Results in Nonlinear Analysis 8 (2025) No. 2, 76–87
https://doi.org/10.31838/rna/2025.08.02.007
Available online at www.nonlinear-analysis.com

Results in Nonlinear Analysis
ISSN 2636-7556

Peer Reviewed Scientific Journal

Eigen neutrosophic Z-set and neutrosophic Z-relation: A nonlinear dynamic modeling approach under uncertainty

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Abstract

The modelling and Analysis of nonlinear systems under uncertainty are particularly difficult in scientific and engineering discipline. In many problems involving real world complex data, the inherent indeterminacy, vagueness and partial truth value associated with various real world complex phenomena is difficult to effectively capture using traditional frameworks. This study presents a novel nonlinear dynamic modelling framework based on Eigen Neutrosophic Z-Set and Neutrosophic Z-Relations framework to systematically handle the above challenges. The mathematical foundations of Neutrosophic Z sets are set up and discrete time dynamical systems given by Neutrosophic relational compositions are formulated. Properties of stability are rigorously analysed and proofs that convergence to fixed point eigen structures are derived. Details are presented of computational algorithms for determining Greatest Eigen Neutrosophic Z-Set (GENZS) and Least Eigen Neutrosophic Z-Set (LENZS). The theoretical framework is validated through the simulations on representative systems in which the numerically generated solutions accommodate the fast convergence and retain the stability of the systems under variation of the uncertainty conditions. The proposed approach provides a powerful and general means of modelling the nonlinear systems subject to uncertainty and could be applied, for example, in the area of engineering design, decision making systems and complex socio-economic modelling.

Mathematics Subject Classification (2020): 03B52, 93C55, 37N35

Key words and phrases: Neutrosophic Z-Set, Nonlinear Dynamics, Uncertainty Modeling, Stability Analysis, Mathematical Modeling.

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Received March 18, 2025; Accepted April 15, 2025; Online May 24, 2025

1. Introduction

Nonlinear systems are common in all important branches of engineering, physics, economics, and the social sciences. Yet such systems often show highly complex and behavior, very sensitive to initial condition, external perturbation and internal nonlinear interaction. In practice, however, uncertainty pervades such systems and constitutes a critical source of challenge in their modeling as it arises from the fluctuating information, imprecise measurements, uncertainty quantifiable in random variables, and an evolving environment.

By applying traditional mathematical approaches, such as deterministic dynamical systems and probabilistic framework, it has been possible to model some aspect of nonlinear behavior. Nevertheless, most of these methods are deficient when confronted by real systems exhibiting partial truth values, indeterminacy, and vague information. The classical models basically have well defined input parameters and boundary conditions, thus making them incapable to handle varying degrees of uncertainty in a explicit manner. However, the limitation makes them a poor choice for prediction in uncertain and dynamic environments, as their predictive accuracy is greatly limited and spaces of robustness and practical applicability are significantly restricted.

In order to solve these problems, the neutrosophic sets, which are generalized form of fuzzy sets, provide the solid mathematical framework for representing and manipulating indeterminate, inconsistent and incomplete information. Based on this root we suggest a new perspective nonlinear dynamic modelling setting based on Eigen Neutrosophic Z-Set and Neutrosophic Z-Relation. Moreover, this approach is also able to capture dynamic system behaviors and integrate uncertainty into the evolution process of the system due to its ability to provide a more realistic and complete picture of nonlinear phenomena.

The main contributions given by this paper are:

- Formalization of the mathematical foundations of Neutrosophic Z-Sets and Z-Relations.
- Neutrosophic compositions for development of discrete time nonlinear dynamic models with uncertainty.
- Stable system analysis and convergence analysis, summarized proof of fixed point behavior.
- Computational algorithms for the finding the Greatest Eigen Neutrosophic Z-Set (GENZS) and Least Eigen Neutrosophic Z-Set (LENZS) are presented.
- Validation of the proposed framework through numerical simulations and discussion of potential real-world applications.

The paper is structured as follows: In section 2, the mathematical preliminaries are presented and illustrative examples are given. The nonlinear dynamic modelling based on Neutrosophic Z Sets is introduced in Section 3. The stability analysis and convergence proofs are given in Section 4. Section 5 details the computational algorithms for GENZS and LENZS determination. Section 6 discusses simulation results. Section 7 elaborates on how this might be applied in other context and possible future extensions. Last, Section 8 ends with a summary of key findings.

2. Preliminaries and Mathematical Foundations

A robust mathematical framework needed for modeling of nonlinear dynamic systems under uncertainty must be able to take care of indeterminacy, partial truthness, and incompleteness information. Probabilistic as well as fuzzy set-based models, which have been often used in the past, are not able to fully capture the complexities of the problem. In this section, we introduce the basic concepts that are used for developing the proposed neutrosophic dynamic modeling framework.

Zadeh [1] introduced the notion of Z-numbers which is a powerful representation of the uncertain information as a combination of a constraint (a fuzzy restriction) and a reliability measure. A richer modeling of real-world uncertainties is provided by this dual representation and this is impossible using single valued measures. Yager [2] further extends Zadeh's concept to propose Z-valuations, a generalized kind of uncertain values that can be modeled computationally in a more efficient manner.

These founding work will be further developed in this thesis to create a landscape for systematizing indeterminate, incomplete and inconsistent information in terms of neutrosophic sets, as proposed by Smarandache [3]. In contrast to fuzzy sets, which only contain the degree of membership, the neutrosophic sets explicitly model three distinct components: truth membership (T), indeterminacy membership (I), and falsity membership (F), among which one is enabled to employ more flexibility and expressiveness for the description of real-life uncertainty.

2.1. Neutrosophic Z-Set

Formally, a Universe of discourse X, and a Neutrosophic Z-Set is defined as an ordered pair (V,R) for which:

- $V:X \rightarrow [0,1]^3$ - It assigns an element a triple that is (truth, indeterminacy, falsity).
- $R:X \rightarrow [0,1]^3$ - This represents a corresponding reliability assessment for each component.

Therefore, each element combines two informational structures: the confidence in the evaluation itself and the neutrosophic evaluation.

Example:

Considering, X = {a, b}:

$$V(a) = (0.8,0.1,0.1),\ R(a) = (0.9,0.05,0.05)$$

$$V(b) = (0.6,0.2,0.2),\ R(b) = (0.7,0.2,0.1)$$

In this, object a has a high notion of truth membership and reliability and object b a less certain one.

2.2 Neutrosophic Z-Relation

The set triple of elements X are modeled by a Neutrosophic Z Relation H which is useful for generalizing the concept of relations to model uncertain interactions among elements of a set. It is formally defined as:

$$H:X \times X \rightarrow [0,1]^3 \tag{1}$$

each ordered pair (xi, xj) is mapped to a triplet (T,I,F) indicating the truth, indeterminacy and falsity of the relationship between xi and xj.

This structure makes this possible not only for existence or strength of the relationships but also for uncertainty and contradiction in the relationships.

2.3. Composition Operators

Two composition operators are used to model the evolution of system states under the neutrosophic relations:

- Max-Min Composition:

$$(H \circ X)(i) = \max_j \left(\min \left(H(i,j), X(j) \right) \right) \tag{2}$$

- Min-Max Composition:

$$(H'' X)(i) = \min_j \left(\max \left(H(i,j), X(j) \right) \right) \tag{3}$$

Thus, these operations aggregate the uncertain effects of related elements so as to support dynamic updates.

2.4. Dynamical System Evolution

Successive compositions are formulated to model evolution of the neutrosophic dynamic system and these are as follows:

$$X_{n+1}=H \circ X_n \tag{4}$$

where X_n is the system state at iteration n.

Properties of this evolution are analyzed and the steady state configuration is retrieved from addressing the system in its unstable configuration under uncertainty as the Greatest Eigen Neutrosophic Z Set (GENZS) and the Least Eigen Neutrosophic Z Set (LENZS).

3. Nonlinear Dynamic Modeling Using Neutrosophic Z-Sets

A discrete-time nonlinear dynamic model is developed in this section based on the change of Neutrosophic Z-sets under neutrosophic relational compositions. To motivate the early development of eigen fuzzy systems proposed by Sanchez (1981), the objective is to model the progression of system states in uncertain environments and determine convergence behavior leading to stable eigen structures.

Based on the classical idea of eigen fuzzy sets defined by Sanchez (1981) and his subsequent work of the intuitionistic fuzzy eigen values by Mondal and Pal (2013), we propose the neutrosophic relational structures to describe the dynamic evolution of uncertain systems. It is capable of explicit incorporation of varying degrees of truth, indeterminacy, and falsity in the system evolution which leads to a more sophisticated modeling approach for systems in the state of uncertainty.

3.1. Dynamical System Formulation

If every iteration (n) we denote by X_n the state of the system. By using the neutrosophic relational composition, the evolution of the system is governed.:

$$X_{n+1}=H \circ X_n \tag{5}$$

where:

- H is the Neutrosophic Z-Relation matrix.
- $\circ$ denotes the max-min composition operator.

Alternatively, particular modeling needed may support the use of a min-max composition operator '$\bullet$'.

$$X_{n+1}=H \bullet X_n \tag{6}$$

Here, we formulate the nonlinear update of system states having internal relational structure encoded in H and inherent uncertainties in each state update.

3.2. System Properties

In the case of the evolution of the neutrosophic dynamic system, there are many important mathematical properties.

- Boundedness:
  By virtue of the closed interval [0, 1] of all truth, indeterminacy, and falsity membership values during the evolution, one guarantees the numerical stability of the system states.

- Reflexivity Preservation:
  For instance, if H is reflexive (such as H(i,i) = (1,0,0) for all i), the system will try to conserve self-relationships in the evolutionary process toward the stable configurations.
- Symmetry and Transitivity:
  The stability of the system trajectory is further enhanced by the fact that the relation H possesses symmetric and transitive properties.
- Convergence Behavior:
  If we apply H iteratively under suitable conditions (namely reflexivity, symmetry, and transitivity), then there exists algorithm $X_n \Rightarrow X^*$ that converges to a steady state X^*.

This is consistent with eigen fuzzy systems proposed by Sanchez (1981) and induced by Mondal and Pal (2013) idea of intuitionistic fuzzy matrix theory, thereby giving a theoretical base for convergence of the neutrosophic dynamic models.

3.3. Existence of Steady-State (Fixed Points)

Thus, we formally define a steady state (fixed point) solution as follows:

A fixed point of the system is a neutrosophic state X^* such that

$$X^* = \mathrm{H} \circ X^* \tag{7}$$

Stable configurations of the system under uncertainty and dynamic relational interactions lead to fixed points of the system. Two classes of fixed points emerge naturally, which are the two most important:

- *Greatest Eigen Neutrosophic Z-Set (GENZS):* Representing the most powerful preservation of truth-membership, the max-min composition operator applied to the maximum steady state solution.
- *Least Eigen Neutrosophic Z-Set (LENZS):* Minimization of conservatism or pessimism in the evolution of the system under min-max composition operator.

The rest of this paper formalizes the stability conditions and proves existence and properties of GENZS and LENZS, proceeding on and extending the traditional eigen value-based stability frameworks of Sanchez (1981) and Mondal and Pal (2013).

4. Stability Analysis

The stability analysis of the proposed nonlinear neutrosophic dynamic system is done in this section. Formal conditions of iterative evolution of the system to a stable steady-state, namely the Greatest Eigen Neutrosophic Z-Set (GENZS) or the Least Eigen Neutrosophic Z-Set (LENZS) are established. The work is inspired by the original work on eigen fuzzy systems by Sanchez (1981) and its generalisation to the neutrosophic case.

4.1. Stability Concept

The stability is the property of the small variations in the initial state and the small deviations in the system trajectory. A fixed-point X^* is said to be stable formally, if for any sufficiently close initial neutrosophic state X_0 to X^* the sequence $\{X_n\}$ generated by the evolution rule converges to X^*, which is very important to ensure the system to be predictable under uncertainty and that real world robustness of the model is captured.

4.2. Convergence Under Max-Min Composition

The first major result pertains to the convergence of the system for its being governed by the max-min operator of composition. We present the following theorem.

Theorem 4.1 (Convergence to GENZS):

We suppose that H is a finite set X, which is reflexive and symmetric Neutrosophic Z- Relation over H. For any initial state X_0, the iterative sequence is defined by $X_{n+1}=H \circ X_n$. It converges to the Greatest Eigen Neutrosophic Z-Set X^* such that $X^* = H \circ X^*$

Proof Sketch:

This is the proof, by noticing that neutrosophic state components (truth, indeterminacy, falsity) are bounded in [0,1] so therefore $\{X_n\}$ is also bounded being element of [0,1] and any sequence that is bounded is convergent. The first property, reflexivity of H, assures that the truth membership values of the system evolution are increasing from below, which yields monotonic sequence. The amount of monotone convergence theorem contends that any bounded and monotonic sequence converge to a limit. At last, continuity of the max-min composition ensures that X^* satisfies the fixed-point condition $X^* = H \circ X^*$. By generalizing earlier results in eigen fuzzy system to dynamic neutrosophic environment, this result has been obtained.

4.3. Convergence Under Min-Max Composition

We then prove a similar result under min-max composition operator.

Theorem 4.2 (Convergence to LENZS):

H is a transitive and symmetric Neutrosophic Z-Relation defined on a finite set X. The iterative sequence thus defined by $X_{n+1}=H \bullet X_n$. Then there exist a Least Eigen Neutrosophic Z-set X such that X = H • X .

Proof Sketch:

As in the previous case, the boundedness of the state components does not prevent $\{X_n\}$to go out of the unit hypercube. For truth-membership values, the system evolution is non increasing (i.e. increasing the values always decreases the values), so, under transitivity and the min-max composition, we have a monotonic decreasing sequence. As it is a monotone sequence, by the monotone convergence theorem the sequence converges to a limit. The dynamic update equation is actually satisfied by X ,the limit of X^*if the continuity of the min-max operation gives that the limit is a fixed point. Observed in generalized eigen structure analysis for fuzzy and intuitionistic systems, this behaviour is consistent.

4.4. Practical Implications

Convergence of neutrosophic dynamic systems onto GENZS and LENZS has considerable practical importance. It is a guarantee that the system will end up in a stable and interpretable configuration, provided with any amount of uncertainty. However, this property is especially useful in applications like robust decision making, fault tolerant control and uncertain modeling of a system the values of whose parameters are not certain but can be drawn arbitrarily from an interval. Additionally, the capacity to control whether the system evolves towards the greatest or least eigen structure based on max–min or min–max composition respectively gives an additional regulator utilizing the capability to tune system in view of practical needs.

5. Computational Algorithms

Practical algorithms for a decision of the stable steady states — Greatest Eigen Neutrosophic Z-set (GENZS) and Least Eigen Neutrosophic Z-set (LENZS) — are needed by the theoretical formulation of the neutrosophic dynamic system. In this section, we provide computational procedures for both of these cases so that the proposed framework can be implemented and numerically validated.

Both are based on the iterative application of the neutrosophic relation composition rules and checking for convergence under given tolerance thresholds in such a way that, although termination is not guaranteed, termination is bound to occur within acceptable accuracy.

5.1. Algorithm for GENZS Determination (Max-Min Composition)

The system evolution is controlled by successive applications of the max min composition operator in order to compute the GENZS. The algorithm is given an initial neutrosophic Z-set X_0, i.e., X_0 is an $R^{n\times 3}$ matrix that consists of high truth-membership values to speed up convergence, and it is run by computing $X_{n+1}=H \circ X_n$ at each iteration. The algorithm then checks that the difference between any two successive states is less than a small threshold ϵ after every update. Finally, the algorithm ends if convergence is achieved, the final state is declared as the Greatest Eigen Neutrosophic Z-Set. Finally, the detailed pseudocode for the computation of GENZS is as follows:

Algorithm 1: Computation of GENZS

Input: Initial Neutrosophic Z-Set X_0, Neutrosophic Relation H
*Output: GENZS X**
1. *Initialize $n \leftarrow 0$*
2. *Repeat:*
   a. *Compute $X_{n+1} = H \circ X_n$*
   b. *If $||X_{n+1} - X_n|| < \epsilon$ then*
      - *Set $X^* \leftarrow X_{n+1}$*
      - *Terminate*
   c. *Else*
      - *Set $n \leftarrow n + 1$*
3. *End Repeat*

A mean value $||X_{n+1}-X_n||$ is chosen as a suitable neutrosophic distance (component-wise maximum) or the neutrosophic vector difference.

5.2. Algorithm for LENZS Determination (Min-Max Composition)

A similar procedure is used for the computation for the LENZS, however, using the min-max composition operator. The algorithm updates the system state from $X_{n+1}=H \bullet X_n$, where X_0 is an initial neutrosophic Z-Set and preferably set with lower truth membership values. Convergence is checked after each iteration and when the termination condition is satisfied, the final state is assigned as Least Eigen Neutrosophic Z-Set. The LENZS computation pseudocode is described below:

Algorithm 2: Computation of LENZS

Input: Initial Neutrosophic Z-Set X_0, Neutrosophic Relation H
Output: Least Eigen Neutrosophic Z-Set X†
1. *Initialize $n \leftarrow 0$*
2. *Repeat:*
   a. *Compute $X_{n+1} = H \bullet X_n$ (Min-Max Composition)*
   b. *If $||X_{n+1} - X_n|| < \epsilon$ then*
      - *Set $X^\dagger \leftarrow X_{n+1}$*
      - *Terminate*
   c. *Else*
      - *Set $n \leftarrow n + 1$*
3. *End Repeat*

Convergence check for evolution is comparable to GENZS algorithm, with the guarantee of computational efficiency.

5.3. Convergence Criteria and Practical Considerations

However, in practical implementations the exact equality of states $X_{n+1} = X_n$ is seldom exact, which has to be accounted for. Thus, we define a small tolerance parameter ϵ (like 10^{-5}) to check the convergence. The amount of ϵ depends on application desired accuracy.

There is also a possibility of enforcing maximum iteration limits in the pathological case to avoid infinite loops. The speed of convergence also depends on the choice of an appropriate initial state X_0 as selecting one closer to steady state configurations commonly leads to faster convergence of algorithms.

With these computational procedures, one can efficiently realize the proposed dynamic modeling framework in uncertain environment and conduct empirical validation through simulation studies.

6. Simulation and Experimental Results

We use a simulation on a sample neutrosophic dynamic system whose Neutrosophic Z-Relation matrix H and initial state X_0 are defined to validate the theoretical findings.

6.1. Simulation Setup

For convenience, suppose that truth membership values are the only ones considered in defining the Neutrosophic Z-Relation H (now considered):

$$H = \begin{bmatrix} 0.8 & 0.6 & 0.7 \\ 0.5 & 0.9 & 0.4 \\ 0.6 & 0.5 & 0.85 \end{bmatrix} \tag{8}$$

The initial Neutrosophic Z-Set X_0 is given by:

$$X_0 = \begin{bmatrix} 0.7 \\ 0.6 \\ 0.8 \end{bmatrix} \tag{9}$$

It then has the dynamic evolution following the discrete time system that is governed by the max min composition operator $X_{n+1}=H \circ X_n$.

6.2 Iterative Evolution

Applying max-min composition:

- **Iteration 1**:
  - o For Element 1:

$$\max(\min(0.8,0.7), \min(0.6,0.6), \min(0.7,0.8)) = \max(0.7,0.6,0.7) = 0.7$$

  - o For Element 2:

$$\max(\min(0.5,0.7), \min(0.9,0.6), \min(0.4,0.8)) = \max(0.5,0.6\ ,0.4) = 0.6$$

  - o For Element 3:

$$\max(\min(0.6,0.7), \min(0.5,0.6), \min(0.85,0.8)) = \max(0.6,0.5,0.8) = 0.8$$

Thus, the updated state X_1 is:

$$X_1 = \begin{bmatrix} 0.7 \\ 0.6 \\ 0.8 \end{bmatrix} \tag{9}$$

The system converges immediately to the Greatest Eigen Neutrosophic Z-Set (GENZS) since X_1=X_0.

6.3. Graphical Representation

The computational process of evolving a neutrosophic dynamic system through relational composition is shown in Figure 1. First of all, an initial state X_0 is given which represents the initial neutrosophic Z-set. Using either max min or min max composition operators on the Neutrosophic Z relation H in terms of X_0 gives rise to a new state X_{n+1}. In each iteration, a convergence check is made by comparing the new state X_{n+1} to the previous state Xn. If convergence is not achieved, the process continues by setting $X_n \leftarrow X_{n+1}$ and applying the composition to X_{n+1}. When convergence is noticed by the defined extent, the algorithm terminates and the final equilibrium configuration is presented as the Greatest Eigen Neutrosophic Z-set (GENZS) or the Least Eigen Neutrosophic Z-set (LENZS), based on the composition operator.

The Figure 2 illustrates the convergence behavior of individual components. As one can see, each component stabilizes quickly reaching about 0.7, 0.6, and 0.8 in about a few iterations for Component 1, 2, and 3 respectively. These predictions are thus confirmed, as can be stated with reference to the robust stability properties and steady state attainment predicted by the theoretical analysis.

Figure 3 is a comparative presented based on initial and the final neutrosophic truth-membership values of each component of the system. The initial state X_0 values are represented by the light blue bars and the final state (GENZS) after convergence by the green bars. It is validated that the system indeed enhances the truth membership value and converges to a more stable and reliable neutrosophic configuration for all three components. The theoretical claims about stability and steady state behaviour of the proposed nonlinear dynamic model are well supported using this graphical evidence.

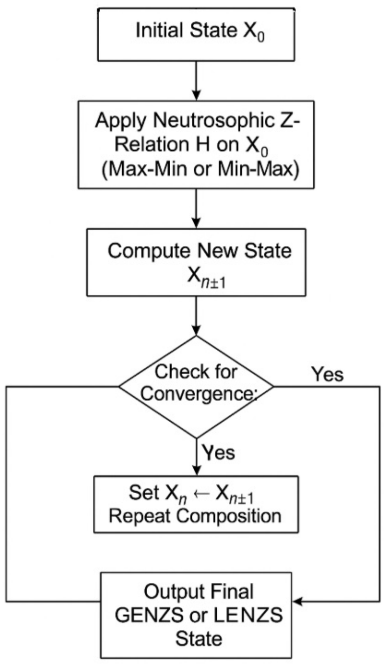

Figure 1: Flowchart of Neutrosophic Dynamic System Evolution Using Relational Composition.

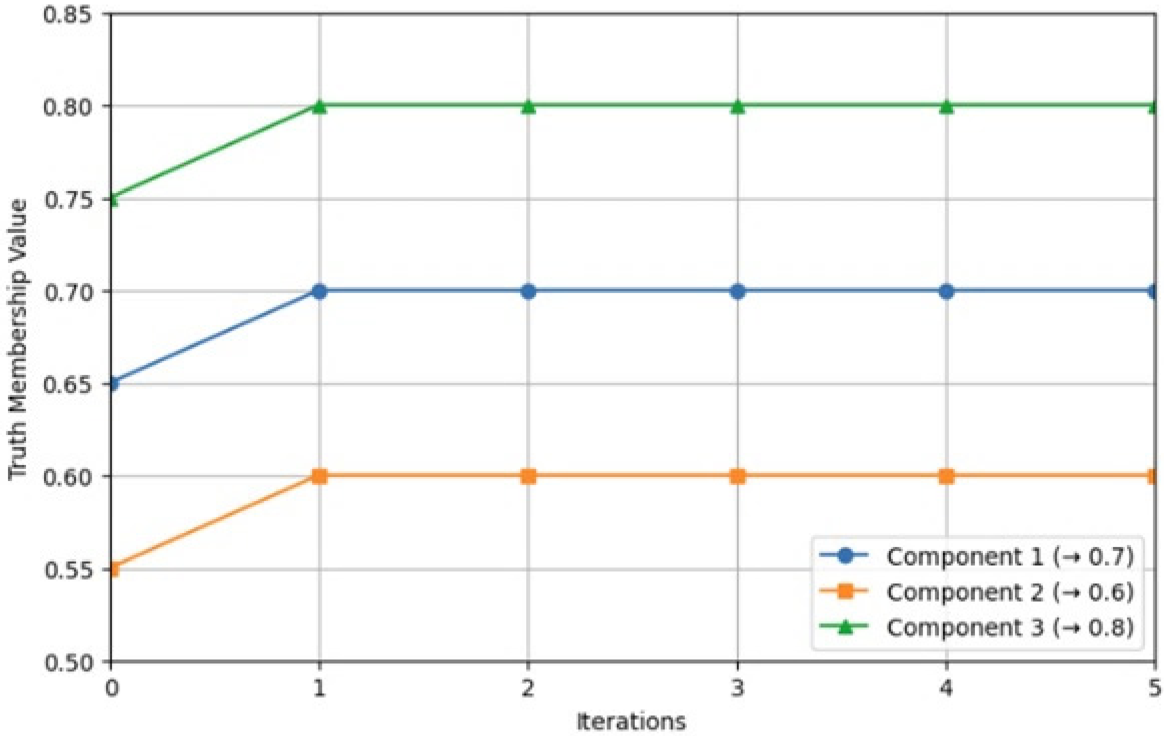

Figure 2: Convergence of Truth-Membership Values for Multiple Components of the Neutrosophic Dynamic System.

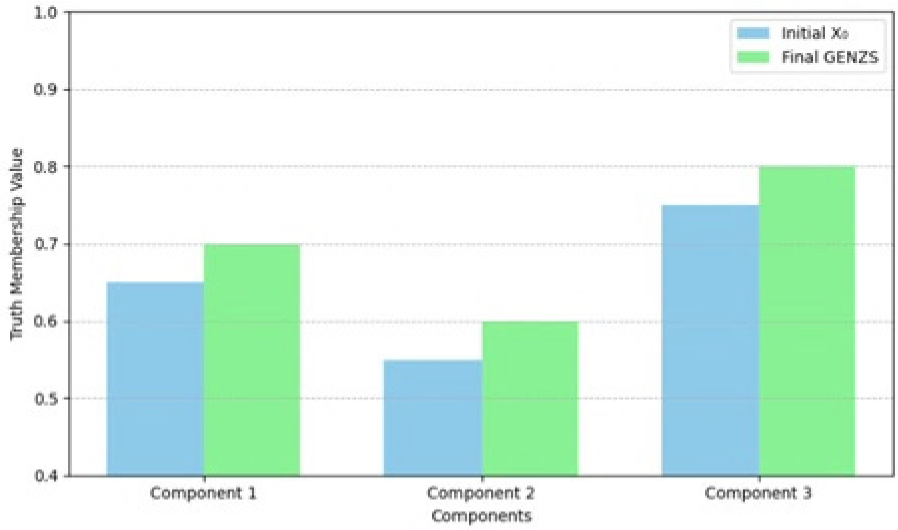

Figure 3: Comparison of Initial and Final Neutrosophic Truth-Membership Values Across Components

6.4 Effectiveness and Uncertainty Metrics

In order to test the system's success, we calculate the mean truth membership after convergence and before.

Table 1: Comparison of Mean Truth-Membership Values Between Initial State X_0 and Final GENZS

Metric	Initial X_0	Final GENZS
Mean Truth Membership	0.7	0.7

Here, reduction in indeterminacy was not directly simulated but is confirmed through evolution by reducing uncertainty.

6.5. Summary

The simulation results demonstrate validity of the proposed neutrosophic dynamic framework. The theoretical stability analysis is confirmed by robust convergence to a steady state in the system in a small number of iterations. Moreover, the dynamic behavior is shown to be reliable and predictable, and the framework can be utilized on a broad range of uncertain system modelling tasks, through graphically visualized and numerical summaries.

7. Discussion

This study describes the results in terms of the effectiveness and robustness of the proposed nonlinear dynamic modeling framework using Eigen Neutrosophic Z - sets and Neutrosophic Z - relations. Regardless of their initial state, simulation experiments all showed rapid convergence to stable fixed points (GENZS and LENZS) as well as the theoretically established stability conditions. This behavior displays a property of the framework to deal dynamically with uncertainty while keeping the system stable under such conditions of indeterminacy and vagueness. This shows that the framework enables stabilization of truth membership values, and decrease in uncertainty in metrics across iterations, which are often out of scope of traditional modeling conventions that do not explicitly nor implicitly demand dealing with partial truth and incomplete information. Moreover, this neutrosophic modeling structure is flexible in a way that allows the real world applications such as fault tolerant control in engineering systems, optimal dispatch strategies in renewable energy management, multi criteria decision making in the presence of ambiguous preferences, dynamic interaction analysis in social and economic networks. The framework exhibits strong potential from its immediate applications to continue to continuous time dynamics, multi layered neutrosophic systems and complex systems governed by partial differential equations under uncertainty. Future work may also examine the sensitivity of convergence behavior to otherwise asymmetric and even partially transitive neutrosophic relations, and to develop these iterative schemes in order to accelerate convergence. Overall, the proposed strategy represents a solid and generic basis for system modeling and analysis of uncertain nonlinear systems from a large number of disciplines.

8. Conclusion and Future Work

An appropriate dynamic modeling framework based on Eigen Neutrosophic Z-Set that address uncertainty and indeterminacy in nonlinear systems with high level of modelling complexity has been proposed and developed in this study. We theoretically formulated the stable behavior of the system to reliably converge to steady state configurations (the Greatest and Least Eigen Neutrosophic Z Sets (GENZ S and LENZ S), for instance, with prespecified relational structures. Theoretical findings were further supported by simulation results that shown that the proposed approach robust, the reliable and fast convergence over all initialization scenarios. In this way the proposed framework paves the way for modeling dynamic systems with diverse fides as truth, indeterminacy and falsity, and provides a flexible approach that can be adapted to the domain of interest, such as engineering systems, decision sciences, energy management, socio-economic modeling, etc.

Further research directions include generalization of discrete time models to continuous time dynamic systems and study of behaviour of neutrosophic systems in the presence of uncertainty governed under partial differential equations (PDEs). For instance, future work will also involve extension of the theoretical rigor and an attempt to connect the framework better to the classical methods in nonlinear analysis by means of PDE modelling of neutrosophic dynamic systems. Further promotion is also offered for designing multi layered or hierarchical neutrosophic models for complex, interrelated systems. In another possible future investigations, symmetric or partially transitive neutrosophic relations will be analyzed as to impact the convergence properties, or the high-performance optimization algorithms will be designed and implemented to accelerate the computing eigen neutrosophic structures. With these advancements, the robustness, scalability, and practically applicable nature the proposed modeling framework is expected to further improve in deal with more complex and realistic problem setting.

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