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Forced oscillations of a plate consisting of ellipses and circles of various sizes with complex contours on machine parts

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Abstract

The article considers forced oscillations of elastic plates in the multilinked area. The surface of the plate undergoes actions of the disturbing force directed along the normal. The task leads to solution of different kinds of differential equations satisfying to initial and boundary conditions of various type. These equations are solved by the Bessel functions and through the method of the theory of functions with complex variables. Thus, the article offers the general methodology to the solutions of the dynamics of the elastic plates that is under action of the disturbing force.

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1. Introduction

The relevance of the topic is unquestionable, since the study of the dynamics of the viewed plate is of particular importance in many fields of modern technology, including in the development of space engines. Also, solving this problem in a partially general form allows for easy solutions for special

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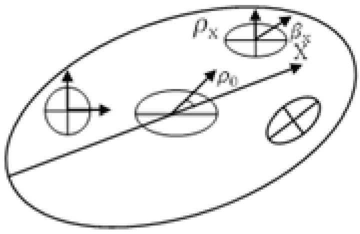

Figure 1. The multi-joint elliptical plate weakened by elliptical holes of different orientations

cases. On the other hand, the issue of finding the solution of the mathematical-physical equations of the complex contour of the region with various boundary conditions in multi-connected regions is always relevant. This research work can be useful for graduate and postgraduate students, teachers, researchers and engineers

The question of constructing a methodology for finding a solution satisfying various boundary conditions on the contours of the equation expressing the forced oscillations of the multi-joint elliptical plate weakened by elliptical holes of different orientations is considered. Assume that some of the internal contours are free of any influence and others are elastically fixed. The outer contour is immovably fixed.

Let's attach a polar coordinate system to each of the elliptic contours (ρ_k,θ_k) and to the outer contour (ρ,θ) and take the parallel and same direction of their polar axes. The equation of the forced oscillations of the plate in the polar coordinate system is as follows:

$$\Delta^2 W(\rho,\theta,t)+\lambda^4 \frac{\partial^2 W(\rho,\theta,t)}{\partial t^2}=\tilde{q}(\rho,\theta,t) \tag{1}$$

Here, W(ρ,θ,t) is the inclination of the plate from the starting plane; Δ - Laplace operator

$$\Delta=\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} \tag{2}$$

$\lambda^4=\dfrac{\mathrm{h}\gamma}{\mathrm{Dg}}$ - h-plate thickness, γ/g-mass density, $\tilde{q}(\rho,\theta,t)$- exciting force to the plane of the plate, t-time, D - is the cylindrical stiffness of the plate and

$$\mathrm{D}=\frac{\mathrm{E_1h^3}}{12(1-\nu^2)} \tag{3}$$

calculated by the formula. Here, $\boldsymbol{E_1}$ is the Young's modulus of the plate material, ν- Poisson's coefficient, the condition imposed on the function $\tilde{q}$(ρ,θ,t)- is that it can be divided into a series of functions during the specific oscillations of the plate. Elastic reinforcement of internal contours can be homogeneous, elliptical tubes with cross-section thickness h_1 fixed at the other end.

In this case, the reaction force

$$\mathrm{q_i}=-C_i\ddot{U}(\rho,\theta,t) \tag{4}$$

in the form of, where

$$C_i=\frac{1}{l_i}\pi\left[(a_i+b_i)h_i+h_i^2\right] i=\overline{1,m_2} \tag{5}$$

Here, $\boldsymbol{C_i}$ – ***i***- is the collapse coefficient of the i-th pipe, h_1- **i**- is the thickness of the ***i***-th support pipe, $\boldsymbol{m_2}$ is the number of elastic supports.

Let us denote the external contour L_0, and the internal contours free of load **– $\mathbf{L_r}$** il. Then a complex outline

$$L = L_0 + \sum_{k=1}^{m_1} L_r + \sum_{r=1}^{m_2} L_r \tag{6}$$

so that $m_1+m_2=m$; m- is the number of all internal contours.

Let us write the following boundary condition:

a) In fixed external contour

$$W(\rho,\theta,t)\big|_{L_0} = \frac{\partial W(\rho,\theta,t)}{\partial \rho}\bigg|_{t_n} = 0 \tag{7}$$

b) Since m**1** internal contours are free from external influence, the bending and twisting moments on them are equal to zero, i.e.

$$M_\rho\,|_{t_n} = -D\left[\frac{\partial^2 W(\rho,\theta,t)}{\partial \rho^2} + \nu\left(\frac{1}{\rho}\frac{\partial^2 W(\rho,\theta,t)}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 W(\rho,\theta,t)}{\partial \theta^2}\right)\right]_{t_n} = 0$$

$$\left(a_p - \frac{1}{\rho}\frac{\partial M_{\rho\theta}}{\partial \theta}\right) = -D\left[\begin{matrix}\frac{\partial^3 W(\rho,\theta,t)}{\partial \rho^3} + \frac{1}{\rho}\frac{\partial^2 W(\rho,\theta,t)}{\partial \rho^3} - \frac{1}{\rho^2}\frac{\partial W(\rho,\theta,t)}{\partial \rho} + (2-\nu)\frac{1}{\rho^2}\frac{\partial^3 W(\rho,\theta,t)}{\partial \rho \partial \theta^2} \\ -(3-\nu)\frac{1}{\rho^3}\frac{\partial^2 W(\rho,\theta,t)}{\partial \theta^2}\end{matrix}\right]_{t_n} = 0 \tag{8}$$

Here, Mρ is bending, Mρθ – twisting moments, Qp – shearing force.

c) The shear force on elastically fixed contours must balance with the reaction force. Given the rigid connection of the plate to the support, the angle between the support (in our example, the support tube) and the external normals of the plate surfaces changes. If transverse and torsional oscillations of the support are not taken into account, the following conditions must be satisfied on these contours:

$$\left(a_P - C_r W(\rho,\theta,t)\right)_{t_p} = D\left\{\begin{matrix}\left[\frac{\partial^3 W(\rho,\theta,t)}{\partial \rho^3} + \frac{1}{\rho}\frac{\partial^2 W(\rho,\theta,t)}{\partial \rho^2} + \frac{1}{\rho^2}\frac{\partial^3 W(\rho,\theta,t)}{\partial \rho \partial \theta^2} - \frac{2}{\rho^3}\frac{\partial^3 W(\rho,\theta,t)}{\partial \theta^2}\right]_{t_2} \\ -C_r W(\rho,\theta,t)\,|_{t_2}\end{matrix}\right\} = 0$$

$$\frac{\partial W(\rho,\theta,t)}{\partial \rho}\,\bigg|_{t_n} = 0 \tag{9}$$

Assume that at the beginning of the oscillation process, all points of the plate are not out of its plane, but the initial velocities are a function of their positions in the plane, i.e., V(ρ,θ)=0. In this case the initial conditions $W(\rho,\theta,0)=0$ and $\frac{\partial W(\rho,\theta,t)}{\partial t}\Big|_{t=0} = V(\rho,\theta)$ canere. The function V(ρ,θ) can also be sorted by eigenfunctions.

Thus, the problem of investigating forced oscillations of a plate with elliptical holes is brought to the problem of finding a solution that satisfies the boundary conditions (7), (8), (9) and initial conditions (10) in the multiple connection region of equation (6).

It should be noted that the goal is to provide a method for solving problems.

Finding custom dances. Arguments are not written for simplicity.

To study specific oscillations, we choose $\tilde{q}$(ρ,θ,0)=0, then (1) corresponding to the following figure

$$\Delta^2 \tilde{W} + \lambda_1^4 \frac{\partial^2 \tilde{W}}{\partial t^2} = 0 \tag{11}$$

Here, W˜- is an unknown function that characterizes the deviation of the plates from the plane state during the specific oscillation. W

$$\tilde{W} = W^*\left(\rho,\theta\right)_{sin}^{cos} n_0 t \tag{12}$$

If we look for it in the picture, we get from (11):

$$\Delta^2 W^* - \lambda_1^4 W^* = 0, \text{bele ki } \lambda_1^4 = \lambda^4 n_0^2 \tag{12}$$

(12) solution of the equation

$$W^* = \sum_{n=0}^{\infty} \bar{W}_n\left(\rho,\theta\right)_{sin}^{cos} n\theta \tag{13}$$

If we look for it, then from (11) we get:

$$\left(\frac{d^2}{d\rho^2} + \frac{1}{\rho}\frac{d}{dp} - \frac{n^2}{p^2}\right)^2 \bar{W}_n - \lambda_1 \bar{W}_n = 0 \tag{14}$$

Let E^0 be the unit differential operator. So $E^0 * \bar{W}_n = \bar{W}_n$. Then, by introducing the unit differentiation operator, equation (14) can be separated into two second order ordinary differential equations:

$$\frac{d^2\bar{W}_n}{dp^2} + \frac{1}{\rho}\frac{d\bar{W}_n}{d\rho} - \left(\frac{n^2}{\rho^2} + \lambda_1^2\right)\bar{W}_n = 0 \tag{15}$$

$$\frac{d^2\bar{W}_n}{dp^2} + \frac{1}{\rho}\frac{d\bar{W}_n}{d\rho} - \left(\frac{n^2}{\rho^2} + \lambda_1^2\right)\bar{W}_n = 0 \tag{16}$$

Let's substitute $\rho = \frac{\alpha}{\lambda_1}$. Then if we multiply (15) byλ_1^{-2} , and (16) by -λ_1^{-2} , we can get Bessel equations:

$$\frac{d^2\bar{W}_n(\alpha)}{d\alpha^2} + \frac{1}{\alpha}\frac{d\bar{W}_n(\alpha)}{d\alpha} - \left(1 + \frac{n^2}{\alpha^2}\right)\bar{W}_n(\alpha) = 0 \tag{17}$$

$$\frac{d^2\bar{W}_n(\alpha)}{d\alpha^2} + \frac{1}{\alpha}\frac{d\bar{W}_n(\alpha)}{d\alpha} - \left(1 - \frac{n^2}{\alpha^2}\right)\bar{W}_n(\alpha) = 0 \tag{18}$$

2. Preliminaries

It is known that each solution of equations (17) and (18) is also a solution of equation (14), and equations (17) and (18) have no common solutions that can satisfy both of them, except for the trivial solution. Therefore, the system of linearly independent fundamental solutions of equations (17) and (18) should be built on the basis of the system of fundamental solutions of equation (14). Equation (17) is modified by the following cylindrical function, the fundamental solutions of which are not linearly dependent on each other:

$$J_n\left(\lambda_1,\rho\right), N_n\left(\lambda_1,\rho\right), I_n\left(\lambda_1,\rho\right), K_n\left(\lambda_1,\rho\right) \tag{19}$$

Note that n must be an integer to satisfy the continuity condition. So, the function W(ρ,θ,t) should take its previous value when it completes the closed circuit. Let's formally look for the solution of equation (12) in the following way:

$$W^* = \sum_{n=0}^{\infty}\left\{\begin{aligned}&\left[A_n^{\pm}J_n(\lambda,\rho) + C_n^{\pm}I_n(\lambda,\rho)\right]_{\text{sin}}^{\text{cos}} n\theta + \sum_{k=1}^{m_1}\left[B_{nk}^{\pm}J_n(\lambda,\rho_k) + +D_{nk}^{\pm}K_n(\lambda,\rho_k)\right]_{\text{sin}}^{\text{cos}} n\theta_k \\ &+ \sum_{r=1}^{m_1}\left[B_{nr}^{\pm}J_n(\lambda,\rho_r) + D_{nr}^{\pm}K_n(\lambda,\rho_r)\right]_{\text{sin}}^{\text{cos}} n\theta_r\end{aligned}\right\} \tag{20}$$

The formality of equation (20) is explained by the fact that the expression written inside the first square brackets and its trigonometric multipliers are expressed in the (ρ,θ) coordinate system, while the remaining terms are expressed in the corresponding (ρ_k,θ_k) and (ρ_r,θ_r) polar coordinate systems. Therefore, the expression for W* must be written in a specific coordinate system.

We apply the theorem of graphical addition. According to this theorem:

$$z_1 - z_2 e^{-i\varphi} = re^{i\varphi} \qquad z_1 - z_2 e^{i\varphi} = re^{-i\varphi} \tag{21}$$

the following equation is true if the conditions are satisfied and

$$|Z_1| > |Z_2|$$

$$Z_\upsilon(r)e^{i\upsilon\psi} = \sum_{\rho=-\infty}^{m} Z_{\upsilon=p}(z_1)J_\rho(z_2)e^{ip\varphi} \tag{22}$$

So, here $Z_v(z) - J_\upsilon(z)$ and $N_\upsilon(z)$ can be any one of the functions. Z_1 and Z_2 are two different points of the complex plane. According to that theorem, if conditions (21) are met and Iz1I > Iz2I

$$I_\upsilon(r)e^{i\upsilon w} = \sum_{\rho=-\infty}^{\infty} (-1)^\rho I_{\upsilon+\rho}(z_1)I_\rho(z_2)e^{i\rho\varphi} \tag{23}$$

$$K_\upsilon(r)e^{i\upsilon w} = \sum_{\rho=-\infty}^{\infty} K_{\upsilon+p}(z_1)I_p(z_2)e^{i\rho\varphi} \tag{24}$$

In the matter under consideration

$$Z_1 = z_q + I_q \bar{\mathrm{k}} \mathrm{e}^{\varphi},\ \bar{\mathrm{k}} = \overline{2,\mathrm{m}},\ \mathrm{q} = \overline{2,\mathrm{m}}, \tag{25}$$

The distance between the centers of Iqk¯-k¯and the q-th contours is the positive angle of the vector *φ**qw*** - *ıqk¯*with respect to the polar axis of the kth system.

$$z_{\bar{k}} = \rho_{\bar{k}^{-e^\theta}},\ z_q = \rho_q e^{\theta q}$$

Let us replace $z_q = e^{-\theta}\omega_q$ in equation (25). Here θ_{qt} q-cu

The origin of the semi-axis of the ellipse is at the center of the outer ellipse located and polar axis with its major semi-axis it is the smallest angle formed by the system with the polar axis.

Sign

$$\theta_q = \begin{cases} +1, 0 < \theta_q < \dfrac{\pi}{2} \text{ when} \\ -1, -\dfrac{\pi}{2} < \theta_q < 0 \text{ when} \end{cases}$$

$$\omega_q = x_q + iy_q \text{ let it be } x_q \frac{a}{b}\bar{x}_a, y_a = \tilde{y}_q \tag{26}$$

let's replace it. Then the canonical equation of the ellipse in the q-th system

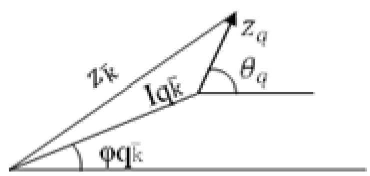

Figure 2. The polar axis of the k-th system

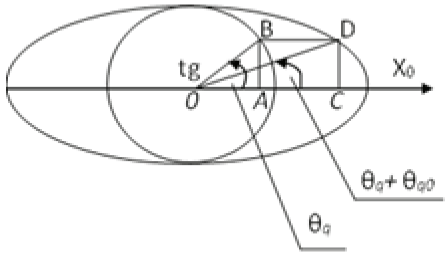

Figure 3. The semi-axis of the ellipse

$$\frac{x_q^2}{a_q^2}+\frac{y_q^2}{b_q^2}=1 \tag{27}$$

when (26) transforms (27) ellipse

$$\tilde{x}_q^2+\tilde{y}_q^2=b^2 \tag{28}$$

looks around.

$z_q=e^{-10_{q0}}\varpi_q$ the transformation Z_k, the complex plane, z_{-kq} is a spiral θ_{q_0} as much as the angle around the point, and then the post-rotation argument of $\boldsymbol{Z_q}$ is the argument of - ϖ_q, $\theta_q+\theta_{q_0}$ happens. t_q=$x_q n y_q$ let's say.

In the third figure, from the OCD triangl $\frac{CD}{OC}=TG\left(\theta_q+\theta_{q_0}\right)$

From here $CD=ODTG\left(\theta_q+\theta_{q_0}\right)=x_q tg\left(\theta_q+\theta_{q_0}\right)$

From the triangle OAB $\frac{AB}{OA}=tg\theta_q$

From here $AB=OAtg\theta_q=x_q tg\theta_q$. $AB=CD=y_q=y_a$ we can write because.

$$arc\ tg\theta_q=arc\ tg\left[\frac{Re\varpi_q}{Retq}tg\left(\theta_q+\theta_{q_0}\right)\right] \tag{29}$$

$$p_q=\left|t_q\right|=\sqrt{x_q^2+y_q^2},\quad \frac{Re\varpi_q}{Ret_q}=\frac{a_q}{b_q} \tag{30}$$

Thus, equation (25) can be written as

$$p_k-e^{i\theta_k}=p_q e^{i\theta_q}+lqke^{i\varphi qk}$$

If we multiply both sides of this equation $\lambda_1 e^{-i\varphi qk}$ by its expression, we get:

$$\lambda_1 p_k e^{i\left(\theta_k-\varphi_{qk}\right)}=\lambda_1 lq\bar{k}-\left(-\lambda p_q\right)e^{-i\left(\varphi_{qk}-\theta_q\right)}$$

Thus $lqk>p_q$, and υ=n expressions (21), (22) and (23) are written as follows:

$$z_n\left(\lambda,p_k\right)e^{in\left(\theta_k-\varphi_{qk}\right)}=\sum^{\infty}Z_{n+1}\left(\lambda,lqk\right)J_p\left(\lambda,p_q\right)e^{ip\left(\varphi_{qk}-\theta_q\right)} \tag{31}$$

$$I_n\left(\lambda,p_k\right)e^{in\left(\theta_k-\varphi_{qk}\right)}=\sum_{p=-\infty}^{\infty}\left(-1\right)^p I_{n+p}\left(\lambda,lqk\right)I_p\left(-\lambda,p_q\right)e^{ip\left(\varphi_{qk}-\theta_q\right)} \tag{32}$$

$$K_n\left(\lambda,p_k\right)e^{in\left(\theta_k-\varphi_{qk}\right)}=\sum_{p=-\infty}^{\infty}K_{n+p}\left(\lambda,lqk\right)I_p\left(-\lambda,p_q\right)e^{ip\left(\varphi_{qk}-\theta_q\right)} \tag{33}$$

Multiplying $e^{in\varphi_{qk}}$ these equalities, we get:

$$Z_n\left(\lambda, p_k\right)_{sin}^{cos} n\theta k=\sum_{p=-\infty}^{\infty}(-1)^p Z_{n+p}(\lambda, lqk) J_p\left(\lambda, p_q\right)_{sin}^{cos}\left[(n+p)\varphi_{qk}-p\theta_q\right] \tag{34}$$

$$Z_n\left(\lambda, p_k\right)_{sin}^{cos} n\theta k=\sum_{p=-\infty}^{\infty}(-1)^p Z_{n+p}(\lambda, lqk) J_p\left(\lambda, p_q\right)_{sin}^{cos}\left[(n+p)\varphi_{qk}-p\theta_q\right] \tag{35}$$

$$K_n\left(\lambda, p_k\right)_{sin}^{cos} n\theta k=\sum_{p=-\infty}^{\infty}(-1)^p K_{n+p}(\lambda, lqk) I_p\left(\lambda, p_q\right)_{sin}^{cos}\left[(n+p)\varphi_{qk}-p\theta_q\right] \tag{36}$$

When we move p,θ_q from one system $\rho\bar{\theta}_k$ to another, we act in the same way. Therefore, to obtain the results of this transition, in relations θk to θ_q, lqk to l_{kq} (34), (35) and (36),

It is enough to replace θ˜q with θ˜k, ρ˜q with ρ˜k, φqk with φbk. Then we get the following relations:

$$Z_a(\lambda,\rho_q)_{sin}^{cos} n\theta_q = \sum_{\rho=-\infty}^{\infty}(-1)^p Z_{n+p}(\lambda, I_{kq}) J_p(\lambda,\rho_k)_{sin}^{cos}\left[(n+p)\varphi_{kq}-\rho\bar{\theta}_k\right] \tag{37}$$

$$I_n(\lambda,\rho_q)_{sin}^{cos} n\theta_q = \sum_{\rho=-\infty}^{\infty} I_{n+p}(\lambda, I_{kq}) J_p(\lambda,\rho_k)_{sin}^{cos}\left[(n+p)\varphi_{kq}-\rho\bar{\theta}_k\right] \tag{38}$$

$$K_n(\lambda,\rho_q)_{sin}^{cos} n\theta_q = \sum_{\rho=-\infty}^{\infty}(-1)^p K_{n+p}(\lambda, I_{kq}) J_p(\lambda,\rho_k)_{sin}^{cos}\left[(n+p)\varphi_{kq}-\rho\bar{\theta}_k\right] \tag{39}$$

To change from the (ρ,θ) system to the (ρ_q,θ_q) or (ρ_k,θ_k) system, we write (20) as follows:

$$Z = z_q + I_k e^{\varphi} \tag{40}$$

We perform transformations (25)-(30) on Z_k. Then the equation

$$\lambda, \rho e^{m(\theta-\varphi)} = \lambda, I_{kq} - \left(-\lambda, \tilde{\rho}_k\right) e^{-1(\theta q)-}\theta k \tag{41}$$

falls into the form of

Thus, according to the Graph theorem, we can accept v=n and write when $\lambda k > \tilde{\rho} k$.

$$Z_a(\lambda,\rho_q) e^{in(\theta-\varphi)} = \sum_{\rho=-\infty}^{\infty}(-1)^p Z_{n+p}(\lambda, I_{kq}) J_p(\lambda,\rho_k)^{n(\theta-\varphi)} \tag{42}$$

If we multiply each side of this equation by $e^{in\varphi}$, (42) will look like this:

$$Z_a(\lambda,\rho_q)_{sin}^{cos} n\theta = \sum_{\rho=-\infty}^{\infty}(-1)^p Z_{n+p}(\lambda, I_{kq}) J_p(\lambda,\rho_k)_{sin}^{cos}\left[(n+p)\varphi_{kq}-\rho\bar{\theta}_k\right] \tag{43}$$

If we perform a similar transformation for the functions I_n and K_n

$$I_n(\lambda,\rho_q)_{sin}^{cos} n\theta_q = \sum_{\rho=-\infty}^{\infty} I_{n+p}(\lambda, I_{kq} J_p(\lambda,\rho_k)_{sin}^{cos}[(n+p)\varphi_{kq}-\rho\bar{\theta}_k \tag{44}$$

$$K_n(\lambda,\rho_q)_{sin}^{cos} n\theta = \sum_{\rho=-\infty}^{\infty}(-1)^n K_{n+p}\left(\lambda, I_{kq}\right) J_p(\lambda,\rho_k)_{sin}^{cos}\left[(n+p)\varphi_{kq}-\rho\bar{\theta}_k\right] \tag{45}$$

we get the relationships.

To switch from (ρ, θ) system to (ρ_q, θ_q) system, it is enough to replace <<k>> index with <<q>> index in relations (43), (44) and (45).

When switching from (ρ_k, θ_k) or (ρ_q,θ_q) system to (ρ,θ) system, two cases are considered:

Case I – the center of the outer contour has a central elliptical hole that coincides with the center, but is not similar to it, that is, the major semi-axes are not equal to the minor semi-axes, or the major semi-axis does not coincide with the major semi-axis of the outer ellipse.

Case II – The elliptical hole considered in case I is not present.

Let's look at the transitive relations for each of these cases.

1-In the third case, to move from the ($\rho_{\mathbf{k}}$, $\theta_{\mathbf{k}}$) system to the (ρ, θ) system, we write (20) as follows:

$$Z_k = z + I_{ka} e^{ifg} \tag{46}$$

we substitute in the $z = e^{-i\theta q_0}$ equation. After that, we subject the W complex plane along the polar axis $\frac{a_r}{b_r}$ to - coefficient compression.$z = \tilde{\rho}_r e^{-i(\theta_{ro} - \theta_o)}$ then it becomes and (46) equation

$$p_k e^{i\theta_k} = p_r^{-i(\theta_{r0} - \theta_2)} + l_{k0} e^{i\varphi_{k0}} \text{ or}$$

$$\lambda_1 \rho_k e^{(\theta_k - \theta_k)} = \lambda_1 l_{\overline{r}o} - \left(-\lambda_1 \overline{\rho}_r\right) e^{-i\left(\theta_{R\varphi} + \theta_{i\varphi} - \overline{\theta}_r\right)} \tag{47}$$

falls into the form of.

Applying the Graph theorem to (47), we get:

$$Z_m(\lambda_1 \overline{\rho}_r) e^{m(\theta - \varphi_{:0})} = \sum_{\rho=-\infty}^{\infty} (-1)^{\rho} Z_{n+p}\left(\lambda_1 \overline{\rho}_{k0}\right) J_{\rho}(\lambda_1 \tilde{\rho}_r) e^{\rho\left(\theta_{R\varphi} + \theta_{i\varphi} - \overline{\theta}_r\right)} \tag{48}$$

We multiply this equation by $e^{mp_{11}}$ and get:

$$(\lambda_1 \overline{\rho}_{\overline{k}})_{sin}^{cos} n\theta_{\overline{k}} = \sum_{p=-\infty}^{\infty} (-1)^p Z_{n+p}\left(\lambda_1 l_{\overline{k}0}\right) J_{\rho}(\lambda_1 \tilde{\rho}_r)_{sin}^{cos}\left[\left(n+p\right)\varphi_{\overline{k}} - p\theta_{r0} - p\tilde{\theta}_r\right]$$

$$I_n(\lambda_1 \overline{\rho}_{\overline{k}})_{sin}^{cos} n\theta_{\overline{k}} = \sum_{p=-\infty}^{\infty} I_{n+p}\left(\lambda_1 l_{\overline{k}0}\right) I_{\rho}(\lambda_1 \tilde{\rho}_r)_{sin}^{cos}\left[\left(n+p\right)\varphi_{\overline{k}0} + p\theta_{r0} - p\tilde{\theta}_r\right] \tag{49}$$

$$K_n(\lambda_1 \overline{\rho}_{\overline{k}})_{sin}^{cos} n\theta_{\overline{k}} = \sum_{p=-\infty}^{\infty} (-1)^p K_{n+p}\left(\lambda_1 l_{\overline{k}0}\right) I_{\rho}(\lambda_1 \tilde{\rho}_r)_{sin}^{cos}\left[\left(n+p\right)\varphi_{\overline{k}0} + p\theta_{r0} - p\tilde{\theta}_r\right] \tag{50}$$

we also get their relations.

When the center coincides with the semi-major axis of the elliptical hole, θk –0=0 is taken in equations (48)-(50).

Let's again use the relationship $z_i = z + l_{\overline{k}} e^{i\varphi}$ to switch to the (ρ,θ) system connected to the external contour. In this case, since the polar axis coincides with the semi-major axis of the outer ellipse, we subject the transformation $z = e^{-i\varphi}$ to compression along the axis. Then $\rho_{\overline{k}} e^{i\theta_1} = \tilde{\rho}_r^{-i\theta_1} + l_{k1} e^{i\varphi}$ and as a result

$$\lambda_1 \rho_k e^{i(\theta_k - \varphi_{k3})} = \lambda_1 \rho_{\overline{k}_1} - \left(-\lambda_1 \overline{\rho}_k\right) e^{-i(\varphi_{k3} - \theta_1)}$$

is taken. Then the expressions (48), (49) and (50) for the system related to the outer contour are as follows:

$$Z_n(\lambda_1 \overline{\rho}_{\overline{k}})_{sin}^{cos} n\theta_{\overline{k}} = \sum_{p=-\infty}^{\infty} (-1)^p Z_{n+p}\left(\lambda_1 l_{\overline{k}0}\right) J_{\rho}(\lambda_1 \tilde{\rho}_r)_{sin}^{cos}\left[\left(n+p\right)\varphi_{\overline{k}} - p\theta_{r0} - p\tilde{\theta}_r\right] \tag{51}$$

$$I_n(\lambda_1 \overline{\rho}_{\overline{k}})_{sin}^{cos} n\theta_{\overline{k}} = \sum_{p=-\infty}^{\infty} I_{n+p}\left(\lambda_1 l_{\overline{k}0}\right) I_{\rho}(\lambda_1 \tilde{\rho}_r)_{sin}^{cos}\left[\left(n+p\right)\varphi_{\overline{k}0} + p\theta_{r0} - p\tilde{\theta}_r\right] \tag{52}$$

$$K_n(\lambda_1\bar{\rho}_{\bar{k}})^{cos}_{sin} n\theta_{\bar{k}} = \sum_{p=-\infty}^{\infty} (-1)^p K_{n+p}\left(\lambda_1 l_{\bar{k}0}\right) I_\rho(\lambda_1\tilde{\rho}_r)^{cos}_{sin}\left[(n+p)\varphi_{\bar{k}0} + p\theta_{r0} - p\tilde{\theta}_r\right] \quad (53)$$

When $\tilde{\rho}_k > l_{i1}$ these relations are as follows:

$$Z_n(\lambda_1\bar{\rho}_{\bar{k}})^{cos}_{sin} n\theta_{\bar{k}} = \sum_{p=-\infty}^{\infty} (-1)^p Z_{n+p}\left(\lambda_1 l_{\bar{k}0}\right) J_\rho(\lambda_1\tilde{\rho}_r)^{cos}_{sin}\left[(n+p)\theta_{\bar{k}} - p\theta_{r1} - p\tilde{\theta}_{k1}\right] \quad (54)$$

$$I_n(\lambda_1\bar{\rho}_{\bar{k}})^{cos}_{sin} n\theta_{\bar{k}} = \sum_{p=-\infty}^{\infty} I_{n+p}\left(\lambda_1 l_{\bar{k}0}\right) I_\rho(\lambda_1\tilde{\rho}_r)^{cos}_{sin}\left[(n+p)\varphi_{\bar{k}0} + p\theta_{r1} - p\tilde{\theta}_{k1}\right] \quad (55)$$

$$K_n(\lambda_1\bar{\rho}_{\bar{k}})^{cos}_{sin} n\theta_{\bar{k}} = \sum_{p=-\infty}^{\infty} (-1)^p K_{n+p}\left(\lambda_1 l_{\bar{k}0}\right) I_\rho(\lambda_1\tilde{\rho}_r)^{cos}_{sin}\left[(n+p)\theta_{\bar{k}0} + p\theta_{k1} - p\tilde{\theta}_r\right] \quad (56)$$

let's assume that the first of the double indices indicates the system to which the expression was imported, and the second indicates the system from which it was imported.

Note that **l**Q_k=**l**k_q h_{es}.

Let's put the expression we received into a more convenient form. First, let's denote the coefficients B^2_{nk} and D^2_{nk} as B^2_n and D^2_n for the case where k=1 in the expression of the eigenfunction in the system (ρ**1**,θ**2**) and include them in the first infinite sum and **k** from 2 to **m** combine the second and third sums by changing.

$$W = \sum_{n=1}^{\infty}\left\{\begin{aligned} &\left[A^2_n J_n\left(\lambda,\tilde{\rho}_1\right) + C^2_n I_n\left(\lambda,\tilde{\rho}_1\right) + B^2_n N_n\left(\lambda,\widetilde{\rho_1}\right) + D^{\pm}_{nk} K_n\right]^{cos}_{sin} n\tilde{\theta} \\ &\sum_{k=2}^{m}\left[B^2_{nk}\sum_{p=-\infty}^{\infty}(-1)^p N_{n+p}\left(\lambda,l_{k1}\right) J_p\left(\lambda,\tilde{\rho}_1\right)\right]^{cos}_{sin}\left[(n+p)\varphi_{\bar{k}1} - p\tilde{\theta}_1\right] \\ &+ D^{\pm}_{nk}\sum_{p=-\infty}^{\infty}(-1)^p\left[K_{n+p}\left(\lambda,l_{k1}\right) J_p\left(\lambda,\tilde{\rho}_1\right)\right]^{cos}_{sin}\left[(n+p)\varphi_{\bar{k}1} - p\tilde{\theta}\right] \end{aligned}\right\} \quad (57)$$

Let us change the sums taken in the integral (-∞,∞) and take into account the following property of cylindrical functions:

$$\sum_{p=-\infty}^{\infty} J_{-v-p}\left(z_1\right) J_{-p}\left(z_2\right) \quad (58)$$

$$\sum_{p=-\infty}^{\infty} (-1)^p N_{n+p}\left(\lambda,l_{k1}\right) J_p\left(\lambda,\tilde{\rho}_1\right)^{cos}_{sin}\left[(n+p)\varphi_{\bar{k}1} p\tilde{\theta}_1\right]$$

$$\sum_{p=0}^{\infty}(-1)^p N_{n+p}\left(\left(\lambda,l_{k1}\right) J_p\left(\lambda,\tilde{\rho}_1\right)^{cos}_{sin}\right)\left[(n+p)\varphi_{\bar{k}1} - p\tilde{\theta}_1\right]$$

$$\sum_{p=-\infty}^{\infty} (-1)^p N_{n+p}\left(\lambda,l_{k1}\right) J_p\left(\lambda,\tilde{\rho}_1\right)^{cos}_{sin}\left[(n+p)\varphi_{\bar{k}1} - p\tilde{\theta}_1\right]$$

$$\sum_{p=0}^{\infty}(-1)^p N_{n+p}\left(\left(\lambda,l_{k1}\right) J_p\left(\lambda,\tilde{\rho}_1\right)^{cos}_{sin}\right)\left[(n+p)\varphi_{\bar{k}1} - p\tilde{\theta}_1\right]$$

$$=(-1)^p N_{n-p}(\lambda, l_{k1}) J_p(\lambda, \tilde{\rho}_1)_{sin}^{cos}\left[(n-p)\varphi_{\bar{k}1} + p\tilde{\theta}_1\right]$$

If we open and regroup the brackets included in the trigonometric functions, we get:

$$\sum_{p=-\infty}^{0} (-1)^p \varepsilon_n N_{n+p}(\lambda, l_{k1}) J_p(\lambda, \tilde{\rho}_1)_{sin}^{cos}\left[(n+p)\varphi_{\bar{k}1} - p\tilde{\theta}_1\right]$$

Here

$$N_{1\ pn}^1 = N_{n+p}\cos(n+p)\varphi_{\bar{k}1} + (-1)^p N_{n-p}\cos(n-p)\varphi_{\bar{k}1}$$

$$N_{2\ pn}^1 = -N_{n+p}\sin(n+p)\varphi_{\bar{k}1} + (-1)^p \frac{N_{3\ pn}^1}{N_{2\ pn}^1}\sin p\tilde{\theta}_1 N_{n-p}\sin(n-p)\varphi_{\bar{k}1}$$

$$N_{3\ pn}^1 = -N_{n+p}\cos(n+p)\varphi_{\bar{k}1} + (-1)^p N_{n-p}\cos(n-p)\varphi_{\bar{k}1} \tag{59}$$

$$N_{4\ pn}^1 = N_{n+p}\sin(n+p)\varphi_{\bar{k}1} + (-1)^p N_{n-p}\sin(n-p)\varphi_{\bar{k}1}$$

marked. We will buy in a similar way

$$\sum_{p=-\infty}^{0} (-1)^p K_{n+p}(\lambda, l_{k1}) I_p(\lambda, \tilde{\rho}_1)_{sin}^{cos}\left[(n+p)\varphi_{\bar{k}1} - p\tilde{\theta}_1\right]$$

$$\sum_{p=0} (-1)^n \varepsilon_n J_p(\lambda, \tilde{\rho}_1)_{sin}^{cos}\left(\frac{K_{1\ pn}^1}{K_{4\ pn}^1}\cos p\tilde{\theta}_1 \pm \frac{K_{3\ pn}^1}{K_{2\ pn}^1}\sin p\tilde{\theta}_1\right) \tag{60}$$

and here

$$K_{1\ pn}^1 = K_{n+p}\cos(n+p)\varphi_{\bar{k}1} + (-1)^p K_{n-p}\cos(n-p)\varphi_{\bar{k}1}$$

$$K_{2\ pn}^1 = -K_{n+p}\sin(n+p)\varphi_{\bar{k}1} + (-1)^p K_{n-p}\sin(n-p)\varphi_{\bar{k}1}$$

$$K_{3\ pn}^1 = -K_{n+p}\cos(n+p)\varphi_{\bar{k}1} + (-1)^p K_{n-p}\cos(n-p)\varphi_{\bar{k}1} \tag{61}$$

$$K_{4\ pn}^1 = K_{n+p}\sin(n+p)\varphi_{\bar{k}1} + (-1)^p K_{n-p}\sin(n-p)\varphi_{\bar{k}1}$$

Thus, the expression of the characteristic function for the case $\tilde{\rho}_1 < l_{k1}$ is:

$$W^* = \sum_{n=0}^{\infty}\Bigg\{\left[A_n^{\pm}J_n(\lambda, \tilde{\rho}_1) + C_n^{\pm}I_n(\lambda, \tilde{\rho}_1) + B_n^{\pm}N_n(\lambda, \tilde{\rho}_1) + D_n^{\pm}K_n(\lambda, \tilde{\rho}_1)\right]_{sin}^{cos} n\theta$$
$$+\sum_{k=2}^{n}\sum_{p=0}^{0}(-1)^n \varepsilon_n \left[\begin{array}{l} B_{pk}^{\pm}J_p(\lambda, \tilde{\rho}_1)\left(\dfrac{N_{1\ pn}^1}{N_{4\ pn}^1}\cos p\tilde{\theta}_1 \pm \dfrac{N_{1\ pn}^1}{N_{4\ pn}^1}\cos p\tilde{\theta}_1\right) \\ \quad + D_{pk}^{\pm}J_1(\lambda, \tilde{\rho}_1)\left(\dfrac{K_{1\ pn}^1}{K_{4\ pn}^1}\cos p\tilde{\theta}_1 \pm \dfrac{K_{3\ pn}^1}{K_{2\ pn}^1}\sin p\tilde{\theta}_1\right)\end{array}\right]\Bigg\} \tag{62}$$

And when $\tilde{\rho}_1 > l_{k1}$

$$W^* = \sum_{n=0}^{\infty} \left\{ \left[A_n^{\pm} J_n\left(\lambda, \tilde{\rho}_1\right) + C_n^{\pm} I_n\left(\lambda, \tilde{\rho}_1\right) + B_n^{\pm} N_n\left(\lambda, \tilde{\rho}_1\right) + D_n^{\pm} K_n\left(\lambda, \tilde{\rho}_1\right) \right]_{sin}^{cos} n\tilde{\theta} \right.$$
$$\left. + \sum_{k=2}^{n} \sum_{p=0}^{0} (-1)^n \varepsilon_n \left[\begin{aligned} & B_{pk}^{\pm} N_p\left(\lambda, \tilde{\rho}_1\right) \left(\frac{J_{1\ pn}^1}{J_{4\ pn}^1} \cos p\tilde{\theta}_1 \pm \frac{J_{1\ pn}^1}{J_{4\ pn}^1} \sin p\tilde{\theta}_1 \right) \\ & + D_{pk}^{\pm} J_1\left(\lambda, \tilde{\rho}_1\right) \left(\frac{I_{1\ pn}^1}{I_{4\ pn}^1} \cos p\tilde{\theta}_1 \pm \frac{I_{3\ pn}^1}{I_{2\ pn}^1} \sin p\tilde{\theta}_1 \right) \end{aligned} \right] \right\} \tag{63}$$

in expressions (61), (62) $\bar{k} = 2,3,\ldots m$; and ε_n is the Neumann factor.

$$\varepsilon_n = \begin{Bmatrix} 0,5 & n = 0 & when \\ 1 & n \neq 0 & when \end{Bmatrix}$$

In order to obtain the expression of the eigenfunction in the system connected to the central hole whose contour is an ellipse, the expression (n+p)φ$\bar{\mathbf{k}}$**1** in the expressions (59) and (60) is changed to (n+p) φ$\bar{\mathbf{k}}$**1**+p$\tilde{\theta}$r0, and $\tilde{\theta}$1 to θr need to be replaced. Then, taking into account the appropriate changes, the expression (61) is considered to be $\tilde{\rho}$**1**> l**k1** on the contour of the central hole, and $\tilde{\rho}$**1**>0 on the contour of the ellipse in the center, since l**k1**=0.

By means of similar mathematical operations, we can show that (pq, θq) (as well as the expression of the eigenfunction in the (pk, θk) system ($\tilde{\rho}_q < l_{qr}, \tilde{\rho}_q < l_{qk}$) is as follows:

$$W^* = \sum_{n=0}^{\infty} \left\{ \begin{aligned} & \left[B_n^{\pm} N_n\left(\lambda, \tilde{\rho}_1\right) + D_n^{\pm} K_n\left(\lambda, \tilde{\rho}_1\right) \right]_{sin}^{cos} n\tilde{\theta} + \sum_{p=0}^{\infty} (-1)^n \varepsilon_n J_p\left(\lambda, \tilde{\rho}_q\right) \\ & A_p^{\pm} \left(\frac{J_{1\ pn}^1}{J_{4\ pn}^1} \cos p\tilde{\theta}_q \pm \frac{J_{1\ pn}^1}{J_{4\ pn}^1} \sin p\tilde{\theta}_q \right) + B_n^{\pm} \left(\frac{N_{1\ pn}^1}{N_{4\ pn}^1} \cos p\tilde{\theta}_q \pm \frac{N_{1\ pn}^1}{N_{4\ pn}^1} \sin p\tilde{\theta}_q \right) + I_n\left(\lambda, \tilde{\rho}_q\right) \\ & \left[\left(\frac{J_{1\ pn}^1}{J_{4\ pn}^1} \cos p\tilde{\theta}_q \pm \frac{J_{1\ pn}^1}{J_{4\ pn}^1} \sin p\tilde{\theta}_q \right) C_p^{\pm} + D_{pk}^{\pm} \left(\frac{K_{1\ pn}^1}{K_{4\ pn}^1} \cos p\tilde{\theta}_q \pm \frac{K_{3\ pn}^1}{K_{2\ pn}^1} \sin p\tilde{\theta}_q \right) \right] \\ & + \sum_{k=2}^{\infty} J_n^{\pm}\left(\lambda, \tilde{\rho}_q\right) B_n^{\pm} \left(\frac{N_{1\ pn}^1}{N_{4\ pn}^1} \cos p\tilde{\theta}_q \pm \frac{N_{1\ pn}^1}{N_{4\ pn}^1} \sin p\tilde{\theta}_q \right) \\ & + D_{pk}^{\pm} I_n\left(\lambda, \tilde{\rho}_q\right) \left(\frac{K_{1\ pn}^1}{K_{4\ pn}^1} \cos p\tilde{\theta}_q \pm \frac{K_{3\ pn}^1}{K_{2\ pn}^1} \sin p\tilde{\theta}_q \right) \end{aligned} \right\} \tag{64}$$

The expression (63) is used only to satisfy the rigidities on the internal contours. Therefore, we do not formulate the expression of the eigenfunction for the cases $\tilde{\rho}_q > l_{qk}$ and $\tilde{\rho}_q > l_{qk}$.

Note that the following signs are used in (63).

$$\frac{Z_{1\,pk}^{0\ k}}{Z_{4\,pk}^{3\ k}} = Z_{p+n}\left(\lambda, lq\bar{k}\right)\cos\left(n+p\right)\varphi_{q\bar{k}} \pm (-1)^n Z_{-n}\left(\lambda, lq\bar{k}\right)\cos\left(p-n\right)\varphi_{q\bar{k}} \tag{65}$$

$$\frac{Z_{3\,pk}^{4\ k}}{Z_{3\,pk}^{2\ k}} = Z_{p+n}\left(\lambda, lq\bar{k}\right)\sin\left(n+p\right)\varphi_{q\bar{k}} \pm (-1)^n Z_{-n}\left(\lambda, lq\bar{k}\right)\sin\left(p-n\right)\varphi_{q\bar{k}} \tag{66}$$

$$\frac{Z_{3\,pk}^{4\ k}}{Z_{3\,pk}^{2\ k}} = (-1)^{\varphi} \left[I_{p+n}\left(\lambda, lq\bar{k}\right)_{sin}^{cos} \left(p+n\right)\varphi_{q\bar{k}} \pm I_{p-n}\left(\lambda, lq\bar{k}\right)_{sin}^{cos} \left(p-n\right)\varphi_{q\bar{k}} \right] \tag{67}$$

$$\frac{Z_{3\,pk}^{4\;k}}{Z_{2\,pk}^{0\;k}}=\left(-1\right)^{\varphi}\left[I_{p+n}\left(\lambda,lq\bar{k}\right)_{sin}^{cos}\left(p+n\right)\varphi_{q\bar{k}}\pm I_{p-n}\left(\lambda,lq\bar{k}\right)_{sin}^{cos}\left(p-n\right)\varphi_{q\bar{k}}\right] \tag{68}$$

$$ip=\begin{cases} p & q=0,1 \quad when \\ n & q>2 \quad when \end{cases}$$

$$\frac{K_{1\,pk}^{0\;k}}{Z_{4\,pk}^{3\;k}}=K_{p+n}\left(\lambda,lq\bar{k}\right)_{sin}^{cos}\left(n+p\right)\varphi_{q\bar{k}}\pm K_{p-n}\left(\lambda,lq\bar{k}\right)_{sin}^{cos}\cos\left(p-n\right)\varphi_{q\bar{k}} \tag{69}$$

$$\frac{K_{1\,pk}^{4\;k}}{Z_{4\,pk}^{3\;k}}=K_{p+n}\left(\lambda,lq\bar{k}\right)_{sin}^{cos}\left(n+p\right)\varphi_{q\bar{k}}\pm K_{p-n}\left(\lambda,lq\bar{k}\right)_{sin}^{cos}\cos\left(p-n\right)\varphi_{q\bar{k}} \tag{70}$$

$$\begin{aligned}&A_n^+J_n\left(\lambda,b_1\right)+C_n^+I_n\left(\lambda,b_1\right)+B_n^+N_n\left(\lambda,b_1\right)+D_n^+K_n\left(\lambda,b_1\right)\\&\quad+\sum_{k=1}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+N_n\left(\lambda,b_1\right)J_{1pn}^{1,k}+D_{p\bar{k}}^+K_n\left(\lambda,b_1\right)J_{1pn}^{1,k}\right]=0\end{aligned} \tag{71}$$

$$\sum_{k=2}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+N_n\left(\lambda,b_1\right)J_{3pn}^{1,k}+D_{p\bar{k}}^+K_n\left(\lambda,b_1\right)J_{3pn}^{1,k}\right]=0 \tag{72}$$

$$\begin{aligned}&A_n^+J_n\left(\lambda,b_1\right)+C_n^+I_n\left(\lambda,b_1\right)+B_n^+N_n\left(\lambda,b_1\right)+D_n^+K_n\left(\lambda,b_1\right)\\&\quad+\sum_{k=2}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+N_n\left(\lambda,b_1\right)J_{1pn}^{1,k}+D_{p\bar{k}}^+K_n\left(\lambda,b_1\right)J_{1pn}^{1,k}\right]=0\end{aligned} \tag{73}$$

$$\sum_{k=2}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+N_n\left(\lambda,b_1\right)J_{3pn}^{1,k}+D_{p\bar{k}}^+K_n\left(\lambda,b_1\right)J_{3pn}^{1,k}\right]=0 \tag{74}$$

The central ellipse is considered load-free. Therefore, we get the following equations on that contour $\tilde{\rho}_2=b_2$:

$$\begin{aligned}&A_n^+J_n\left(\lambda,b_2\right)+C_n^+I_n\left(\lambda,b_2\right)+B_n^+N_n\left(\lambda,b_2\right)+D_n^+K_n\left(\lambda,b_2\right)\\&\quad+\sum_{k=2}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+N_n\left(\lambda,b_2\right)J_{1pn}^{2,k}+D_{p\bar{k}}^+K_n\left(\lambda,b_1\right)J_{1pn}^{2,k}\right]=0\end{aligned} \tag{75}$$

$$\sum_{k=2}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+N_n\left(\lambda,b_2\right)J_{3pn}^{2,k}+D_{p\bar{k}}^+K_n\left(\lambda,b_1\right)J_{3pn}^{2,k}\right]=0 \tag{76}$$

$$\begin{aligned}&A_n^+M_1\left[J_n\left(\lambda,b_2\right)\right]+C_n^+M_1\left[I_n\left(\lambda,b_2\right)\right]+B_n^+M_1\left[N_n\left(\lambda,b_2\right)\right]+D_n^+M_1\left[K_n\left(\lambda,b_2\right)\right]\\&+\sum_{k=2}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+M_1\left[J_n\left(\lambda,b_2\right)\right]N_{3pn}^{2,k}+D_{p\bar{k}}^+M_1\left[I_n\left(\lambda,b_2\right)\right]K_{3pn}^{2,k}\right]=0\end{aligned} \tag{77}$$

$$\sum_{k=2}^{m}\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[B_{p\bar{k}}^+M_1\left[J_n\left(\lambda,b_2\right)\right]N_{3pn}^{2,k}+D_{p\bar{k}}^+M_1\left[I_n\left(\lambda,b_2\right)\right]K_{3pn}^{2,k}\right]=0 \tag{78}$$

Boundary conditions on elastically fixed contours ($\tilde{\rho}_q=b_r$) are expressed by the following equations:

$$B_{p\bar{k}}^{+}N_n\left(\lambda,b_2\right)+D_{p\bar{k}}^{+}K_n\left(\lambda,b_1\right)+\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\begin{bmatrix}A_p^{+}J_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+C_p^{+}I_n\left(\lambda,b_1\right)J_{1pn}^{q,1}\\+B_n^{+}J_n\left(\lambda,b_1\right)N_{3pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{1pn}^{q,1}\end{bmatrix}$$
$$+\sum_{r=1}^{m}\sum_{p=0}^{m}(-1)^n\varepsilon_n\left[B_n^{+}J_n\left(\lambda,b_1\right)N_{1pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{3pn}^{q,1}\right]=0 \tag{79}$$

$$\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[A_p^{+}J_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+C_p^{+}I_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+B_n^{+}J_n\left(\lambda,b_1\right)N_{3pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{1pn}^{q,1}\right]$$
$$+\sum_{r=1}^{m}\sum_{p=0}^{m}(-1)^n\varepsilon_n\left[B_n^{+}J_n\left(\lambda,b_1\right)N_{1pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{3pn}^{q,1}\right]=0 \tag{80}$$

$$A_n^{+}M_1\left[N_n\left(\lambda,b_r\right)\right]+D_n^{+}M_1\left[K_n\left(\lambda,b_r\right)\right]+\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\begin{bmatrix}A_p^{+}J_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+C_p^{+}I_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+\\+B_n^{+}J_n\left(\lambda,b_1\right)N_{3pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{1pn}^{q,1}\end{bmatrix}$$
$$+\sum_{r=1}^{m}\sum_{p=0}^{m}(-1)^n\varepsilon_n\left[B_n^{+}J_n\left(\lambda,b_1\right)N_{1pn}^{q,1}++D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{3pn}^{q,1}\right]=0 \tag{81}$$

$$\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[A_p^{+}J_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+C_p^{+}I_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+B_n^{+}J_n\left(\lambda,b_1\right)N_{3pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{1pn}^{q,1}\right]$$
$$+\sum_{r=1}^{m}\sum_{p=0}^{m}(-1)^n\varepsilon_n\left[B_n^{+}J_n\left(\lambda,b_1\right)N_{1pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{3pn}^{q,1}\right]=0 \tag{82}$$

Here

$$M_1\left[\widetilde{z_n}\left(\lambda,b_r\right)\right]=\lambda_1^3 z_n\left(\lambda,b_r\right)+\frac{\lambda_1^2}{b_r}\tilde{z}\left(\lambda,b_r\right)-\frac{\lambda_1}{b_r^2}\left(1-\mathrm{n}^2\right)\tilde{z}\left(\lambda,b_r\right)+\left(\frac{2n^2}{b_r^3}-\frac{C_r}{D}\right)\tilde{z}_n\left(\lambda,b_r\right) \tag{83}$$

Thus, z˜ (λ,b_r) can be any one of the four cylindrical functions. On internal contours free of load, we get:

$$B_{nq}^{+}M_r\left[N_n\left(\lambda,b_r\right)\right]+D_{nq}^{+}M_r\left[K_n\left(\lambda,b_r\right)\right]$$
$$+\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n A_p^{+}M_r\left[I_n\left(\lambda,b_1\right)J_{1pn}^{q,1}++B_n^{+}J_n\left(\lambda,b_1\right)N_{3pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{1pn}^{q,1}\right]$$
$$+\sum_{r=1}^{m}\sum_{p=0}^{m}(-1)^n\varepsilon_n\left[B_n^{+}J_n\left(\lambda,b_1\right)N_{1pn}^{q,1}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{3pn}^{q,1}\right] \tag{84}$$

$$\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n\left[A_p^{+}J_n\left(\lambda,b_1\right)J_{1pn}^{k,q}+C_p^{+}I_n\left(\lambda,b_1\right)J_{1pn}^{k,q}+B_n^{+}J_n\left(\lambda,b_1\right)N_{3pn}^{k,q}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{1pn}^{k,q}\right]$$
$$+\sum_{k=1}^{m}\sum_{p=0}^{m}(-1)^n\varepsilon_n\left[B_n^{+}J_n\left(\lambda,b_1\right)N_{1pn}^{k,q}+D_{p\bar{k}}^{+}I_1\left(\lambda,b_1\right)K_{3pn}^{k,q}\right]=0 \tag{85}$$

$$B_{nq}^{+}M_3\left[N_n\left(\lambda,b_r\right)\right]+D_{nq}^{+}M_3\left[K_n\left(\lambda,b_r\right)\right]$$
$$+\sum_{p=0}^{\infty}\left(-1\right)^n\varepsilon_n A_p^{+}M_3\left[I_n\left(\lambda,b_1\right)J_{1pn}^{q,1}+B_n^{+}M_3\left(\lambda,b_1\right)N_{3pn}^{q,1}+D_{p\bar{k}}^{+}M_3\left(\lambda,b_1\right)K_{1pn}^{q,1}\right]$$
$$+\sum_{k=1}^{m}\sum_{p=0}^{m}(-1)^n\varepsilon_n\left[B_n^{+}M_3\left(\lambda,b_1\right)N_{1pn}^{q,1}+D_{p\bar{k}}^{+}M_3\left(\lambda,b_1\right)K_{3pn}^{q,1}\right] \tag{86}$$

$$\sum_{p=0}^{\infty}\left(-1\right)^{n}\varepsilon_{n}A_{p}^{+}M_{3}\begin{bmatrix}J_{n}\left(\lambda,b_{1}\right)]J_{1pn}^{q,1}+C_{p}^{+}M_{3}\left[J_{n}\left(\lambda,b_{1}\right)\right]J_{1pn}^{q,1}+B_{n}^{+}M_{3}\left[J_{n}\left(\lambda,b_{1}\right)\right]N_{3pn}^{q,1}\\ +D_{p\bar{k}}^{+}M_{3}\left[J_{n}\left(\lambda,b_{1}\right)\right]K_{1pn}^{q,1}\end{bmatrix}$$
$$+\sum_{k=1}^{m}\sum_{p=0}^{m}(-1)^{n}\varepsilon_{n}\left[B_{n}^{+}M_{3}\left[J_{n}\left(\lambda,b_{1}\right)\right]N_{1pn}^{q,1}+D_{p\bar{k}}^{+}M_{3}[J_{n}\left(\lambda,b_{1}\right)]K_{3pn}^{q,1}\right] \tag{87}$$

Here

$$M_{2}\left[\widetilde{z_{n}}\left(\lambda,b_{k}\right)\right]=\lambda_{1}^{2}z_{n}\left(\lambda,b_{k}\right)+\frac{\nu\lambda_{1}}{b_{k}}\tilde{z}\left(\lambda,b_{k}\right)-\frac{\nu n^{2}}{b_{k}}\tilde{z}_{n}\left(\lambda,b_{k}\right) \tag{88}$$

$$M_{3}\left[\widetilde{z_{n}}\left(\lambda,b_{k}\right)\right]=\lambda_{1}^{2}z_{n}\left(\lambda,b_{k}\right)+\frac{\nu\lambda_{1}}{b_{k}}\tilde{z}\left(\lambda,b_{k}\right)-\left[\frac{1}{b_{r}^{2}}-\frac{n^{2}\left(2-\nu\right)}{b_{k}}\right]\lambda_{1}\tilde{z}_{n}\left(\lambda,b_{k}\right)+\frac{n^{3}\left(2-\nu\right)}{b_{r}^{3}}z_{n}\left(\lambda,b_{k}\right) \tag{89}$$

To obtain an infinite system of linear algebraic equations including unknown coefficients with a minus sign on them, in the system (70) - (86) you can compare the (+) sign above the unknowns with the (-) sig

$$\begin{aligned}&z_{1_{pn}}^{1,k}-ni\,z_{2_{pn}}^{1,k}\ for,z_{3_{pn}}^{1,k}-ni\,z_{4_{pn}}^{1,k}\ for,z_{1_{pn}}^{q,k}-ni\,z_{2_{pn}}^{q,k}\ for,z_{3_{pn}}^{q,k}-ni\,z_{1_{pn}}^{q,k}\\&for,z_{1_{pn}}k,q-ni\,z_{2_{pn}}^{k,q}\ for,z_{3_{pn}}^{k,q}-ni\,z_{1_{pn}}^{k,q}\end{aligned} \tag{90}$$

can be replaced by -. Let's call that system (89) as a whole. Systems (70) - (86) and (89) are independent systems and can be solved separately. If (70) - (86) and (89) if the systems are regular systems (the regularity of the analogous system is partially shown in (2)), then they can be brought to a "cut" finite system and solved in EHM.

3. Main Results

Finding mandatory dances

Let's point out the special functions $\overset{\grave{U}}{W_{q}}\left(q=\overline{1,\infty}\right)$. To solve the problem of forced dance, let's make a system of orthonormalized functions from eigenfunctions as follows, let $\psi=W_{1}^{\circ}$, then be a normalized function. $\Phi=\frac{\psi_{1}}{\left\|\psi_{1}\right\|}$ After that, we write the sequential course of the process, and we hope that the dear reader can understand everything himself:

$$\begin{aligned}&\psi_{2}=W_{2}^{*}-\left(W_{2}^{*}\phi_{1}\right)\phi_{1}\quad\phi_{2}=\frac{\psi_{2}}{\psi_{2}}\\&\psi_{3}=W_{3}^{*}-\left(W_{3}^{*}\phi_{2}\right)\phi_{2}-\left(W_{3}^{*}\phi_{1}\right)\phi_{1}\quad\phi_{3}=\frac{\psi_{3}}{\psi_{3}}\\&\psi_{if}=W_{q}^{*}-\sum_{j=1}^{\overline{if}}\left(W_{if}^{*}-\phi_{j}\right)\quad\phi_{q}=\frac{\psi_{if}}{\psi_{if}}\end{aligned} \tag{91}$$

Here $\left(W_{q}^{*},\phi_{j}\right)$ is the scalar product of functions. ψ_{q} denotes the norm of the function ψ_{q} in L**2** space.

$$\begin{aligned}&\psi_{n}=\iint\left|\psi_{n}\right|^{2}dS\\&\phi_{ij}=\frac{W_{q}^{*}}{\left\|\psi_{3}\right\|}-\sum_{j=1}^{q-1}a_{ij}W^{*}\end{aligned} \tag{92}$$

$$a_{ij} = \frac{1}{\psi}\left[-\frac{\left(W_q^{*}\phi_j\right)}{\psi_i} + \sum_{j=1}^{q-1} a_{ij}\right] \qquad q \neq j \qquad when,$$

$$a_{ij} = \psi_3^{\ -1} \qquad q = j \qquad when$$

$$a_k^n = (-1)^{1+h}\frac{1}{\|\psi_k\|}\sum_{j=1}^{q-1}\frac{\left(W_q^{*}\phi_j\right)}{\|\psi_3\|}\sum^{q=1}\frac{\left(W_q^{*}\phi_j\right)}{\|\psi_3\|} - \int_{q-j+1=j+1}^{q_{r2}-1}\frac{\left(W_{a^{-1}}^{*}\phi_{a^n j}\right)\left(W_{a^{-1}}^{*}\phi_j\right)}{\|\psi_{q-j+1}\|} \tag{93}$$

$$\tilde{q} = \overline{1,\infty}, \quad j = 1, q$$

in the form of
Let's write the expression (90) as follows:

$$\phi_{q_1} = \frac{1}{\psi_k}\left[W_{\overline{q_1}}^{*} - \sum_{q_2=1}^{q_2-1}\left(W_{\bar{q}}^{*}\phi_{q_1}\right)\phi_{q_1}\right] \tag{94}$$

In (93), let's write the following expression in place of the ϕq word outside the scalar product:

$$\phi_{q_2} = \frac{1}{\psi_k}\left[W_{\bar{q}}^{*} - \sum_{q_2=1}^{q_2-1}\left(W_{\bar{q}}^{*}\phi_{q_2}\right)\phi_{q_2}\right]$$

The we buy

$$\begin{aligned}\$_q &= \frac{W_q^{*}}{\psi_{\bar{q}}} - \frac{W_{\bar{q}}}{\psi_{\bar{q}}}\left[\sum_{q_2=1}^{q_1-1}\left(W_{q_1}^{*}\$_{q_1}\right)W_{q_1}^{*} - \sum_{q_1-1}^{q_1-2}\left(W_{q_2}^{*}\$_{q_2}\right)\$_{q_2}\right]\frac{1}{\psi_{\bar{q}}} \\ &= \frac{W_q^{*}}{\psi_{\bar{q}}} - \frac{1}{\psi_{\bar{q}}}\sum_{q_1-1}^{1}\left(W_{q_1}^{*}\$_{q_1}\right)\frac{1}{\psi_{\bar{q}}} + \frac{1}{\psi_{\bar{q}}}\sum_{q_1-1}^{q_1-2}\left(\frac{W_q^{*}\$_{q_2}}{\psi_{\bar{q}}}\right)\sum_{q_1-1}^{q_1-2}\left(W_{q_2}^{*}\$_{q_2}\right)\$_{q_2}\end{aligned} \tag{95}$$

Now, under the last sum sign, in place of the product Φq_2 outside the scalar product, we write its analogous expressions and continue the process until $q_1 = 1\left(i = \overline{1,\bar{q}}\right)$. After that, regrouping the expressions, we get:

$$\begin{aligned}\$_q = \frac{W_q^{*}}{\psi_{\bar{q}}} - \frac{W_{\bar{q}}^{*}}{\psi_{\bar{q}}}\Bigg\{ & W_{q_1}^{*}\$_{q_1} - \sum_{q_2=1}^{q_1-1}\left(W_{q_1}^{*}\$_{q_1}\right)\frac{\left(W_{q_2}^{*}\$_{q_2}\right)\$_{q_2}}{\psi_{\bar{q}}} + \\ & \sum_{q_2=1}^{q_1-1}\frac{W_q^{*}\$_{q_2}}{\psi_{\bar{q}}}\sum_{q_2=1}^{q_1-1}\frac{W_q^{*}\$_{q_2}}{\psi_{\bar{q}}}\left(W_{q_2}^{*}\$_{q_2}\right)\sum_{q_2=3}^{q_1-1}\frac{W_{q_1}^{*}\$_{q_2}}{\psi_{\bar{q}}} + \ldots + \frac{W_q^{*}}{\psi_{\bar{q}} * \psi_2}\left\{\left(W_{q_2}^{*}\$_{q_2}\right) + \ldots\right\}\Bigg\}\end{aligned} \tag{96}$$

From here, the truth of statement (92) is evident. Considering the properties of multilayer sums, let's consider special cases, for example ≪q̃=4, j=3≫; Let's look at the cases ≪q̃=4, j=2≫ and ≪q̃=4, j=1≫, when j>q̃-2 the sum $\sum_{J=j}^{q-2}\alpha_a^1$ does not exist and when q̃=4, j=3 we get:

$$\propto_{43} = \left[-\frac{1}{\psi_4}\left(W_{q_2}^{*}\$_{q_2}\right)\right]\frac{1}{\psi_3} = -\frac{\left(W_4^{*}\$_3\right)}{\psi_4 * \psi_3}.$$

By the same rule

$$\propto_{43}=\left[-\frac{1}{\psi_4}\left(W_{q_2}^{*}\$_{\ q_2}\right)+\sum_{2}^{2}\alpha_{42}^{2}\right]\frac{1}{\psi_4};\alpha_{42}^{2}=(-1)^4\frac{1}{\psi_4}\sum_{3}^{3}\sum_{3}^{2}\frac{W_{q_1}^{*}\$_{\ q_2}}{\psi_{\bar q}}\left(W_{q_2}^{*}\$_{\ q_2}\right) \quad (97)$$

And finally

$$\propto_{43}=\frac{\left(W_4^{*}\$_{\ 2}\right)}{\|\psi_4\|*\|\psi_2\|}+\frac{\left(W_4^{*}\$_{\ 3}\right)\left(W_3^{*}\$_{\ 2}\right)}{\|\psi_4\|*\|\psi_3\|*\|\psi_2\|}$$

When $\tilde q=4, j=1\tilde q-2=2$, j=1. Therefore, if the straight sum $\tilde q_{j=1}=1$

$$\propto_{43}=\frac{1}{\psi_4 \ * \ \psi_2}\left[\left(W^{*}\$_{\ 1}\right)\right]-\sum_{J_1=1}^{2}\propto_{41}^{J_1}$$

happens.

$$\propto_{41}^{1}=(-1)^2\sum_{q_1=2}^{3}\sum_{q_2=1}^{q_1=1}\frac{\left(W_4^{*}\$_{\ q_1}\right)\left(W_{q_1}^{*}\$_{\ q_1}\right)}{\psi_1}=\frac{\left(W_4^{*}\$_{\ 2}\right)\left(W_2^{*}\$_{\ 1}\right)}{\psi_2}+\frac{\left(W_4^{*}\$_{\ 3}\right)\left(W_3^{*}\$_{\ 1}\right)}{\psi_3}$$

$$\propto_{41}^{3}=(-1)^3\sum_{q_1=2}^{3}\sum_{q_2=1}^{2}\sum_{q_2=1}^{1}\frac{\left(W_4^{*}\$_{\ q_1}\right)\left(W_q^{*}\$_{\ q_2}\right)\left(W_{q_1}^{*}\$_{\ q_3}\right)}{\psi_3 \ * \ \psi_2}=-\frac{\left(W_4^{*}\$_{\ 1}\right)\left(W_q^{*}\$_{\ 2}\right)\left(W_{q_1}^{*}\$\ \right)}{\psi_3 \ * \ \psi_2}.$$

Thus,

$$\propto_{41}^{1}=-\frac{\left(W_4^{*}\$_{\ q_1}\right)}{\psi_4 \ * \ \psi_1}+\frac{\left(W_4^{*}\$_{\ 3}\right)\left(W_3^{*}\$_{\ 1}\right)}{\psi_4 \ * \ \psi_3 \ * \ \psi_2}-\frac{\left(W_4^{*}\$_{\ 3}\right)\left(W_3^{*}\$_{\ 1}\right)}{\psi_4 \ * \ \psi_3 \ * \ \psi_2}$$
$$-\frac{\left(W_4^{*}\$_{\ 1}\right)\left(W_q^{*}\$_{\ 2}\right)\left(W_{q_1}^{*}\$_{\ 1}\right)}{\psi_4 \ * \ \psi_3 \ * \ \psi_2 \ * \ \psi_1}$$

It has been verified that the coefficients are obtained in a sufficiently large amount. It is clear from expressions (56) and (58) that all Φq̄ functions satisfy all the boundary conditions that the eigenfunctions satisfy. Therefore $\tilde q(\rho,\theta,t)=\sum_{\bar q}^{\infty}T_{\bar q}(t)\phi_{\bar q}$

So, here

$$T_{\bar q}(t)=\int_{\bar q}\int\tilde q(\rho,\theta,t)\phi_{\bar q}ds$$

$\bar D$ is the solution of (1) when the solution we are looking for is multi-connected

$$W=\sum_{\bar q=1}^{\infty}\beta_{\bar q}(t)\phi_{\bar q}$$

If we search in the picture, the border conditions will be automatically paid. Here, $\beta_{\bar q}(t)$ is an unknown function.

Thus, if we write (94) and (96) in (1), we get:

$$\sum_{\bar q=1}^{\infty}\left\{(\beta_q(t)+n_{0q}^{2}\beta_q(t)-\lambda^{-4}T_q-\left[\beta_q(T)\sum_{j=1}^{q-1}\alpha_{qj}n_{0j}^{2}W_j^{*}+\beta_q^{*}\sum_{j=1}^{q-1}\alpha_{qj}W_j^{*}-\lambda^{-4}T_q(t)\right]\right\}=0 \quad (98)$$

By equating the coefficients of the same eigenfunctions, we get an infinite system of linear differential equations with inhomogeneous constant coefficients in which the functions$\beta_{\bar{\theta}}(t)$ are internal:

$$\alpha_{\overline{qq}}\left[\beta_{\bar{q}}''(t)+n_{0\bar{q}}^{2}\beta_{\bar{q}}(t)-\lambda^{-4}T_{q}(t)\right]-\sum_{i=\bar{q}+1}^{m}\left[\beta_{i}''(t)+n_{0\bar{q}}^{2}\beta_{j}(t)-\lambda^{-4}T_{j}\left(t\right)\right]\alpha_{\bar{q}j}=0 \tag{99}$$

If we denote here $\gamma_i\left(t\right)=\beta_t''(t)+n_{0j}^2\beta_i(t)-\lambda^{-4}T_j(t)$ we will arrive at the following system of algebraic equations:

$$\alpha_{\overline{qq}}\gamma_{\bar{q}}-\sum_{j=\bar{g}+1}^{\infty}\alpha_{\bar{q}}\gamma_{j}=0 \tag{100}$$

or if we write openly

$$\alpha_{11}\gamma-\alpha_{\overline{12}}\gamma_{2}-\alpha_{\overline{13}}\gamma_{\bar{3}}-\alpha_{\overline{14}}\gamma_{\bar{4}}-\ldots-\ldots=0 \tag{101}$$

$$\alpha_{\overline{22}}\gamma_{\bar{2}}-\alpha_{\overline{23}}\gamma_{\bar{3}}-\alpha_{\overline{24}}\gamma_{\bar{4}}-\ldots-\ldots=0$$

$$\alpha_{\overline{33}}\gamma_{\bar{3}}-\alpha_{\overline{34}}\gamma_{\bar{4}}-\alpha_{\overline{35}}\gamma_{\bar{5}}-\ldots-\ldots=0$$

$$\alpha_{\overline{44}}\gamma_{\bar{4}}-\alpha_{\overline{45}}\gamma_{\bar{5}}-\ldots-\ldots=0$$

If we cut and solve the system (70)-(86), we will find functions belonging to a finite number, then the system (100) becomes a finite system. In this case, the last equation of the system, let's call it the N - th, takes the form$\bar{\alpha}_{NN}\gamma_N$=0 and we can determine all γ_i(i=$\overline{1,N}$) starting from that equation. That is, in this case, we

$$\beta_t''(t)+n_{0j}^2\beta_i(t)=\lambda^{-4}T_j(t) \tag{102}$$

we have to solve the independent equations of the form (100) If we find the non-trivial solution (101), it is obtained as follows:

$$\beta_t''(t)+n_{0j}^2\beta_i(t)=\lambda^{-4}T_j(t)+\gamma_i(t) \tag{103}$$

The general solutions of equations (101) and (102) are, respectively:

$$\beta_i(t)=C_{1i}\cos n_{0i}t+C_{2i}\sin n_{0i}tn_{0i}^{-1}\int_0^i\lambda^{-4}T_j\left(\xi\right)\sin n_{0i}\left(t-\xi\right)d\xi \tag{104}$$

$$\beta_i\left(t\right)=C_{1i}\cos n_{0i}t+C_{2i}\sin n_{0i}t+n_{0i}^{-1}\int_0^i\begin{bmatrix}\gamma_i\left(\xi\right)\\ \lambda^{-4}T_j\left(\xi\right)\end{bmatrix}\sin\left(t-\xi\right)d\xi \tag{105}$$

To find the arbitrary constants C_{1i}. and C_{2i} we also divide the function V(ρ, θ) into series by means of the function $\Phi_{\bar{q}}$:

$$V\left(\rho,\theta\right)=\sum_{\bar{q}=1}^{\infty}H_{\bar{q}}\Phi_{\bar{q}} \tag{106}$$

so that,

$$H_{\bar{q}}=\iint_D W(\rho,\theta)\Phi_{\bar{q}}\mathrm{ds} \tag{107}$$

Then (10) initial conditions

$$\beta_{\bar{q}}(0)=0;\beta_{\bar{q}}(0)=H_{\bar{q}} \tag{108}$$

they fall into the picture. We solve them and find: $C_{1\bar{q}}=0;C_{1\bar{q}}=n_{0\bar{q}}^{-1}H_{\bar{q}}$
Thus,

$$\beta_{\bar{q}}(t)=n_{0\bar{q}}^{-1}H_{\bar{q}}\sin n_{0i}t+n_{0i}^{-1}\int_0^1\left[\gamma_i(\xi)+\lambda^{-4}T_j(\xi)\right]\sin n_{0\bar{q}}(t-\xi)d\xi$$

$$\beta_{\bar{q}}(t)=n_{0\bar{q}}^{-1}H_{\bar{q}}\sin n_{0i}t+n_{0i}^{-1}\int_0^1\lambda^{-4}T_j(J)\sin n_{0\bar{q}}(t-\xi)d\xi$$

After that, the law of departure of each point of the board from the plane position at the desired moment during the forced dance is known to us

$$W=\sum_{q=1}^{N}\beta_{\bar{q}}(t)\left[\frac{W_{\bar{q}}^*}{\psi_{\bar{q}}}-\sum_{j=1}^{q-1}\alpha_{\bar{q}j}W_j^*\right]$$

Now we can determine the laws of change of the bending and twisting moments Mρ, Mθ and $M_{\rho\theta}$ as well as the cutting force in the oscillation process with the following well-known formulas:

$$M_\rho=-D\left[\frac{\partial^2W}{\partial\rho^2}+\nu\left(\frac{1}{\rho}\frac{\partial W}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2W}{\partial\theta^2}\right)\right];M_\theta=-D\left[\frac{1}{\rho}\frac{\partial W}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2W}{\partial\theta^2}+\nu\frac{\partial^2W}{\partial\rho^2}\right]$$

$$M_{\rho\theta}=(1-\nu)D\left(\frac{1}{\rho}\frac{\partial^2W}{\partial\rho\partial\theta}-\frac{1}{\rho^2}\frac{\partial W}{\partial\rho}\right);Q_\rho=-D\frac{\partial}{\partial\rho}\Delta W;Q_\theta=-\frac{1}{\rho}D\frac{\partial}{\partial\theta}\Delta W$$

Considering (101), these formulas can be written as follows:

$$M_\rho=-D\sum_{q=1}^{N}\beta_{\bar{q}}(t)\widetilde{L_1}\left[\frac{W_{\bar{q}}^*}{\psi_{\bar{q}}}-\sum_{j=1}^{q-1}\alpha_{\bar{q}j}W_j^*\right];$$

$$M_\theta=-D\sum_{q=1}^{N}\beta_{\bar{q}}(t)\widetilde{L_2}\left[\frac{W_{\bar{q}}^*}{\psi_{\bar{q}}}-\sum_{j=1}^{q-1}\alpha_{\bar{q}j}W_j^*\right];$$

$$M_{\rho\theta}=(1-\nu)D\sum_{q=1}^{N}\beta_{\bar{q}}(t)\widetilde{L_3}\left[\frac{W_{\bar{q}}^*}{\psi_{\bar{q}}}-\sum_{j=1}^{q-1}\alpha_{\bar{q}j}W_j^*\right];$$

$$Q_\rho=-D\sum_{q=1}^{N}\beta_{\bar{q}}(t)\widetilde{L_4}\left[\frac{W_{\bar{q}}^*}{\psi_{\bar{q}}}-\sum_{j=1}^{q-1}\alpha_{\bar{q}j}W_j^*\right];$$

$$Q_\theta=-D\sum_{q=1}^{N}\beta_{\bar{q}}(t)\widetilde{L_5}\left[\frac{W_{\bar{q}}^*}{\psi_{\bar{q}}}-\sum_{j=1}^{q-1}\alpha_{\bar{q}j}W_j^*\right];$$

Here the following differential operators are denoted by $L_1\left(i=\overline{1.5}\right)$

$$\widetilde{L_1}=\frac{\partial^2}{\partial\rho^2}+\nu\left(\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2}\right);$$

$$\widetilde{L_2} = \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + \nu\frac{\partial^2}{\partial\rho^2};$$

$$\widetilde{L_3} = \frac{1}{\rho}\frac{\partial^2}{\partial\rho\partial\theta} - \frac{1}{\rho^2}\frac{\partial}{\partial\rho};$$

$$\widetilde{L_4} = \frac{\partial^3}{\partial\rho^3} + \frac{1}{\rho}\frac{\partial^2}{\partial\rho^2} - \frac{1}{\rho^2}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\rho\partial\theta^2};$$

$$\widetilde{L_5} = \frac{\partial^3}{\partial\rho\partial\theta^2} + \frac{1}{\rho}\frac{\partial^2}{\partial\rho\partial\theta} + \frac{1}{\rho^2}\frac{\partial^3}{\partial\theta^3}$$

As a result, it turns out that when $a_{\bar{k}} = b_{\bar{k}}\left(k = \overline{1,n}\right)$ the problem of investigating the forced oscillations of a circular plate with circular holes and its solution is taken as a special case of this problem. This method can be applied to all cases where contours can be projected onto a circle.

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