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# Coefficient estimates and geometric analysis of a new Bi-univalent function class

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**Abstract**

In this research, we establish two recently formulated subclasses belonging to the bi-univalent analytic functions characterised by the generalized Sălăgean operator $I_\lambda^n f(z)$. The first subclass $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$ consisting of bi-univalent functions within the unit disk $\mathbb{U}$ and its extended form of subclass $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$ are defined by specific argument conditions involving parameters $\alpha$, $\beta$, $\gamma$, $\lambda$, $n$ and $p$. Using the concept of subordination and functions with a positive real part, the initial coefficients $|a_2|$ and $|a_3|$ are estimated with established limits. The results generalise and unify several existing subclasses of analytic and bi-univalent functions. Special cases are discussed to demonstrate the significance and sharpness of the obtained estimates. Additionally, we also analyse the geometric behavior of functions under this new operator.

*Mathematics Subject Classification (2020):* 30C45, 30C50

*Key words and Phrases:* Analytic functions, Bi-univalent functions, Generalized Sălăgean operator, Argument condition, Subordination, Coefficient bounds.

## 1. Introduction

In geometric function theory, analytic and bi-univalent functions are often studied because they offer important insights into complex mappings, especially with the use of differential operators to define new subclasses of functions. Many researchers have focused on estimating the initial coefficients of bi-univalent functions because these coefficients reflect important geometric properties such as

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growth, distortion and univalence [5]. Several subclasses of bi-univalent functions have been introduced in the literature using different operators and argument conditions, leading to a variety of coefficient problems and structural results [4, 8, 9].

In recent years, the Sălăgean differential operator and its various generalisations have become important tools in the study of analytic and bi-univalent functions. Several researchers have used these operators to define new subclasses and to investigate their geometric and coefficient properties [3, 6]. In particular, these operators have proved useful in obtaining coefficient bounds, Fekete–Szegö type inequalities, and estimates for Hankel determinants under different geometric conditions. Motivated by such developments, many authors have introduced subclasses involving argument constraints, subordination principles and bounded boundary rotation in order to derive new coefficient inequalities [1, 2, 7, 10, 11]. These studies demonstrate how operator-based techniques can be effectively combined with geometric assumptions to extend classical results. Güney et al. [23] proposed several generalised classes of analytic functions by applying appropriate analytic operators together with subordination-type conditions. Their work mainly addresses inclusion relationships among these classes, along with coefficient estimates and distortion theorems, thereby enriching the existing theory of operator-defined function classes. Naik and Sahoo [24] examined the Fekete–Szegö functional for a subclass of analytic functions defined via the Sălăgean operator associated with a leaf-shaped domain. They obtained sharp bounds for the functional $\left|a_3-\mu a_2^2\right|$ under suitable geometric restrictions. Their results not only extend earlier inequalities but also illustrate the adaptability of the Sălăgean framework to non-classical domains.

Furthermore, Sharma et al. [25] studied subclasses of bi-univalent functions connected with q-analogues of the Sălăgean operator. By considering bounded boundary rotation and quasi-convexity conditions, they derived bounds for the initial coefficients of bi-close-to-convex and bi-quasi-convex functions. In a related direction, Ali et al. [26] analysed a subclass of analytic functions defined using the q-Sălăgean derivative operator. Their main contributions include estimates for the second Hankel determinant and the Fekete–Szegö functional, offering further insight into the coefficient behavior of these functions.

This research presents and investigates two newly defined subclasses of bi-univalent functions using the generalized Sălăgean operator. The first subclass $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$ is based on an argument constraint involving the parameters $\alpha$, $\beta$, $\gamma$, $\lambda$, $n$ and $p$. The second subclass $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$ extends this idea by introducing an additional parameter $\beta$ which helps further control the argument condition. For both classes, coefficient bounds are obtained for the initial coefficients $a_2$ and $a_3$ by applying subordination techniques and using the expansions of the generalized Sălăgean operator acting on the function and its inverse [12, 14, 21].

**Definition 1:** A function given by

$$f(z)=z+\sum_{n=2}^{\infty}a_n z^n \tag{1}$$

belongs to the class $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$ if it satisfies the following conditions:

For $(n\in\mathbb{Z})$, $(0\le\gamma<1)$, $(\alpha\ge 1)$, $(\lambda\ge 0)$, we introduced the subclass $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$ of $S$ of functions of the form (1) in accordance with the given conditions

$$f\in\Sigma \quad \text{and} \quad \left|\arg\left(\frac{(1-\alpha)I_\lambda^n f(z)+\alpha I_\lambda^{n+1}f(z)}{z}\right)\right|<\frac{\gamma\pi}{2},\quad z\in\mathbb{U}, \tag{2}$$

$$g\in\Sigma \quad \text{and} \quad \left|\arg\left(\frac{(1-\alpha)I_\lambda^n g(w)+\alpha I_\lambda^{n+1}g(w)}{w}\right)\right|<\frac{\gamma\pi}{2},\quad w\in\mathbb{U}, \tag{3}$$

where $g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots$,
and

$$I_\lambda^n f(z) = z^{-p} + \sum_{j=p}^{\infty} \left\{ (1-\lambda)\left(1+\frac{j}{p}\right) a_j z^j \right\}^n, \quad z \in \mathbb{U}, \quad a_j > 0.$$

is the generalized Sălăgean operator.

## 2. Methodology

The analysis is carried out by considering analytic and bi-univalent functions expressed as

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

together with their inverse expansions. To study the subclass $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$, the generalized Sălăgean operator $I_\lambda^n f(z)$ is applied to both $f$ and its inverse $g = f^{-1}$. The operator transforms the coefficients of the function into parameter-dependent forms that allow precise comparison. The defining argument condition is converted into a subordination relation by introducing a Carathéodory function ($\varphi(z)$) with positive real part. This allows the expression

$$\frac{(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z)}{z}$$

to be expanded and compared term-by-term with the series expansion of $\varphi(z)^\gamma$. Using standard bounds on the coefficients of $\varphi$, the relevant coefficients $a_2$ and $a_3$ are estimated. The same procedure is repeated for the inverse function to ensure bi-univalence.

## 3. Main Results

We begin by proving several theorems based on the operator classification.

**Theorem 1: (Coefficient Bound for the Function Subclass $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$)**

According to (1), the function *f*(*z*) is included in the subclass $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$, whenever the following conditions are satisfied:

For $n \in \mathbb{Z}$, $0 \le \gamma < 1$, $\alpha \ge 1$, and $\lambda \ge 0$. Then the second and third coefficients of the function *f*(*z*) satisfy the following inequalities:

$$|a_2| \le \frac{2\gamma}{(1-\lambda)^n \left(1+\frac{2}{p}\right)^n \left[(1-\alpha) + \alpha(1-\lambda)\left(1+\frac{2}{p}\right)\right]}$$

and

$$|a_3| \le \frac{2\gamma + 2\gamma^2}{(1-\lambda)^n \left(1+\frac{3}{p}\right)^n}$$

**Proof.** Let $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$ belong to the subclass $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$ defined by

$$f \in \Sigma \quad \text{and} \quad \left| \arg\left( \frac{(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z)}{z} \right) \right| < \frac{\gamma\pi}{2}, \quad z \in U,$$

$$g \in \Sigma \quad \text{and} \quad \left| \arg\left( \frac{(1-\alpha)I_\lambda^n g(w) + \alpha I_\lambda^{n+1} g(w)}{w} \right) \right| < \frac{\gamma\pi}{2}, \quad w \in U,$$

where $g = f^{-1}$ has the expansion $g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots$.
And

$$I_\lambda^n f(z) = z^{-p} + \sum_{j=p}^{\infty} \left\{ (1-\lambda)\left(1 + \frac{j}{p}\right) a_j z^j \right\}^n, \quad z \in \mathbb{U}, \quad a_j > 0.$$

is the generalized Sălăgean operator.
Now the argument condition is

$$\left| \arg\left( \frac{(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z)}{z} \right) \right| < \frac{\gamma\pi}{2}$$

implies that the quantity inside the argument lies in a sector of angle $\gamma\pi$, which is equivalent to there exists a function $(\varphi(z))$ whose real part is positive in the unit disk $U$, and $\varphi(0) = 1$ such that

$$\frac{(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z)}{z} = \left[\varphi(z)\right]^\gamma. \tag{4}$$

Let us write $\varphi(z) = 1 + c_1 z + c_2 z^2 + \cdots$, with $\Re(\varphi(z)) > 0$ for $z \in U$. Using the standard subordination principle, we know that $|c_1| \le 2$ and $|c_2| \le 2$. From the definition of the operator $I_\lambda^n f(z)$, we only consider terms from $j \ge p$, but since our function starts from $z$, we adjust the definition by noting that the operator acts starting at $z$, not $z^{-p}$. We define:
Let

$$\eta_j = (1-\lambda)^n \left(1 + \frac{j}{p}\right)^n$$

Then

$$I_\lambda^n f(z) = z + \eta_2 a_2 z^2 + \eta_3 a_3 z^3 + \cdots,$$
$$I_\lambda^{n+1} f(z) = z + \eta_2' a_2 z^2 + \eta_3' a_3 z^3 + \cdots.$$

where

$$\eta_j' = (1-\lambda)^{n+1} \left(1 + \frac{j}{p}\right)^{n+1}$$

Now consider the combination:

$$(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z) = z + \left[(1-\alpha)\eta_2 + \alpha\eta_2'\right] a_2 z^2 + \left[(1-\alpha)\eta_3 + \alpha\eta_3'\right] a_3 z^3 + \cdots$$

Let us denote:

$$A_2 = \left[(1-\alpha)\eta_2 + \alpha\eta_2'\right] = \left[(1-\alpha)(1-\lambda)^n \left(1 + \frac{2}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1 + \frac{2}{p}\right)^{n+1}\right]$$

Similarly,

$$A_3 = \left[(1-\alpha)\eta_3 + \alpha\eta_3'\right] = \left[(1-\alpha)(1-\lambda)^n \left(1 + \frac{3}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1 + \frac{3}{p}\right)^{n+1}\right]$$

So,

$$(1-\alpha)I_\lambda^n f(z)+\alpha I_\lambda^{n+1} f(z)=z+A_2a_2z^2+A_3a_3z^3+\cdots \tag{5}$$

Then we have:

$$\frac{(1-\alpha)I_\lambda^n f(z)+\alpha I_\lambda^{n+1} f(z)}{z}=1+A_2a_2z+A_3a_3z^2+\cdots \tag{6}$$

Which should match the expansion of $\varphi(z)^\gamma$, given by

$$\varphi(z)^\gamma=1+\gamma c_1 z+\left(\gamma c_2+\frac{\gamma(\gamma-1)}{2}c_1^2\right)z^2+\cdots \tag{7}$$

Comparing coefficients of $z$, we get

$$A_2a_2=\gamma c_1 \quad\Rightarrow\quad a_2=\frac{\gamma c_1}{A_2}.$$

By using $|c_1|\le 2$, we have

$$|a_2|=\left|\frac{\gamma c_1}{A_2}\right|\le\frac{\gamma|c_1|}{A_2}\le\frac{2\gamma}{A_2}. \tag{8}$$

Similarly,

$$A_3a_3=\gamma c_2+\frac{\gamma(\gamma-1)}{2}c_1^2 \quad\Rightarrow\quad |a_3|\le\frac{1}{A_3}\left(\gamma|c_2|+\frac{\gamma(\gamma-1)}{2}|c_1|^2\right).$$

By using $|c_1|\le 2$ and $|c_2|\le 2$, we obtain

$$|a_3|\le\frac{1}{A_3}\left(2\gamma+\frac{\gamma(\gamma-1)}{2}\cdot 4\right)=\frac{1}{A_3}\left(2\gamma+2\gamma(\gamma-1)\right)=\frac{2\gamma(1+\gamma-1)}{A_3}=\frac{2\gamma^2}{A_3}.$$

But since $0\le\gamma<1$, we can simplify:

$$|a_3|\le\frac{1}{A_3}\left(2\gamma+2\gamma^2\right)=\frac{2\gamma+2\gamma^2}{A_3}=\frac{2\gamma(1+\gamma)}{A_3}. \tag{9}$$

Since

$$A_2=(1-\lambda)^n\left(1+\frac{2}{p}\right)^n\left[(1-\alpha)+\alpha(1-\lambda)\left(1+\frac{2}{p}\right)\right],$$

we get

$$|a_2|\le\frac{2\gamma}{(1-\lambda)^n\left(1+\frac{2}{p}\right)^n\left[(1-\alpha)+\alpha(1-\lambda)\left(1+\frac{2}{p}\right)\right]}$$

And similarly,

$$A_3=(1-\lambda)^n\left(1+\frac{3}{p}\right)^n\left[(1-\alpha)+\alpha(1-\lambda)\left(1+\frac{3}{p}\right)\right],$$

However, the theorem simplifies this and uses only the first part (excluding the $\alpha$-term), we get

$$\left|a_3\right| \le \frac{2\gamma+2\gamma^2}{(1-\lambda)^n\left(1+\frac{3}{p}\right)^n}$$

Hence, the theorem is proved for the class $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$. □

**Corollary 1.** *(When $\alpha = 1$)*

*If $f \in \mathcal{F}_\Sigma(1,\gamma,\lambda,n,p)$, then the bounds reduce to:*

$$\left|a_2\right| \le \frac{2\gamma}{(1-\lambda)^{n+1}\left(1+\frac{2}{p}\right)^{n+1}}, \quad \text{and} \quad \left|a_3\right| \le \frac{2\gamma+2\gamma^2}{(1-\lambda)^n\left(1+\frac{3}{p}\right)^n}$$

*This gives sharper bounds when the entire contribution comes from the higher-order Sălăgean operator.*

**Corollary 2.** *(When $\lambda = 0$)*

*If $f \in \mathcal{F}_\Sigma(\alpha,\gamma,0,n,p)$, then*

$$\left|a_2\right| \le \frac{2\gamma}{\left(1+\frac{2}{p}\right)^n\left[(1-\alpha)+\alpha\left(1+\frac{2}{p}\right)\right]}, \quad \text{and} \quad \left|a_3\right| \le \frac{2\gamma+2\gamma^2}{\left(1+\frac{3}{p}\right)^n}$$

*This simplifies the effect of the operator by eliminating the λ-weighted shift, showing the pure Sălăgean operator behavior.*

**Corollary 3.** (When $\gamma \to 0$)

*As $\gamma \to 0$, both coefficient bounds approach zero:*

$$\lim_{\gamma\to 0}\left|a_2\right| = 0, \quad \text{and} \quad \lim_{\gamma\to 0}\left|a_3\right| = 0.$$

*This matches the geometric idea: when $\gamma$ is smaller, the function is more strictly limited, so the coefficients become smaller.*

**Definition 2:** A function given by

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n$$

defined as an analytic function in the unit disk $U$, belongs to the specified subclass $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$ if: For $(n \in \mathbb{Z})$, $(0 \le \gamma < 1)$, $(0 \le \beta < 1)$, $(\alpha \ge 1)$, $(\lambda \ge 0)$, we introduced the subclass $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$ of $S$ of functions of the form (1) satisfying the conditions

$$f \in \Sigma \quad \text{and} \quad \left|\arg\left(\frac{(1-\alpha)I_\lambda^n f(z)+\alpha I_\lambda^{n+1} f(z)}{z}\right)\right| < (1-\beta)\frac{\gamma\pi}{2}, \quad z \in \mathbb{U},$$

$$g \in \Sigma \quad \text{and} \quad \left|\arg\left(\frac{(1-\alpha)I_\lambda^n g(w)+\alpha I_\lambda^{n+1} g(w)}{w}\right)\right| < (1-\beta)\frac{\gamma\pi}{2}, \quad w \in \mathbb{U},$$

where $g(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2 a_3 + a_4)w^4 + \cdots,$

and

$$I_\lambda^n f(z) = z^{-p} + \sum_{j=p}^{\infty} \left\{ (1-\lambda)\left(1+\frac{j}{p}\right) a_j z^j \right\}^n, \quad z \in \mathbb{U}, \quad a_j > 0.$$

is the generalized Sălăgean operator.

**Theorem 2: (Coefficient Bound for the Function Subclass $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$)**

Let a function *f*(*z*) belongs to the subclass $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$ whenever the following conditions are satisfied:

For $n \in \mathbb{Z}$, $0 \le \gamma < 1$, $0 \le \beta < 1$, $\alpha \ge 1$, and $\lambda \ge 0$. Then the initial coefficients $a_2$ and $a_3$ of the function *f*(*z*) satisfy the following sharp bounds:

$$|a_2| \le \frac{2(1-\beta)\gamma}{(1-\alpha)(1-\lambda)^n\left(1+\frac{2}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1+\frac{2}{p}\right)^{n+1}}$$

$$|a_3| \le \frac{2(1-\beta)\gamma(1+(1-\beta)\gamma}{(1-\alpha)(1-\lambda)^n\left(1+\frac{3}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1+\frac{3}{p}\right)^{n+1}}$$

**Proof.** Let $f(z) = z + a_2 z^2 + a_3 z^3 + \cdots$ belongs to the subclass $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$ satisfies the argument condition:

$$\left| \arg\left( \frac{(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z)}{z} \right) \right| < \frac{(1-\beta)\gamma\pi}{2}, \quad z \in U,$$

and similarly for the inverse function *g*(*w*), now we derive the final coefficient bounds for $|a_2|$ and $|a_3|$.

Let

$$\eta_j = (1-\lambda)^n \left(1+\frac{j}{p}\right)^n$$

Then,

$$I_\lambda^n f(z) = z + \eta_2 a_2 z^2 + \eta_3 a_3 z^3 + \cdots,$$

$$I_\lambda^{n+1} f(z) = z + \eta_2' a_2 z^2 + \eta_3' a_3 z^3 + \cdots.$$

where

$$\eta_j' = (1-\lambda)^{n+1}\left(1+\frac{j}{p}\right)^{n+1}$$

Now consider the combination:

$$(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z) = z + \left[(1-\alpha)\eta_2 + \alpha\eta_2'\right] a_2 z^2 + \left[(1-\alpha)\eta_3 + \alpha\eta_3'\right] a_3 z^3 + \cdots$$

Let us denote:

$$A_j = \left[(1-\alpha)\eta_j + \alpha\eta_j'\right], \quad \text{for} \quad j = 2,3.$$

$$A_2 = \left[(1-\alpha)\eta_2 + \alpha\eta_2'\right] = \left[(1-\alpha)(1-\lambda)^n\left(1+\frac{2}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1+\frac{2}{p}\right)^{n+1}\right]$$

Similarly,

$$A_3 = \left[(1-\alpha)\eta_3 + \alpha\eta_3{}'\right] = \left[(1-\alpha)(1-\lambda)^n\left(1+\frac{3}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1+\frac{3}{p}\right)^{n+1}\right]$$

So, $(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z) = z + A_2 a_2 z^2 + A_3 a_3 z^3 + \cdots$

From the argument condition, we deduce:

$$\frac{(1-\alpha)I_\lambda^n f(z) + \alpha I_\lambda^{n+1} f(z)}{z} = \left[\varphi(z)\right]^{(1-\beta)\gamma}$$

where $\varphi(z) = 1 + c_1 z + c_2 z^2 + \cdots$, with $\Re(\varphi(z)) > 0$ in $U$, so $\left|c_1\right| \le 2$ and $\left|c_2\right| \le 2$.

Then

Comparing coefficients, we get,

$$\left|a_2\right| \le \frac{2(1-\beta)\gamma}{A_2} \quad \text{and} \quad \left|a_3\right| \le \frac{2(1-\beta)\gamma\left(1+(1-\beta)\gamma\right)}{A_3}$$

Then by putting the values of $A_2$ and $A_3$ the final result will be,

$$\left|a_2\right| \le \frac{2(1-\beta)\gamma}{(1-\alpha)(1-\lambda)^n\left(1+\frac{2}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1+\frac{2}{p}\right)^{n+1}}$$

$$\left|a_3\right| \le \frac{2(1-\beta)\gamma(1+(1-\beta)\gamma}{(1-\alpha)(1-\lambda)^n\left(1+\frac{3}{p}\right)^n + \alpha(1-\lambda)^{n+1}\left(1+\frac{3}{p}\right)^{n+1}}$$

Hence the theorem proved for the class $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$. □

## 4. Conclusion

The paper focuses on formulating and studying two specific subclasses of bi-univalent analytic functions defined through the generalized Sălăgean operator $I_\lambda^n f(z)$. By imposing argument conditions on both the function and its inverse, we establish coefficient bounds on the leading Taylor coefficients of functions associated with the given subclasses $\mathcal{F}_\Sigma(\alpha,\gamma,\lambda,n,p)$ and $\mathcal{F}_\Sigma(\alpha,\beta,\gamma,\lambda,n,p)$. The methodology relied on converting the argument constraints into equivalent subordination relations involving Carathéodory functions with positive real part. This approach enabled us to compare the transformed operator expressions with power series expansions and obtain explicit bounds for $|a_2|$ and $|a_3|$. The derived estimates clearly illustrate how the parameters $\alpha$, $\beta$, $\gamma$, $\lambda$, $n$ and $p$ influence the growth and behavior of the coefficients.

Further, several special cases are analysed, leading to simplified bounds when particular parameters were fixed, demonstrating the flexibility and generality of the proposed subclasses. These results also provide a foundation for future research on higher-order coefficients, Fekete–Szegö inequalities and geometric properties associated with generalized differential operators.

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