R Y]$. The ideal property, multiplication-module representation, and the socle relation imply that one of $(r_1r_2)B$, $(r_1r_3)B$, and $(r_2r_3)B$ is contained in $A + Soc(Y)$. Thus A is pseudo quasi-2-absorbing. □

Lemma 4.5 *Let Y be a cyclic Z-regular module, and let I be a proper ideal of R with $ann_R(Y) \subseteq I$. Then I is a pseudo quasi-2-absorbing ideal of R if and only if IY is a pseudo quasi-2-absorbing submodule of Y.*

Proof. If I is pseudo quasi-2-absorbing and $(r_1r_2r_3)B \subseteq IY$, write $B = JY$. Then $(r_1r_2r_3)J \subseteq I$, so one of $(r_1r_2)J$, $(r_1r_3)J$, and $(r_2r_3)J$ is contained in $I + Soc(R)$. Multiplying by Y and applying the Z-regular socle relation proves that IY is pseudo quasi-2-absorbing.

Conversely, if $(r_1r_2r_3)J \subseteq I$, then $(r_1r_2r_3)JY \subseteq IY$. Applying the submodule property and passing back through the multiplication-module and socle relations yields one of $(r_1r_2)J$, $(r_1r_3)J$, and $(r_2r_3)J$ contained in $I + Soc(R)$. Hence Y is pseudo quasi-2-absorbing. □

Theorem 4.6 *If Y is a cyclic Z-regular module and A is a proper submodule of Y, then the following statements are equivalent:*

1. *A is S-pseudo quasi-2-absorbing;*
2. *$[A :_R Y]$ is an S-pseudo quasi-2-absorbing ideal of R;*
3. *$A = IY$ for some S-pseudo quasi-2-absorbing ideal I of R satisfying $ann_R(Y) \subseteq I$.*

Proof. By Remark 2.2(4), every S-pseudo quasi-2-absorbing submodule or ideal is pseudo quasi-2-absorbing. Conversely, in the cyclic setting under consideration, Corollary 3.2 identifies the pseudo quasi-2-absorbing and S-pseudo quasi-2-absorbing notions. Thus the equivalence of (1) and (2) follows from Lemma 4.4. The equivalence with (3) follows by taking $I = [A : (R)\ Y]$ and applying Lemma 4.5. □

5. Conclusion

In this paper, we introduced S-pseudo quasi-2-absorbing submodules as a generalization of End-2-absorbing submodules and as a stronger form of pseudo quasi-2-absorbing submodules. The main conclusions are summarized as follows:

1. Every End-2-absorbing submodule is S-pseudo quasi-2-absorbing, and every S-pseudo quasi-2-absorbing submodule is pseudo quasi-2-absorbing; neither converse holds in general.
2. The notion admits the colon characterizations in Propositions 2.3 and 2.4 and the submodule characterization in Proposition 2.6.
3. In scalar, cyclic, finitely generated, and multiplication modules, the S-pseudo quasi-2-absorbing and pseudo quasi-2-absorbing notions coincide under the stated hypotheses.
4. The inverse image under an essential monomorphism preserves the property under the conditions of Proposition 3.4.
5. The property behaves well with respect to quotient modules and extensions in semisimple multiplication modules, as shown in Propositions 3.8 and 3.9.
6. For cyclic projective and cyclic Z-regular modules, the property is characterized by the corresponding residual ideal $[A :_R Y]$ and by representations of the form $A = IY$.

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