



Spin Three-Body Problem of Classical Electrodynamics with Radiation Terms – (I) Derivation of Spin Equations

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Abstract

In the present paper spin equations for three-body problem of classical electrodynamics are introduced. They should be considered jointly with 3-body equations of motion derived in a previous paper of the author. The system of spin equations is an overdetermined one. It is shown that the independent spin equations are nine in number as many as the components of the unknown spin functions. The system obtained will be solved by the fixed-point method in a next paper.

Keywords: Spin three-body problem, Classical electrodynamics, Spin equations.

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1. Introduction

The present paper is one of the series devoted to study of the three-body problem of classical electrodynamics. In [5] we have formulated 3-dimensional three-body problem with radiation terms and in [6] we have proved the existence-uniqueness of periodic solution based on the previous results for two-body problem [2], [3]. The system of equations of motion is of neutral type with delays depending on unknown trajectories. We have derived spin equations for two-body problem in [4] using the Corben's considerations of classical spinning particles [10], [11]. The problem for spinning particles in many papers has been considered (cf. for instance [7], [14], [16]-[20]). Here in a similar way we extend the results from [4] applying the interaction presentation for N-body problem from [1] and derive spin equations for three charged particles. We show that to every particle correspond six equations but three of them are consequences from the first three ones

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in the following sense: every such equation differs from the first one by a summand which is a cross product of spin vector and arbitrary constant vector perpendicular to the spin vector. So, we obtain nine equations for nine unknown spin functions, namely the component of the three spin vectors. We note the papers D.-A. Deckert et al. [8], [9], [12], [13], [15] based on the Wheeler- Feynman approach. The paper consists of five sections, Appendix and References. In Section 2 preliminary results for 3-body problem equations of motion with radiation terms and their relations to spin equations are considered. In Section 3 the spin equations are derived and the elements of the spin tensor are defined. In Section 4 a vector form of the Lorentz part and radiation parts of the first three spin equations is presented. Lorentz parts we call the summands in the right-hand sides of the spin equations which take into account for a given particle the influence of the rest ones. The terms which take into account self-interaction of every particle we call radiation terms. In Section 5 we show that the last three equations are consequences of the first three ones. In this way we obtain nine equations for nine unknown spin functions, or we get as many equations as spin functions are. The existence-uniqueness of a periodic solution of the general system in the second part of the present paper will be proved.

2. Preliminary Results for 3-Body Problem Equations of Motion with Radiation Terms and their Relations to Spin Equations

First, we recall some denotations from [2] - [6]. The investigations are in the Minkowski space. Roman suffixes run over 1, 2, 3, 4, while Greek – over 1, 2, 3 with Einstein summation convention. By $\langle \cdot, \cdot \rangle_4$ we denote the scalar product in the Minkowski space, while by $\langle \cdot, \cdot \rangle$ – the scalar product in three-dimensional Euclidean subspace.

Following the approach from [4] and [10] we introduce spin equations jointly with equations of motion with radiation terms derived in [5] for three-body problem:

$$\begin{aligned} \frac{d\lambda_r^{(1)}}{ds_1} &= \frac{e_1}{m_1 c^2} \left(\left(F_{rs}^{(12)} + F_{rs}^{(13)} \right) \lambda_s^{(1)} + F_{rs}^{(1)rad} \lambda_s^{(1)} \right); \\ \frac{d\lambda_r^{(2)}}{ds_2} &= \frac{e_2}{m_2 c^2} \left(\left(F_{rs}^{(21)} + F_{rs}^{(23)} \right) \lambda_s^{(2)} + F_{rs}^{(2)rad} \lambda_s^{(2)} \right); \\ \frac{d\lambda_r^{(3)}}{ds_3} &= \frac{e_3}{m_3 c^2} \left(\left(F_{rs}^{(31)} + F_{rs}^{(32)} \right) \lambda_s^{(3)} + F_{rs}^{(3)rad} \lambda_s^{(3)} \right); \end{aligned} \tag{2.1}$$

$$\begin{aligned} \frac{d\sigma_{ij}^{(1)}}{ds_1} &= \frac{e_1}{m_1 c^2} \left(\left(F_{im}^{(12)} + F_{im}^{(13)} + F_{im}^{(1)rad} \right) \sigma_{mj}^{(1)} - \sigma_{im}^{(1)} \left(F_{mj}^{(12)} + F_{mj}^{(13)} + F_{mj}^{(1)rad} \right) \right); \\ \frac{d\sigma_{ij}^{(2)}}{ds_2} &= \frac{e_2}{m_2 c^2} \left(\left(F_{im}^{(21)} + F_{im}^{(23)} + F_{im}^{(2)rad} \right) \sigma_{mj}^{(2)} - \sigma_{im}^{(2)} \left(F_{mj}^{(21)} + F_{mj}^{(23)} + F_{mj}^{(2)rad} \right) \right); \\ \frac{d\sigma_{ij}^{(3)}}{ds_3} &= \frac{e_3}{m_3 c^2} \left(\left(F_{im}^{(31)} + F_{im}^{(32)} + F_{im}^{(3)rad} \right) \sigma_{mj}^{(3)} - \sigma_{im}^{(3)} \left(F_{mj}^{(31)} + F_{mj}^{(32)} + F_{mj}^{(3)rad} \right) \right). \end{aligned} \tag{2.2}$$

The electromagnetic tensors $F_{rs}^{(kn)}$ in the right-hand sides we call Lorentz parts. For each particle, they take into account the influence of other particles. The quantities relating to the particles are:

$(x_1^{(k)}(t), x_2^{(k)}(t), x_3^{(k)}(t), x_4^{(k)} = ict) \equiv (\vec{x}^{(k)}, ict)$, $(k = 1, 2, 3)$ – space-time coordinates of the moving particles; c – the vacuum speed of light; L_k – the world lines; m_k – proper masses; e_k – charges $(k = 1, 2, 3)$.

The elements of the electromagnetic tensors $F_{rl}^{(kn)} = \frac{\partial \Phi_l^{(n)}}{\partial x_r^{(k)}} - \frac{\partial \Phi_r^{(n)}}{\partial x_l^{(k)}}$ can be calculated by the retarded

Lienard-Wiechert potentials $\Phi_r^{(n)} = -\frac{e_n \lambda_r^{(n)}}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4}$, where

$$\begin{aligned} \xi^{(kn)} &= \left(\xi_1^{(kn)}, \xi_2^{(kn)}, \xi_3^{(kn)}, \xi_4^{(kn)} \right) \\ &= \left(x_1^{(k)}(t) - x_1^{(n)}(t - \tau_{kn}), x_2^{(k)}(t) - x_2^{(n)}(t - \tau_{kn}), x_3^{(k)}(t) - x_3^{(n)}(t - \tau_{kn}), ic\tau_{kn}(t) \right); \\ \lambda^{(k)} &= \left(\lambda_1^{(k)}, \lambda_2^{(k)}, \lambda_3^{(k)}, \lambda_4^{(k)} \right) = \left(\vec{\lambda}^{(k)}, \lambda_4^{(k)} \right) = \left(\frac{u_1^{(k)}}{\Delta_k}, \frac{u_2^{(k)}}{\Delta_k}, \frac{u_3^{(k)}}{\Delta_k}, \frac{ic}{\Delta_k} \right) = \left(\frac{\vec{u}^{(k)}}{\Delta_k}, \frac{ic}{\Delta_k} \right); \\ \Delta_k &= \sqrt{c^2 - \langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \rangle}; \Delta_n = \sqrt{c^2 - \langle \vec{u}^{(n)}(t - \tau_{kn}), \vec{u}^{(n)}(t - \tau_{kn}) \rangle}. \end{aligned}$$

Since $\xi^{(kn)}$ are isotropic 4-vectors, i.e. $\langle \xi^{(kn)}, \xi^{(kn)} \rangle_4 = 0$, the retarded functions $\tau_{kn}(t)$ can be defined as solutions of the functional equations,

$$\tau_{kn}(t) = \frac{1}{c} \sqrt{\langle \vec{\xi}^{(kn)}, \vec{\xi}^{(kn)} \rangle} \equiv \frac{1}{c} \sqrt{\sum_{\alpha=1}^3 \left[x_\alpha^{(k)}(t) - x_\alpha^{(n)}(t - \tau_{kn}(t)) \right]^2}, \tag{2.3}$$

where $(kn) = (12), (13), (21), (23), (31), (32)$.

Following denotations from [5] we obtain

$$\begin{aligned} \frac{d\lambda_r^{(k)}}{ds_k} &= \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 F_{rm}^{(kn)} \lambda_m^{(k)}; F_{rm}^{(kn)} = \frac{\partial \Phi_m^{(n)}}{\partial x_r^{(k)}} - \frac{\partial \Phi_r^{(n)}}{\partial x_m^{(k)}} = e_n \left(P_r^{(kn)} \xi_m^{(kn)} - P_m^{(kn)} \xi_r^{(kn)} \right); \\ \frac{d\lambda_r^{(k)}}{ds_k} &= \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 e_n \left(P_r^{(kn)} \xi_m^{(kn)} - P_m^{(kn)} \xi_r^{(kn)} \right) \lambda_m^{(k)}; \\ \frac{d\vec{\lambda}^{(k)}}{ds_k} &= -\frac{e_k^2}{m_k c^2} \sum_{n=1, n \neq k}^3 e_n \left[\vec{P}^{(kn)} \langle \lambda^{(k)}, \xi^{(kn)} \rangle_4 - \vec{\xi}^{(kn)} \langle \lambda^{(k)}, P^{(kn)} \rangle_4 \right]; \\ \frac{d\vec{\lambda}^{(k)}}{ds_k} &= \frac{1}{\Delta_k^2} \frac{d\vec{u}^{(k)}(t)}{dt} + \frac{\vec{u}^{(k)}(t)}{\Delta_k^4} \left\langle \vec{u}^{(k)}(t), \frac{d\vec{u}^{(k)}(t)}{dt} \right\rangle; \frac{d\lambda_4^{(k)}}{ds_k} = \frac{ic}{\Delta_k^4} \left\langle \vec{u}^{(k)}(t), \frac{d\vec{u}^{(k)}(t)}{dt} \right\rangle; \\ D_{kn} &= \frac{c^2 \tau_{kn} - \langle \vec{\xi}^{(kn)}, \vec{u}^{(n)} \rangle}{c^2 \tau_{kn} - \langle \vec{\xi}^{(kn)}, \vec{u}^{(k)} \rangle}; \frac{d\vec{\lambda}^{(n)}}{ds_n} = \frac{D_{kn}}{\Delta_n^2} \left(\frac{d\vec{u}^{(n)}}{dt} + \frac{\vec{u}^{(n)}}{\Delta_{kn}^2} \left\langle \vec{u}^{(n)}, \frac{d\vec{u}^{(n)}}{dt} \right\rangle \right); \\ \frac{d\lambda_4^{(n)}}{ds_n} &= \frac{ic D_{kn}}{\Delta_{kn}^4} \left\langle \vec{u}^{(n)}, \frac{d\vec{u}^{(n)}}{dt} \right\rangle; P_r^{(kn)} = -\frac{\lambda_r^{(n)}}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^3} \left[1 + \left\langle \xi^{(kn)}, \frac{d\lambda^{(n)}}{ds_n} \right\rangle_4 \right] + \frac{1}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^2} \frac{d\lambda_r^{(n)}}{ds_n}; \\ \vec{P}^{(kn)} &= -\frac{\vec{\lambda}^{(n)} M_{kn}}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^3} + \frac{1}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^2} \frac{d\vec{\lambda}^{(n)}}{ds_n}; M_{kn} = 1 + \left\langle \xi^{(kn)}, \frac{d\lambda^{(n)}}{ds_n} \right\rangle_4; \\ P_4^{(kn)} &= -\frac{\lambda_4^{(n)} M_{kn}}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^3} + \frac{1}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^2} \frac{d\lambda_4^{(n)}}{ds_n} = -ic \left(\frac{M_{kn}}{\Delta_n \langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^3} - \frac{D_{kn} \langle \vec{u}^{(n)}, \dot{\vec{u}}^{(n)} \rangle}{\langle \lambda^{(n)}, \xi^{(kn)} \rangle_4^2 \Delta_n^4} \right) \equiv -ic L_{kn}; \\ \frac{i}{c} P_4^{(kn)} &= L_{kn} \quad (k = 1, 2, 3; r = 1, 2, 3, 4). \end{aligned}$$

The radiation terms are defined as a half of the difference of retarded and advanced potentials in accordance with the Dirac assumption:

$$F_{ml}^{(k)rad} = \frac{1}{2} \left[\left(\frac{\partial \Phi_l^{(k)ret}}{\partial x_m^{(k)ret}} - \frac{\partial \Phi_m^{(n)ret}}{\partial x_l^{(k)ret}} \right) - \left(\frac{\partial \Phi_l^{(k)adv}}{\partial x_m^{(k)adv}} - \frac{\partial \Phi_m^{(n)adv}}{\partial x_l^{(k)adv}} \right) \right],$$

where $\Phi_r^{(k)ret} = -\frac{e_k \lambda_r^{(k)ret}}{\langle \lambda^{(k)ret}, \xi^{(k)ret} \rangle_4}$, $\Phi_r^{(k)adv} = -\frac{e_k \lambda_r^{(k)adv}}{\langle \lambda^{(k)adv}, \xi^{(k)adv} \rangle_4}$ and

$$F_{rs}^{(k)ret} = \frac{\partial \Phi_s^{(k)ret}}{\partial x_r^{(k)}} - \frac{\partial \Phi_r^{(k)ret}}{\partial x_s^{(k)}} = e_k \left(P_r^{(k)ret} \xi_s^{(k)ret} - P_s^{(k)ret} \xi_r^{(k)ret} \right);$$

$$F_{rs}^{(k)adv} = \frac{\partial \Phi_s^{(k)adv}}{\partial x_r^{(k)}} - \frac{\partial \Phi_r^{(k)adv}}{\partial x_s^{(k)}} = e_k \left(P_r^{(k)adv} \xi_s^{(k)adv} - P_s^{(k)adv} \xi_r^{(k)adv} \right),$$

where

$$P_r^{(k)ret} = -\frac{\lambda_r^{(k)ret}}{\langle \lambda^{(k)ret}, \xi^{(k)ret} \rangle_4} \left[1 + \left\langle \xi^{(k)ret}, \frac{d\lambda^{(k)ret}}{ds_k} \right\rangle_4 \right] + \frac{1}{\langle \lambda^{(k)ret}, \xi^{(k)ret} \rangle_4^2} \frac{d\lambda_r^{(k)ret}}{ds_k};$$

$$P_r^{(k)adv} = -\frac{\lambda_r^{(k)adv}}{\langle \lambda^{(k)adv}, \xi^{(k)adv} \rangle_4} \left[1 + \left\langle \xi^{(k)adv}, \frac{d\lambda^{(k)adv}}{ds_k} \right\rangle_4 \right] + \frac{1}{\langle \lambda^{(k)adv}, \xi^{(k)adv} \rangle_4^2} \frac{d\lambda_r^{(k)adv}}{ds_k};$$

$$F_{rs}^{(k)rad} = \frac{F_{rs}^{(k)ret} - F_{rs}^{(k)adv}}{2} = e_k \frac{P_r^{(k)ret} \xi_s^{(k)ret} - P_s^{(k)ret} \xi_r^{(k)ret} - \left(P_r^{(k)adv} \xi_s^{(k)adv} - P_s^{(k)adv} \xi_r^{(k)adv} \right)}{2}.$$

In the previous paper [5] it is proved that every fourth equation of (2.1) is a consequence of the first three ones. So under the assumption $\sqrt{\langle \vec{u}^{(k)}(t), \vec{u}^{(k)}(t) \rangle} \leq \bar{c} < c$ ($k = 1, 2, 3$) the equations of motion (2.1) are 12 in number but it is proved that three of them are consequence of the rest ones so we obtain nine in number independent equations. In [6] an existence-uniqueness of a periodic solution of (2.1) is proved.

3. Derivation of the Spin Equations

Recalling the summation in repeated Latin indices we introduce an explicit form of the spin equations:

$$\frac{d\sigma_{ij}^{(1)}}{ds_1} = \frac{e_1}{m_1 c^2} \left[\left(F_{im}^{(12)} + F_{im}^{(13)} + F_{im}^{(1)rad} \right) \sigma_{mj}^{(1)} - \sigma_{im}^{(1)} \left(F_{mj}^{(12)} + F_{mj}^{(13)} + F_{mj}^{(1)rad} \right) \right];$$

$$\frac{d\sigma_{ij}^{(2)}}{ds_2} = \frac{e_2}{m_2 c^2} \left[\left(F_{im}^{(21)} + F_{im}^{(23)} + F_{im}^{(2)rad} \right) \sigma_{mj}^{(2)} - \sigma_{im}^{(2)} \left(F_{mj}^{(21)} + F_{mj}^{(23)} + F_{mj}^{(2)rad} \right) \right];$$

$$\frac{d\sigma_{ij}^{(3)}}{ds_3} = \frac{e_3}{m_3 c^2} \left[\left(F_{im}^{(31)} + F_{im}^{(32)} + F_{im}^{(3)rad} \right) \sigma_{mj}^{(3)} - \sigma_{im}^{(3)} \left(F_{mj}^{(31)} + F_{mj}^{(32)} + F_{mj}^{(3)rad} \right) \right]$$

or in abbreviated form

$$\frac{d\sigma_{ij}^{(k)}}{ds_k} = \frac{e_1}{m_1 c^2} \left[\left(\sum_{n=1, n \neq k}^3 F_{im}^{(kn)} + F_{im}^{(k)rad} \right) \sigma_{mj}^{(1)} - \sigma_{im}^{(1)} \left(\sum_{n=1, n \neq k}^3 F_{mj}^{(kn)} + F_{mj}^{(1)rad} \right) \right].$$

Introduce denotations following [4]:

$$\vec{\theta}^{(k)} = \left(\theta_1^{(k)}(t), \theta_2^{(k)}(t), \theta_3^{(k)}(t) \right); \vec{\sigma}^{(k)} = \left(\sigma_1^{(k)}, \sigma_2^{(k)}, \sigma_3^{(k)} \right);$$

$$\vec{\theta}^{(k)} = \frac{1}{c} \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) = \frac{1}{c} \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \frac{1}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \vec{e}_1 - \frac{1}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \vec{e}_2 + \frac{1}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \vec{e}_3$$

that is,

$$\vec{\theta}^{(k)} = \left(\frac{1}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix}; -\frac{1}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix}, \frac{1}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \right)$$

we define the spin tensor

$$\sigma_{\mu\nu}^{(k)} = \begin{pmatrix} 0 & \sigma_3^{(k)} & -\sigma_2^{(k)} & i\theta_1^{(k)} \\ -\sigma_3^{(k)} & 0 & \sigma_1^{(k)} & i\theta_2^{(k)} \\ \sigma_2^{(k)} & -\sigma_1^{(k)} & 0 & i\theta_3^{(k)} \\ -i\theta_1^{(k)} & -i\theta_2^{(k)} & -i\theta_3^{(k)} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & \sigma_3^{(k)} & -\sigma_2^{(k)} & \frac{i}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \\ -\sigma_3^{(k)} & 0 & \sigma_1^{(k)} & -\frac{i}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \\ \sigma_2^{(k)} & -\sigma_1^{(k)} & 0 & \frac{i}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \\ -\frac{i}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} & \frac{i}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} & -\frac{i}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} & 0 \end{pmatrix}.$$

Thus, we introduce the following equations describing the spin of three charged particles:

$$\frac{d\sigma_{12}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\left(\sum_{n=1, n \neq k}^3 F_{1m}^{(kn)} + F_{1m}^{(k)rad} \right) \sigma_{m2}^{(k)} - \sigma_{1m}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{m2}^{(kn)} + F_{m2}^{(k)rad} \right) \right];$$

$$\frac{d\sigma_{13}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\left(\sum_{n=1, n \neq k}^3 F_{1m}^{(kn)} + F_{1m}^{(k)rad} \right) \sigma_{m3}^{(k)} - \sigma_{1m}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{m3}^{(kn)} + F_{m3}^{(k)rad} \right) \right];$$

$$\frac{d\sigma_{23}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\left(\sum_{n=1, n \neq k}^3 F_{2m}^{(kn)} + F_{2m}^{(k)rad} \right) \sigma_{m3}^{(k)} - \sigma_{2m}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{m3}^{(kn)} + F_{m3}^{(k)rad} \right) \right];$$

$$\frac{d\sigma_{14}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\left(\sum_{n=1, n \neq k}^3 F_{1m}^{(kn)} + F_{1m}^{(k)rad} \right) \sigma_{m4}^{(k)} - \sigma_{1m}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{m4}^{(kn)} + F_{m4}^{(k)rad} \right) \right];$$

$$\frac{d\sigma_{24}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\left(\sum_{n=1, n \neq k}^3 F_{2m}^{(kn)} + F_{2m}^{(k)rad} \right) \sigma_{m4}^{(k)} - \sigma_{2m}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{m4}^{(kn)} + F_{m4}^{(k)rad} \right) \right];$$

$$\frac{d\sigma_{34}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\left(\sum_{n=1, n \neq k}^3 F_{3m}^{(kn)} + F_{3m}^{(k)rad} \right) \sigma_{m4}^{(k)} - \sigma_{3m}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{m4}^{(kn)} + F_{m4}^{(k)rad} \right) \right]$$

or

$$\frac{d\sigma_{12}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m2}^{(k)} - F_{m2}^{(kn)} \sigma_{1m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m2}^{(k)} - F_{m2}^{(k)rad} \sigma_{1m}^{(k)} \right); \tag{3.1}$$

$$\frac{d\sigma_{13}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{1m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{1m}^{(k)} \right); \tag{3.2}$$

$$\frac{d\sigma_{23}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{2m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{2m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{2m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{2m}^{(k)} \right); \tag{3.3}$$

$$\frac{d\sigma_{14}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{1m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m4}^{(k)} - F_{m4}^{(k)rad} \sigma_{1m}^{(k)} \right); \tag{3.4}$$

$$\frac{d\sigma_{24}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{2m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{2m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{2m}^{(k)rad} \sigma_{m4}^{(k)} - F_{m4}^{(k)rad} \sigma_{2m}^{(k)} \right); \tag{3.5}$$

$$\frac{d\sigma_{34}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{3m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{3m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{3m}^{(k)rad} \sigma_{m4}^{(k)} - F_{m4}^{(k)rad} \sigma_{3m}^{(k)} \right). \tag{3.6}$$

4. Vector Form of the Lorentz and Radiation Parts of the First Three Spin Equations

The primary goal of the present section is to transform equations (3.1) - (3.6) and to obtain a vector form of the spin equations. Recall that $F_{im}^{(kn)}$ are the Lorentz parts while $F_{im}^{(k)rad}$ - the radiation parts.

Let us consider the cross products with respect to some base $\vec{e}_1, \vec{e}_2, \vec{e}_3$:

$$\vec{\sigma}^{(k)} \times \vec{\xi}^{\rightarrow(k)} = \left(\left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{array} \right|, - \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{array} \right|, \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{array} \right| \right);$$

$$\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} = \left(\left| \begin{array}{cc} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{array} \right|, - \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{array} \right|, \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{array} \right| \right);$$

$$\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} = \left(\left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{array} \right|, - \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{array} \right|, \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{array} \right| \right)$$

and

$$\begin{aligned} \vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{\rightarrow(kn)} \right) &= \left| \begin{array}{ccc} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{array} \right| & - \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{array} \right| & \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{array} \right| \end{array} \right| = \\ &= \left(P_2^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{array} \right| + P_3^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{array} \right| \right) \vec{e}_1 - \\ &- \left(P_1^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{array} \right| - P_3^{(kn)} \left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{array} \right| \right) \vec{e}_2 + \\ &+ \left(-P_1^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{array} \right| - P_2^{(kn)} \left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{array} \right| \right) \vec{e}_3. \end{aligned}$$

We infer

$$\begin{aligned} P_2^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{array} \right| + P_3^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{array} \right| &= \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{\rightarrow(k)} \right) \right]_1; \\ -P_1^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{array} \right| + P_3^{(kn)} \left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{array} \right| &= \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{\rightarrow(k)} \right) \right]_2; \\ -P_1^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{array} \right| - P_2^{(kn)} \left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{array} \right| &= \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{\rightarrow(k)} \right) \right]_3 \end{aligned}$$

where the right subscripts mean the number of the coordinate. In the same way

$$\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) = \left| \begin{array}{ccc} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \left| \begin{array}{cc} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{array} \right| & - \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{array} \right| & \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{array} \right| \end{array} \right| =$$

$$= \left(P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_1 - \left(P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - P_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_2 +$$

$$+ \left(-P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - P_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_3$$

that means

$$P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_1;$$

$$-P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2;$$

$$-P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - P_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} = \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3.$$

For the next term

$$\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} & - \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} & \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} \end{vmatrix} =$$

$$= \left(\xi_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \xi_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} \right) \vec{e}_1 -$$

$$- \left(\xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} - \xi_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} \right) \vec{e}_2 +$$

$$+ \left(-\xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - \xi_2^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} \right) \vec{e}_3.$$

Therefore

$$\xi_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \xi_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} = \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)} \right) \right]_1;$$

$$-\xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + \xi_3^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} = \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)} \right) \right]_2;$$

$$-\xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - \xi_2^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} = \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)} \right) \right]_3$$

and finally

$$\vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) = \begin{vmatrix} \vec{e}_1 & \vec{e}_2 & \vec{e}_3 \\ \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} & - \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} & \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \end{vmatrix} =$$

$$= \left(\xi_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + \xi_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_1 - \left(\xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - \xi_3^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_2 +$$

$$+ \left(-\xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \xi_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \vec{e}_3$$

that is,

$$\begin{aligned} \xi_2^{(kn)} \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{array} \right| + \xi_3^{(kn)} \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{array} \right| &= \left[\vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_1; \\ -\xi_1^{(kn)} \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{array} \right| + \xi_3^{(kn)} \left| \begin{array}{cc} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{array} \right| &= \left[\vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2; \\ -\xi_1^{(kn)} \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{array} \right| - \xi_2^{(kn)} \left| \begin{array}{cc} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{array} \right| &= \left[\vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3. \end{aligned}$$

Transformation of the Lorentz Part of Equation (3.1)

First, we transform the Lorentz part, because the radiation parts can be transformed in the same way. So instead of

$$\frac{d\sigma_{12}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m2}^{(k)} - \sigma_{1m}^{(k)} F_{m2}^{(kn)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m2}^{(k)} - \sigma_{1m}^{(k)} F_{m2}^{(k)rad} \right)$$

we consider only

$$\begin{aligned} \frac{d\sigma_{12}^{(k)}}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left[\sigma_{m2}^{(k)} \left(P_1^{(kn)} \xi_m^{(kn)} - P_m^{(kn)} \xi_1^{(kn)} \right) - \sigma_{1m}^{(k)} \left(P_m^{(kn)} \xi_2^{(kn)} - P_2^{(kn)} \xi_m^{(kn)} \right) \right] = \\ &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(P_1^{(kn)} \sigma_{m2}^{(k)} \xi_m^{(kn)} - \xi_1^{(kn)} \sigma_{m2}^{(k)} P_m^{(kn)} - \xi_2^{(kn)} \sigma_{1m}^{(k)} P_m^{(kn)} + P_2^{(kn)} \sigma_{1m}^{(k)} \xi_m^{(kn)} \right). \end{aligned}$$

But

$$\begin{aligned} P_1^{(kn)} \sigma_{m2}^{(k)} \xi_m^{(kn)} &= P_1^{(kn)} \left(\sigma_{12}^{(k)} \xi_1^{(kn)} + \sigma_{32}^{(k)} \xi_3^{(kn)} + \sigma_{42}^{(k)} \xi_4^{(kn)} \right) = P_1^{(kn)} \left(\sigma_3^{(k)} \xi_1^{(kn)} - \sigma_1^{(k)} \xi_3^{(kn)} + \sigma_{42}^{(k)} \xi_4^{(kn)} \right) = \\ &= -P_1^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{array} \right| - P_1^{(kn)} \tau_{kn} \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{array} \right|; \\ -\xi_1^{(kn)} \sigma_{m2}^{(k)} P_m^{(kn)} &= -\xi_1^{(kn)} \left(\sigma_{12}^{(k)} P_1^{(kn)} + \sigma_{32}^{(k)} P_3^{(kn)} + \sigma_{42}^{(k)} P_4^{(kn)} \right) \\ &= -\xi_1^{(kn)} \left(\sigma_3^{(k)} P_1^{(kn)} - \sigma_1^{(k)} P_3^{(kn)} + \sigma_{42}^{(k)} P_4^{(kn)} \right) = \xi_1^{(kn)} \left| \begin{array}{cc} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{array} \right| - \xi_1^{(kn)} L_{kn} \left| \begin{array}{cc} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{array} \right|; \\ -\xi_2^{(kn)} \sigma_{1m}^{(k)} P_m^{(kn)} &= -\xi_2^{(kn)} \left(\sigma_{12}^{(k)} P_2^{(kn)} + \sigma_{13}^{(k)} P_3^{(kn)} + \sigma_{14}^{(k)} P_4^{(kn)} \right) = \\ &= -\xi_2^{(kn)} \left(\sigma_3^{(k)} P_2^{(kn)} - \sigma_2^{(k)} P_3^{(kn)} + \sigma_{14}^{(k)} P_4^{(kn)} \right) = \xi_2^{(kn)} \left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{array} \right| - \xi_2^{(kn)} L_{kn} \left| \begin{array}{cc} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{array} \right|; \\ P_2^{(kn)} \sigma_{1m}^{(k)} \xi_m^{(kn)} &= P_2^{(kn)} \left(\sigma_{12}^{(k)} \xi_2^{(kn)} + \sigma_{13}^{(k)} \xi_3^{(kn)} + \sigma_{14}^{(k)} \xi_4^{(kn)} \right) = P_2^{(kn)} \left(\sigma_3^{(k)} \xi_2^{(kn)} - \sigma_2^{(k)} \xi_3^{(kn)} + \sigma_{14}^{(k)} \xi_4^{(kn)} \right) = \\ &= -P_2^{(kn)} \left| \begin{array}{cc} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{array} \right| - \tau_{kn} P_2^{(kn)} \left| \begin{array}{cc} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{array} \right| \end{aligned}$$

and then in view of $\frac{d\sigma_{12}^{(k)}}{ds_k} = \frac{d\sigma_3^{(k)}}{ds_k}$ we obtain

$$\frac{d\sigma_3^{(k)}}{ds_k} = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(-P_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} - \tau_{kn} P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - \right. \\ \left. -L_{kn} \xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \xi_2^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} - L_{kn} \xi_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \right. \\ \left. -P_2^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} - \tau_{kn} P_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right)$$

or

$$\frac{d\sigma_3^{(k)}}{ds_k} = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(-P_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} - P_2^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} - \tau_{kn} P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \right. \\ \left. -\tau_{kn} P_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} + \xi_2^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} - L_{kn} \xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \right. \\ \left. -L_{kn} \xi_2^{(kn)} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right)$$

and consequently

$$\frac{d\sigma_3^{(k)}}{ds_k} = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left\{ \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) \right]_3 + \tau_{kn} \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3 - \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) \right]_3 + \right. \\ \left. + L_{kn} \left[\vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3 \right\}.$$

Transformation of the Lorentz Part of Equation (3.2)

Here

$$\frac{d\sigma_{13}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m3}^{(k)} - \sigma_{1m}^{(k)} F_{m3}^{(kn)} \right) + \frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m3}^{(k)} - \sigma_{1m}^{(k)} F_{m3}^{(k)rad} \right);$$

$$\frac{d\sigma_{13}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m3}^{(k)} - \sigma_{1m}^{(k)} F_{m3}^{(kn)} \right)$$

$$= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left[\sigma_{m3}^{(k)} \left(P_1^{(kn)} \xi_m^{(kn)} - P_m^{(kn)} \xi_1^{(kn)} \right) - \sigma_{1m}^{(k)} \left(P_m^{(kn)} \xi_3^{(kn)} - P_3^{(kn)} \xi_m^{(kn)} \right) \right]$$

$$= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(P_1^{(kn)} \sigma_{m3}^{(k)} \xi_m^{(kn)} - \sigma_{m3}^{(k)} P_m^{(kn)} \xi_1^{(kn)} - \sigma_{1m}^{(k)} P_m^{(kn)} \xi_3^{(kn)} + P_3^{(kn)} \sigma_{1m}^{(k)} \xi_m^{(kn)} \right);$$

$$P_1^{(kn)} \sigma_{m3}^{(k)} \xi_m^{(kn)} = P_1^{(kn)} \left(\sigma_{13}^{(k)} \xi_1^{(kn)} + \sigma_{23}^{(k)} \xi_2^{(kn)} + \sigma_{43}^{(k)} \xi_4^{(kn)} \right) = P_1^{(kn)} \left(-\sigma_2^{(k)} \xi_1^{(kn)} + \sigma_1^{(k)} \xi_2^{(kn)} + \sigma_{43}^{(k)} \xi_4^{(kn)} \right)$$

$$= P_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} + \tau_{kn} P_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix};$$

$$-\xi_1^{(kn)} \sigma_{m3}^{(k)} P_m^{(kn)} = -\xi_1^{(kn)} \left(\sigma_{13}^{(k)} P_1^{(kn)} + \sigma_{23}^{(k)} P_2^{(kn)} + \sigma_{43}^{(k)} P_4^{(kn)} \right) = -\xi_1^{(kn)} \left(-\sigma_2^{(k)} P_1^{(kn)} + \sigma_1^{(k)} P_2^{(kn)} + \sigma_{43}^{(k)} P_4^{(kn)} \right)$$

$$= -\xi_1^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + L_{kn} \xi_1^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix};$$

$$-\xi_3^{(kn)} \sigma_{1m}^{(k)} P_m^{(kn)} = -\xi_3^{(kn)} \left(\sigma_{12}^{(k)} P_2^{(kn)} + \sigma_{13}^{(k)} P_3^{(kn)} + \sigma_{14}^{(k)} P_4^{(kn)} \right) = -\xi_3^{(kn)} \left(\sigma_3^{(k)} P_2^{(kn)} - \sigma_2^{(k)} P_3^{(kn)} + \sigma_{14}^{(k)} P_4^{(kn)} \right)$$

$$\begin{aligned}
 &= \xi_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{matrix} \right| - L_{kn} \xi_3^{(kn)} \left| \begin{matrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{matrix} \right|; \\
 P_3^{(kn)} \sigma_1^{(k)} \xi_m^{(kn)} &= P_3^{(kn)} \left(\sigma_{12}^{(k)} \xi_2^{(kn)} + \sigma_{13}^{(k)} \xi_3^{(kn)} + \sigma_{14}^{(k)} \xi_4^{(kn)} \right) = P_3^{(kn)} \left(\sigma_3^{(k)} \xi_2^{(kn)} - \sigma_2^{(k)} \xi_3^{(kn)} + \sigma_{14}^{(k)} \xi_4^{(kn)} \right) = \\
 &= -P_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{matrix} \right| - \tau_{kn} P_3^{(kn)} \left| \begin{matrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{matrix} \right|.
 \end{aligned}$$

Then

$$\begin{aligned}
 &\sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m3}^{(k)} - \sigma_{1m}^{(k)} F_{m3}^{(kn)} \right) = \\
 = \sum_{n=1, n \neq k}^3 e_n &\left(P_1^{(kn)} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{matrix} \right| + \tau_{kn} P_1^{(kn)} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{matrix} \right| - \xi_1^{(kn)} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{matrix} \right| + \right. \\
 &+ L_{kn} \xi_1^{(kn)} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{matrix} \right| + \xi_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{matrix} \right| - L_{kn} \xi_3^{(kn)} \left| \begin{matrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{matrix} \right| - P_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{matrix} \right| - \\
 &\left. - \tau_{kn} P_3^{(kn)} \left| \begin{matrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{matrix} \right| \right).
 \end{aligned}$$

In view of $\frac{d\sigma_{13}^{(k)}}{ds_k} = -\frac{d\sigma_2^{(k)}}{ds_k}$ we have

$$\begin{aligned}
 -\frac{d\sigma_2^{(k)}}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(P_1^{(kn)} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{matrix} \right| - P_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{matrix} \right| + \tau_{kn} P_1^{(kn)} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{matrix} \right| - \right. \\
 &- \tau_{kn} P_3^{(kn)} \left| \begin{matrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{matrix} \right| - \xi_1^{(kn)} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{matrix} \right| + \xi_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{matrix} \right| + \\
 &\left. + L_{kn} \xi_1^{(kn)} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{matrix} \right| - L_{kn} \xi_3^{(kn)} \left| \begin{matrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{matrix} \right| \right).
 \end{aligned}$$

Therefore

$$\begin{aligned}
 P_1^{(kn)} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{matrix} \right| - P_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{matrix} \right| &= - \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)} \right) \right]_2; \\
 \tau_{kn} P_1^{(kn)} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{matrix} \right| - \tau_{kn} P_3^{(kn)} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{matrix} \right| &= -\tau_{kn} \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2; \\
 \xi_1^{(kn)} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{matrix} \right| - \xi_3^{(kn)} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{matrix} \right| &= \left[\vec{\xi}^{(k)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) \right]_2; \\
 L_{kn} \xi_1^{(kn)} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{matrix} \right| - L_{kn} \xi_3^{(kn)} \left| \begin{matrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{matrix} \right| &= -L_{kn} \left[\vec{\xi}^{(k)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2
 \end{aligned}$$

that is,

$$\begin{aligned}
 -\frac{d\sigma_2^{(k)}}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left\{ - \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)} \right) \right]_2 - \tau_{kn} \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\lambda}^{(k)} \right) \right]_2 + \right. \\
 &\left. + \left[\vec{\xi}^{(k)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) \right]_2 - L_{kn} L_{kn} \left[\vec{\xi}^{(k)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 \right\}
 \end{aligned}$$

or

$$\frac{d\sigma_2^{(k)}}{ds_k} = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left\{ \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) \right]_2 + \tau_{kn} \tau_{kn} \left[\vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 - \left[\vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) \right]_2 + L_{kn} L_{kn} \left[\vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 \right\}.$$

Transformation of the Lorentz part of equation (3.3)

For the Lorentz part of the third equation

$$\frac{d\sigma_{23}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \sum_{n=1, n \neq k}^3 \left(F_{2m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{2m}^{(k)} \right) + \frac{e_k}{m_k c^2} \left(F_{2m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{2m}^{(k)} \right)$$

we have

$$\begin{aligned} \sum_{n=1, n \neq k}^3 \left(F_{2m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{2m}^{(k)} \right) &= \sum_{n=1, n \neq k}^3 e_n \left(\left(P_2^{(kn)} \xi_m^{(kn)} - P_m^{(kn)} \xi_2^{(kn)} \right) \sigma_{m3}^{(k)} - \left(P_m^{(kn)} \xi_3^{(kn)} - P_3^{(kn)} \xi_m^{(kn)} \right) \sigma_{2m}^{(k)} \right) \\ &= \sum_{n=1, n \neq k}^3 e_n \left(P_2^{(kn)} \sigma_{m3}^{(k)} \xi_m^{(kn)} - \sigma_{m3}^{(k)} P_m^{(kn)} \xi_2^{(kn)} - P_m^{(kn)} \sigma_{2m}^{(k)} \xi_3^{(kn)} + P_3^{(kn)} \sigma_{2m}^{(k)} \xi_m^{(kn)} \right); \end{aligned}$$

$$P_2^{(kn)} \sigma_{m3}^{(k)} \xi_m^{(kn)} = P_2^{(kn)} \left(\sigma_{13}^{(k)} \xi_1^{(kn)} + \sigma_{23}^{(k)} \xi_2^{(kn)} + \sigma_{43}^{(k)} \xi_4^{(kn)} \right) = P_2^{(kn)} \left(-\sigma_2^{(k)} \xi_1^{(kn)} + \sigma_1^{(k)} \xi_2^{(kn)} + \sigma_{43}^{(k)} \xi_4^{(kn)} \right)$$

$$= P_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} + \tau_{kn} P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix};$$

$$-\xi_2^{(kn)} \sigma_{m3}^{(k)} P_m^{(kn)} = -\xi_2^{(kn)} \left(\sigma_{13}^{(k)} P_1^{(kn)} + \sigma_{23}^{(k)} P_2^{(kn)} + \sigma_{43}^{(k)} P_4^{(kn)} \right) = -\xi_2^{(kn)} \left(-\sigma_2^{(k)} P_1^{(kn)} + \sigma_1^{(k)} P_2^{(kn)} + \sigma_{43}^{(k)} P_4^{(kn)} \right)$$

$$= -\xi_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + L_{kn} \xi_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix};$$

$$-\xi_3^{(kn)} P_m^{(kn)} \sigma_{2m}^{(k)} = -\xi_3^{(kn)} \left(P_1^{(kn)} \sigma_{21}^{(k)} + P_3^{(kn)} \sigma_{23}^{(k)} + P_4^{(kn)} \sigma_{24}^{(k)} \right) = -\xi_3^{(kn)} \left(-P_1^{(kn)} \sigma_3^{(k)} + P_3^{(kn)} \sigma_1^{(k)} + P_4^{(kn)} \sigma_{24}^{(k)} \right)$$

$$= -\xi_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} + L_{kn} \xi_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix};$$

$$P_3^{(kn)} \sigma_{2m}^{(k)} \xi_m^{(kn)} = P_3^{(kn)} \left(\sigma_{21}^{(k)} \xi_1^{(kn)} + \sigma_{23}^{(k)} \xi_3^{(kn)} + \sigma_{24}^{(k)} \xi_4^{(kn)} \right) = P_3^{(kn)} \left(-\sigma_3^{(k)} \xi_1^{(kn)} + \sigma_1^{(k)} \xi_3^{(kn)} + \sigma_{24}^{(k)} \xi_4^{(kn)} \right)$$

$$= P_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} + \tau_{kn} P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix}.$$

Further on we have

$$\begin{aligned} \sum_{n=1, n \neq k}^3 \left(F_{2m}^{(kn)} \sigma_{m3}^{(k)} - F_{m3}^{(kn)} \sigma_{2m}^{(k)} \right) &= e_n \left(P_2^{(kn)} \sigma_{m3}^{(k)} \xi_m^{(kn)} - \sigma_{m3}^{(k)} P_m^{(kn)} \xi_2^{(kn)} - P_m^{(kn)} \sigma_{2m}^{(k)} \xi_3^{(kn)} + P_3^{(kn)} \sigma_{2m}^{(k)} \xi_m^{(kn)} \right) = \\ &= e_n \left(P_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} + \tau_{kn} P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - \xi_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + L_{kn} \xi_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} - \right. \\ &\quad \left. - \xi_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} + L_{kn} \xi_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} - \tau_{kn} P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right). \end{aligned}$$

Then

$$\begin{aligned} \frac{d\sigma_1^{(k)}}{ds_k} = & \sum_{\substack{n=1, \\ n \neq k}}^3 \frac{e_k e_n}{m_k c^2} \left(P_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} + \tau_{kn} P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + \right. \\ & + \tau_{kn} P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \xi_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} - \xi_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} + \\ & \left. + L_{kn} \xi_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + L_{kn} \xi_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right). \end{aligned}$$

But

$$\begin{aligned} P_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} + P_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} &= \left[\vec{P}^{(kn)} \times (\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)}) \right]_1; \\ \tau_{kn} P_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + \tau_{kn} P_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} &= \tau_{kn} \left[\vec{P}^{(kn)} \times (\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}) \right]_1; \\ -\xi_2^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} - \xi_3^{(kn)} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} &= - \left[\vec{\xi}^{(kn)} \times (\vec{\sigma}^{(k)} \times \vec{P}^{(kn)}) \right]_1; \\ L_{kn} \xi_2^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} + L_{kn} \xi_3^{(kn)} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} &= L_{kn} \left[\vec{\xi}^{(kn)} \times (\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}) \right]_1 \end{aligned}$$

and then

$$\begin{aligned} \frac{d\sigma_1^{(k)}}{ds_k} = & \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left\{ \left[\vec{P}^{(kn)} \times (\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)}) \right]_1 + \tau_{kn} \left[\vec{P}^{(kn)} \times (\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}) \right]_1 - \right. \\ & \left. - \left[\vec{\xi}^{(kn)} \times (\vec{\sigma}^{(k)} \times \vec{P}^{(kn)}) \right]_1 + L_{kn} \left[\vec{\xi}^{(kn)} \times (\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)}) \right]_1 \right\}. \end{aligned}$$

Finally, we rewrite the Lorentz parts of the first three spin equation in a vector form

$$\begin{aligned} \frac{d\vec{\sigma}^{(k)}}{ds_k} = & \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) + \tau_{kn} \vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + \right. \\ & \left. + L_{kn} \vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]. \end{aligned}$$

Having in mind that

$$\frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m2}^{(k)} - \sigma_{1m}^{(k)} F_{m2}^{(k)rad} \right) = \frac{e_k^2}{2m_k c^2}$$

$$\begin{aligned} & \left\{ \left[\vec{P}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) \right]_3 + \tau_k^{ret} \left[\vec{P}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3 - \left[\vec{\xi}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) \right]_3 + \right. \\ & \left. + L_{k,ret} \left[\vec{\xi}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3 \right\} - \\ & - \left\{ \left[\vec{P}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) \right]_3 + \tau_k^{adv} \left[\vec{P}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3 - \left[\vec{\xi}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) \right]_3 + \right. \\ & \left. + L_{k,adv} \left[\vec{\xi}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_3 \right\}; \end{aligned}$$

$$\frac{e_k}{m_k c^2} \left(F_{1m}^{(k)rad} \sigma_{m3}^{(k)} - \sigma_{1m}^{(k)} F_{m3}^{(k)rad} \right) = \frac{e_k^2}{2m_k c^2}$$

$$\left\{ \left[\vec{P}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) \right]_2 + \tau_k^{ret} \left[\vec{P}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 - \left[\vec{\xi}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) \right]_2 + \right.$$

$$\left. + L_{k,ret} \left[\vec{\xi}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 \right\} - \left\{ \left[\vec{P}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) \right]_2 + \tau_k^{adv} \left[\vec{P}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 - \right.$$

$$\left. - \left[\vec{\xi}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) \right]_2 + L_{k,adv} \left[\vec{\xi}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_2 \right\};$$

$$\frac{e_k}{m_k c^2} \left(F_{2m}^{(k)rad} \sigma_{m3}^{(k)} - F_{m3}^{(k)rad} \sigma_{2m}^{(k)} \right) = \frac{e_k^2}{2m_k c^2}$$

$$\left\{ \left[\vec{P}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) \right]_1 + \tau_k^{ret} \left[\vec{P}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_1 - \left[\vec{\xi}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) \right]_1 + \right.$$

$$\left. + L_{k,ret} \left[\vec{\xi}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_1 \right\} -$$

$$- \left\{ \left[\vec{P}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) \right]_1 + \tau_k^{adv} \left[\vec{P}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_1 - \left[\vec{\xi}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) \right]_1 + \right.$$

$$\left. + L_{k,adv} \left[\vec{\xi}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]_1 \right\}$$

we obtain

$$\frac{d\vec{\sigma}^{(k)}}{ds_k} = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) + \tau_{kn} \vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + \right.$$

$$\left. + L_{kn} \vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] + \frac{e_k^2}{2m_k c^2} \left\{ \left[\vec{P}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) + \tau_k^{ret} \vec{P}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \right.$$

$$\left. - \vec{\xi}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) + L_{k,ret} \vec{\xi}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] -$$

$$- \left[\vec{P}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) + \tau_k^{adv} \vec{P}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \vec{\xi}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) + \right.$$

$$\left. + L_{k,adv} \vec{\xi}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] \right\}.$$

Transformation of the spin equation (3.4)

$$\frac{d\sigma_{14}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\sum_{n=1, n \neq k}^3 \left(F_{1m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{1m}^{(k)} \right) + F_{1m}^{(k)rad} \sigma_{m4}^{(k)} - F_{m4}^{(k)rad} \sigma_{1m}^{(k)} \right];$$

$$F_{1m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{1m}^{(k)} = e_n \left(P_1^{(kn)} \xi_m^{(kn)} \sigma_{m4}^{(k)} - P_m^{(kn)} \sigma_{m4}^{(k)} \xi_1^{(kn)} - P_m^{(kn)} \sigma_{1m}^{(k)} \xi_4^{(kn)} + P_4^{(kn)} \sigma_{1m}^{(k)} \xi_m^{(kn)} \right);$$

$$P_1^{(kn)} \xi_m^{(kn)} \sigma_{m4}^{(k)} = P_1^{(kn)} \left(\xi_1^{(kn)} \sigma_{14}^{(k)} + \xi_2^{(kn)} \sigma_{24}^{(k)} + \xi_3^{(kn)} \sigma_{34}^{(k)} \right)$$

$$= P_1^{(kn)} \left(\xi_1^{(kn)} \frac{i}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \xi_2^{(kn)} \frac{i}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \xi_3^{(kn)} \frac{i}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \right)$$

$$= \frac{i P_1^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix};$$

$$\begin{aligned}
 -\xi_1^{(kn)} P_m^{(kn)} \sigma_{m4}^{(k)} &= -\xi_1^{(kn)} \left(P_1^{(kn)} \sigma_{14}^{(k)} + P_2^{(kn)} \sigma_{24}^{(k)} + P_3^{(kn)} \sigma_{34}^{(k)} \right) \\
 &= -\xi_1^{(kn)} \left(\frac{iP_1^{(kn)}}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \frac{iP_2^{(kn)}}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \frac{iP_3^{(kn)}}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \right) \\
 &= -\frac{i\xi_1^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} ; \\
 -\xi_4^{(kn)} P_m^{(kn)} \sigma_{1m}^{(k)} &= -\xi_4^{(kn)} \left(P_2^{(kn)} \sigma_{12}^{(k)} + P_3^{(kn)} \sigma_{13}^{(k)} + P_4^{(kn)} \sigma_{14}^{(k)} \right) \\
 &= -\xi_4^{(kn)} \left(P_2^{(kn)} \sigma_3^{(k)} - P_3^{(kn)} \sigma_2^{(k)} + P_4^{(kn)} \sigma_{14}^{(k)} \right) \\
 &= -\xi_4^{(kn)} \left(- \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} + \frac{iP_4^{(kn)}}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \\
 &= i c \tau_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} - i c \tau_{kn} L_{kn} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} ; \\
 P_4^{(kn)} \sigma_{1m}^{(k)} \xi_m^{(kn)} &= P_4^{(kn)} \left(\sigma_{12}^{(k)} \xi_2^{(kn)} + \sigma_{13}^{(k)} \xi_3^{(kn)} + \sigma_{14}^{(k)} \xi_4^{(kn)} \right) \\
 &= P_4^{(kn)} \left(\sigma_3^{(k)} \xi_2^{(kn)} - \sigma_2^{(k)} \xi_3^{(kn)} + \sigma_{14}^{(k)} \xi_4^{(kn)} \right) \\
 &= -P_4^{(kn)} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} + P_4^{(kn)} i c \tau_{kn} \frac{i}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \\
 &= i c L_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} + i c L_{kn} \tau_{kn} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} .
 \end{aligned}$$

Then

$$\begin{aligned}
 F_{1m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{1m}^{(k)} &= e_n \left(\frac{iP_1^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \frac{i\xi_1^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \right. \\
 &\quad \left. + i c \tau_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} - i c \tau_{kn} L_{kn} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + i c L_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} \right. \\
 &\quad \left. + i c L_{kn} \tau_{kn} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \\
 &= e_n \left(\frac{iP_1^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \frac{i\xi_1^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right. \\
 &\quad \left. + i c \tau_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} + i c L_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} \right)
 \end{aligned}$$

or

$$\begin{aligned} \frac{i}{c} \frac{d(\vec{\lambda} \times \vec{\sigma})_1}{ds_k} &= \\ &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(\frac{i P_1^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_2 & \sigma_3 \end{vmatrix} - \frac{i \xi_1^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_2 & \sigma_3 \end{vmatrix} + \right. \\ &\quad \left. + i \tau_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ P_2^{(kn)} & P_3^{(kn)} \end{vmatrix} + i c L_{kn} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \xi_2^{(kn)} & \xi_3^{(kn)} \end{vmatrix} \right) \end{aligned}$$

Transformation of the spin equation (3.5)

$$\frac{d\sigma_{24}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\sum_{n=1, n \neq k}^3 \left(F_{2m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{2m}^{(k)} \right) + F_{2m}^{(k)rad} \sigma_{m4}^{(k)} - F_{m4}^{(k)rad} \sigma_{2m}^{(k)} \right];$$

$$F_{2m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{2m}^{(k)} = e_n \left(P_2^{(kn)} \xi_m^{(kn)} \sigma_{m4}^{(k)} - P_m^{(kn)} \sigma_{m4}^{(k)} \xi_2^{(kn)} - P_m^{(kn)} \sigma_{2m}^{(k)} \xi_4^{(kn)} + P_4^{(kn)} \xi_m^{(kn)} \sigma_{2m}^{(k)} \right);$$

$$\begin{aligned} P_2^{(kn)} \xi_m^{(kn)} \sigma_{m4}^{(k)} &= P_2^{(kn)} \left(\xi_1^{(kn)} \sigma_{14}^{(k)} + \xi_2^{(kn)} \sigma_{24}^{(k)} + \xi_3^{(kn)} \sigma_{34}^{(k)} \right) \\ &= P_2^{(kn)} \left(\frac{i \xi_1^{(kn)}}{c} \begin{vmatrix} \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \frac{i \xi_2^{(kn)}}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_3^{(k)} \end{vmatrix} + \frac{i \xi_3^{(kn)}}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1 & \sigma_2^{(k)} \end{vmatrix} \right) \\ &= \frac{i P_2^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_2 & \sigma_3 \end{vmatrix}; \end{aligned}$$

$$\begin{aligned} \xi_2^{(kn)} P_m^{(kn)} \sigma_{m4}^{(k)} &= \xi_2^{(kn)} \left(P_1^{(kn)} \sigma_{14}^{(k)} + P_2^{(kn)} \sigma_{24}^{(k)} + P_3^{(kn)} \sigma_{34}^{(k)} \right) \\ &= \xi_2^{(kn)} \left(\frac{i P_1^{(kn)}}{c} \begin{vmatrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \lambda_2^{(k)} & \lambda_3^{(k)} \end{vmatrix} - \frac{i P_2^{(kn)}}{c} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \lambda_1 & \lambda_3^{(k)} \end{vmatrix} + \frac{i P_3^{(kn)}}{c} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} \end{vmatrix} \right) \\ &= \frac{i \xi_2^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_2 & \sigma_3 \end{vmatrix}; \end{aligned}$$

$$\begin{aligned} \xi_4^{(kn)} P_m^{(kn)} \sigma_{2m}^{(k)} &= \xi_4^{(kn)} \left(P_1^{(kn)} \sigma_{21}^{(k)} + P_3^{(kn)} \sigma_{23}^{(k)} + P_4^{(kn)} \sigma_{24}^{(k)} \right) \\ &= \xi_4^{(kn)} \left(\begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - \frac{i P_4^{(kn)}}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_3 \end{vmatrix} \right) \\ &= \xi_4^{(kn)} \left(\begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - L_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_3 \end{vmatrix} \right); \end{aligned}$$

$$\begin{aligned} P_4^{(kn)} \xi_m^{(kn)} \sigma_{2m}^{(k)} &= P_4^{(kn)} \left(\xi_1^{(kn)} \sigma_{21}^{(k)} + \xi_3^{(kn)} \sigma_{23}^{(k)} + \xi_4^{(kn)} \sigma_{24}^{(k)} \right) \\ &= P_4^{(kn)} \left(\begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} - \frac{i \xi_4^{(kn)}}{c} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_3 \end{vmatrix} \right) \\ &= P_4^{(kn)} \left(\begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} + \tau_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1 & \sigma_3 \end{vmatrix} \right); \end{aligned}$$

$$\begin{aligned}
 F_{2m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{2m}^{(k)} &= e_n \left(-\frac{iP_2^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \frac{i\xi_2^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \right. \\
 &\quad \left. -i c \tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - i c \tau_{kn} L_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} - i c L_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} \right) + \\
 &\quad \left. + i c \tau_{kn} L_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right) \\
 &= e_n \left(-\frac{iP_2^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \frac{i\xi_2^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \right. \\
 &\quad \left. -i c \tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - i c L_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} \right)
 \end{aligned}$$

or

$$\begin{aligned}
 -\frac{i}{c} \frac{d}{\Delta_k dt} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ u_1^{(k)} & u_3^{(k)} \end{vmatrix} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(-\frac{iP_2^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \frac{i\xi_2^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \right. \\
 &\quad \left. -i c \tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} - i c L_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} \right);
 \end{aligned}$$

$$\begin{aligned}
 \frac{i}{c} \frac{d(\vec{\lambda} \times \vec{\sigma})_2}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(\frac{iP_2^{(kn)}}{c} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \frac{i\xi_2^{(kn)}}{c} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + \right. \\
 &\quad \left. + i c \tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ P_1^{(kn)} & P_3^{(kn)} \end{vmatrix} + i c L_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \xi_1^{(kn)} & \xi_3^{(kn)} \end{vmatrix} \right).
 \end{aligned}$$

Transformation of the spin equation (3.6)

$$\frac{d\sigma_{34}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left[\sum_{n=1, n \neq k}^3 \left(F_{3m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{3m}^{(k)} \right) + F_{3m}^{(k)rad} \sigma_{m4}^{(k)} - F_{m4}^{(k)rad} \sigma_{3m}^{(k)} \right];$$

$$F_{3m}^{(kn)} \sigma_{m4}^{(k)} - F_{m4}^{(kn)} \sigma_{3m}^{(k)} = e_n \left(P_3^{(kn)} \xi_m^{(kn)} \sigma_{m4}^{(k)} - P_m^{(kn)} \sigma_{m4} \xi_3^{(kn)} - P_m^{(kn)} \sigma_{3m} \xi_4^{(kn)} + P_4^{(kn)} \xi_m^{(kn)} \sigma_{3m}^{(k)} \right);$$

$$P_3^{(kn)} \xi_m^{(kn)} \sigma_{m4}^{(k)} = P_3^{(kn)} \left(\xi_1^{(kn)} \sigma_{14}^{(k)} + \xi_2^{(kn)} \sigma_{24}^{(k)} + \xi_3^{(kn)} \sigma_{34}^{(k)} \right) = \frac{i}{c} P_3^{(kn)} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix};$$

$$P_m^{(kn)} \sigma_{m4} \xi_3^{(kn)} = \left(P_1^{(kn)} \sigma_{14}^{(k)} + P_2^{(kn)} \sigma_{24}^{(k)} + P_3^{(kn)} \sigma_{34}^{(k)} \right) \xi_3^{(kn)} = \frac{i}{c} \xi_3^{(kn)} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix};$$

$$\begin{aligned} \xi_4^{(kn)} P_m^{(kn)} \sigma_{3m}^{(k)} &= \xi_4^{(kn)} \left(P_1^{(kn)} \sigma_{31}^{(k)} + P_2^{(kn)} \sigma_{32}^{(k)} + P_4^{(kn)} \sigma_{34}^{(k)} \right) \\ &= \xi_4^{(kn)} \left(-i c \tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + i c \tau_{kn} L_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \right) \\ P_4^{(kn)} \xi_m^{(kn)} \sigma_{3m}^{(k)} &= P_4^{(kn)} \left(\xi_1^{(kn)} \sigma_{31}^{(k)} + \xi_2^{(kn)} \sigma_{32}^{(k)} + \xi_4^{(kn)} \sigma_{34}^{(k)} \right) \\ &= P_4^{(kn)} \left(i c L_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} - i c \tau_{kn} L_{kn} \begin{vmatrix} \lambda_1^{(k)} & \lambda_2^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} \end{vmatrix} \right) \end{aligned}$$

or

$$\begin{aligned} \frac{i}{c} \frac{d(\vec{\lambda} \times \vec{\sigma})_3}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(\frac{i}{c} P_3^{(kn)} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \frac{i}{c} \xi_3^{(kn)} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} \right. \\ &\quad \left. + i c \tau_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ P_1^{(kn)} & P_2^{(kn)} \end{vmatrix} + i c L_{kn} \begin{vmatrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \xi_1^{(kn)} & \xi_2^{(kn)} \end{vmatrix} \right). \end{aligned}$$

Now we are able to write in a vector form of the last three spin equations with radiation terms.

The last three equations can be written in a vector form

$$\begin{aligned} \frac{d(\vec{\lambda} \times \vec{\sigma}^{(k)})}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(\vec{P}^{(kn)} \langle \vec{\xi}^{(kn)} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \rangle - \vec{\xi}^{(kn)} \langle \vec{P}^{(kn)} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \rangle + \right. \\ &\quad \left. + c^2 \tau_{kn} \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + c^2 L_{kn} \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) \right) + \\ &\quad + \frac{e_k^2}{2m_k c^2} \left\{ \left(\vec{P}^{(k)ret} \langle \vec{\xi}^{(k)ret} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \rangle - \vec{\xi}^{(k)ret} \langle \vec{P}^{(k)ret} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \rangle \right) + \right. \\ &\quad \left. + c^2 \tau_k^{ret} \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) + c^2 L_{k,ret} \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) \right) - \\ &\quad - \left(\vec{P}^{(k)adv} \langle \vec{\xi}^{(k)adv} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \rangle - \vec{\xi}^{(k)adv} \langle \vec{P}^{(k)adv} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \rangle \right) + \\ &\quad \left. + c^2 \tau_k^{adv} \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) + c^2 L_{k,adv} \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) \right\}. \end{aligned}$$

5. The Last Three Equations are Consequences of the First Three Ones

First, we rewrite the equations of motion in the form

$$\begin{aligned} \frac{d\vec{\lambda}^{(k)}}{ds_k} &= -\frac{e_k^2}{m_k c^2} \left[\vec{P}^{(kn)} \langle \lambda^{(k)}, \xi^{(kn)} \rangle_4 - \vec{\xi}^{(kn)} \langle \lambda^{(k)}, P^{(kn)} \rangle_4 \right]; \\ \frac{d\vec{\lambda}^{(k)}}{ds_k} &= -\frac{e_k^2}{m_k c^2} \left[\vec{P}^{(kn)} \frac{\langle \vec{u}^{(k)}, \vec{\xi}^{(kn)} \rangle - c^2 \tau_{kn}}{\Delta_k} - \vec{\xi}^{(kn)} \frac{\langle \vec{u}^{(k)}, \vec{P}^{(kn)} \rangle + c^2 L_{kn}}{\Delta_k} \right] \end{aligned} \tag{5.1}$$

and

$$\begin{aligned} \frac{d\vec{\sigma}^{(k)}}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(kn)} \right) + \tau_{kn} \vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \vec{\xi}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + \right. \\ &\quad \left. + L_{kn} \vec{\xi}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] + \\ &\quad + \frac{e_k^2}{2m_k c^2} \left\{ \left[\vec{P}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)ret} \right) + \tau_k^{ret} \vec{P}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \vec{\xi}^{(k)ret} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)ret} \right) + \right. \right. \\ &\quad \left. \left. + L_{k,ret} \vec{\xi}^{(k)ret} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] - \left[\vec{P}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{\xi}^{(k)adv} \right) + \tau_k^{adv} \vec{P}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \right. \right. \\ &\quad \left. \left. - \vec{\xi}^{(k)adv} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(k)adv} \right) + L_{k,adv} \vec{\xi}^{(k)adv} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] \right\}. \end{aligned} \tag{5.2}$$

Our main goal is to check that if the last two systems have a solution then possess a solution $(\vec{\lambda}^{(k)}, \vec{\sigma}^{(k)})$, i.e.

$$\frac{d(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)})}{ds_k} = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(\vec{P}^{(kn)} \begin{vmatrix} \xi_1^{(kn)} & \xi_2^{(kn)} & \xi_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} - \xi^{(kn)} \begin{vmatrix} P_1^{(kn)} & P_2^{(kn)} & P_3^{(kn)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \end{vmatrix} + c^2 \tau_{kn} (\vec{\sigma}^{(k)} \times \vec{P}^{(kn)}) + c^2 L_{kn} (\vec{\sigma}^{(k)} \times \xi^{(kn)}) \right) \equiv \vec{R}_2$$

has a solution too. We verify this implication just for the Lorentz part.

Indeed, multiplying (5.1) from the right by $\vec{\sigma}^{(k)}$ and (5.2) from the left by $\vec{\lambda}^{(k)}$ we get:

$$\frac{d\vec{\lambda}^{(k)}}{ds_k} \times \vec{\sigma}^{(k)} = -\frac{e_k^2}{m_k c^2} \left[\left(\vec{P}^{(kn)} \times \vec{\sigma}^{(k)} \right) \frac{\langle \vec{u}^{(k)}, \xi^{(kn)} \rangle - c^2 \tau_{kn}}{\Delta_k} - \left(\xi^{(kn)} \times \vec{\sigma}^{(k)} \right) \frac{\langle \vec{u}^{(k)}, \vec{P}^{(kn)} \rangle + c^2 L_{kn}}{\Delta_k} \right];$$

$$\begin{aligned} \vec{\lambda}^{(k)} \times \frac{d\vec{\sigma}^{(k)}}{ds_k} &= \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \vec{\lambda}^{(k)} \times \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \xi^{(kn)} \right) + \tau_{kn} \vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \right. \\ &\quad \left. - \xi^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + L_{kn} \xi^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]. \end{aligned}$$

We add the above last equations

$$\begin{aligned} &\frac{d\vec{\lambda}^{(k)}}{ds_k} \times \vec{\sigma}^{(k)} + \vec{\lambda}^{(k)} \times \frac{d\vec{\sigma}^{(k)}}{ds_k} \\ &= -\frac{e_k^2}{m_k c^2} \left[\left(\vec{P}^{(kn)} \times \vec{\sigma}^{(k)} \right) \frac{\langle \vec{u}^{(k)}, \xi^{(kn)} \rangle - c^2 \tau_{kn}}{\Delta_k} - \left(\xi^{(kn)} \times \vec{\sigma}^{(k)} \right) \frac{\langle \vec{u}^{(k)}, \vec{P}^{(kn)} \rangle + c^2 L_{kn}}{\Delta_k} \right] + \\ &+ \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \vec{\lambda}^{(k)} \times \left[\vec{P}^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \xi^{(kn)} \right) + \tau_{kn} \vec{P}^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \right. \\ &\quad \left. - \xi^{(kn)} \times \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) + L_{kn} \xi^{(kn)} \times \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right] \vec{R}_1 \end{aligned}$$

or

$$\begin{aligned} &\frac{d\vec{\lambda}^{(k)}}{ds_k} \times \vec{\sigma}^{(k)} + \vec{\lambda}^{(k)} \times \frac{d\vec{\sigma}^{(k)}}{ds_k} = -\frac{e_k^2}{m_k c^2} \left[\left(\vec{P}^{(kn)} \times \vec{\sigma}^{(k)} \right) \frac{\langle \vec{u}^{(k)}, \xi^{(kn)} \rangle - c^2 \tau_{kn}}{\Delta_k} - \left(\xi^{(kn)} \times \vec{\sigma}^{(k)} \right) \frac{\langle \vec{u}^{(k)}, \vec{P}^{(kn)} \rangle + c^2 L_{kn}}{\Delta_k} \right] + \\ &+ \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left[\left\langle \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \times \xi^{(kn)} \right\rangle \vec{P}^{(kn)} - \left\langle \vec{\lambda}^{(k)}, \vec{P}^{(kn)} \right\rangle \left(\vec{\sigma}^{(k)} \times \xi^{(kn)} \right) - \tau_{kn} \left\langle \vec{\lambda}^{(k)}, \vec{P}^{(kn)} \right\rangle \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) - \right. \\ &\quad \left. - \left\langle \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right\rangle \xi^{(kn)} + \left\langle \vec{\lambda}^{(k)}, \xi^{(kn)} \right\rangle \left(\vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right) - L_{kn} \left\langle \vec{\lambda}^{(k)}, \xi^{(kn)} \right\rangle \left(\vec{\lambda}^{(k)} \times \vec{\sigma}^{(k)} \right) \right]. \end{aligned}$$

Denote by $\vec{R}_1^{(k)}$ the right-hand side of the last equality and form the scalar product

$$\left\langle \vec{R}_1^{(k)}, \vec{\sigma}^{(k)} \right\rangle = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(\left\langle \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \times \xi^{(kn)} \right\rangle \left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle - \left\langle \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \times \vec{P}^{(kn)} \right\rangle \left\langle \xi^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \right).$$

We have to show that the right-hand side $\vec{R}_1^{(k)}$ of the last equation coincides (in weak sense) with the right-hand side $\vec{R}_2^{(k)}$ of the following vector equation corresponding to the last three equations. For $\vec{R}_2^{(k)}$ we have

$$\left\langle \vec{R}_2^{(k)}, \vec{\sigma}^{(k)} \right\rangle = \sum_{n=1, n \neq k}^3 \frac{e_k e_n}{m_k c^2} \left(\left\langle \vec{P}^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \xi^{(kn)} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle - \left\langle \xi^{(kn)}, \vec{\sigma}^{(k)} \right\rangle \left\langle \vec{P}^{(kn)} \times \vec{\lambda}^{(k)}, \vec{\sigma}^{(k)} \right\rangle \right).$$

Then we conclude that $\langle \vec{R}_1^{(k)}, \vec{\sigma}^{(k)} \rangle - \langle \vec{R}_2^{(k)}, \vec{\sigma}^{(k)} \rangle = \langle \vec{R}_1^{(k)} - \vec{R}_2^{(k)}, \vec{\sigma}^{(k)} \rangle = 0$. It is easy to see that the solutions of the last equation are $\vec{R}_1^{(k)} - \vec{R}_2^{(k)} = \vec{0}$ or $\vec{R}_1^{(k)} - \vec{R}_2^{(k)} = \vec{q} \times \frac{\vec{\sigma}^{(k)}}{\langle \vec{\sigma}^{(k)}, \vec{\sigma}^{(k)} \rangle}$, where \vec{q} is an arbitrary chosen vector.

The transformation of the radiation parts can be accomplished in a similar way.

6. Appendix

(A1) We verify that $\sigma_{ij}^{(k)} \lambda_j^{(k)} = 0$ (cf. [10]. [11]) ($i = 1, 2, 3, 4$) in view of $\sigma_{ii}^{(k)} = 0$:

$$\begin{aligned} & \sigma_{11}^{(p)} \lambda_1^{(p)} + \sigma_{12}^{(p)} \lambda_2^{(p)} + \sigma_{13}^{(p)} \lambda_3^{(p)} + \sigma_{14}^{(p)} \lambda_4^{(p)} = \sigma_{12}^{(p)} \lambda_2^{(p)} + \sigma_{13}^{(p)} \lambda_3^{(p)} + \sigma_{14}^{(p)} \lambda_4^{(p)} = \\ & = \sigma_3^{(k)} \lambda_2^{(k)} - \sigma_2^{(k)} \lambda_3^{(k)} + i \frac{\lambda_2^{(k)} \sigma_3^{(k)} - \lambda_3^{(k)} \sigma_2^{(k)}}{c} ic = - \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \lambda_2^{(k)} & \lambda_3^{(k)} \end{matrix} \right| + \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \lambda_2^{(k)} & \lambda_3^{(k)} \end{matrix} \right| = 0; \\ & \sigma_{21}^{(k)} \lambda_1^{(k)} + \sigma_{22}^{(k)} \lambda_2^{(k)} + \sigma_{23}^{(k)} \lambda_3^{(k)} + \sigma_{24}^{(k)} \lambda_4^{(k)} = \sigma_{21}^{(k)} \lambda_1^{(k)} + \sigma_{23}^{(k)} \lambda_3^{(k)} + \sigma_{24}^{(k)} \lambda_4^{(k)} = \\ & = \sigma_2^{(k)} \lambda_1^{(k)} + \sigma_1^{(k)} \lambda_3^{(k)} + i \frac{\lambda_3^{(k)} \sigma_1^{(k)} - \lambda_1^{(k)} \sigma_3^{(k)}}{c} ic = \left| \begin{matrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \lambda_1^{(k)} & \lambda_3^{(k)} \end{matrix} \right| - \left| \begin{matrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \lambda_1^{(k)} & \lambda_3^{(k)} \end{matrix} \right| = 0; \\ & \sigma_{31}^{(k)} \lambda_1^{(k)} + \sigma_{32}^{(k)} \lambda_2^{(k)} + \sigma_{33}^{(k)} \lambda_3^{(k)} + \sigma_{34}^{(k)} \lambda_4^{(k)} = \sigma_{31}^{(k)} \lambda_1^{(k)} + \sigma_{32}^{(k)} \lambda_2^{(k)} + \sigma_{34}^{(k)} \lambda_4^{(k)} = \\ & = \sigma_2^{(k)} \lambda_1^{(k)} - \sigma_1^{(k)} \lambda_2^{(k)} + i \frac{\lambda_1^{(k)} \sigma_2^{(k)} - \lambda_2^{(k)} \sigma_1^{(k)}}{c} ic = - \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} \end{matrix} \right| + \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} \end{matrix} \right| = 0; \\ & \sigma_{41}^{(k)} \lambda_1^{(k)} + \sigma_{42}^{(k)} \lambda_2^{(k)} + \sigma_{43}^{(k)} \lambda_3^{(k)} + \sigma_{44}^{(k)} \lambda_4^{(k)} = \sigma_{41}^{(k)} \lambda_1^{(k)} + \sigma_{42}^{(k)} \lambda_2^{(k)} + \sigma_{43}^{(k)} \lambda_3^{(k)} = \\ & = - \frac{i(\lambda_2^{(k)} \sigma_3^{(k)} - \lambda_3^{(k)} \sigma_2^{(k)}) \lambda_1^{(k)}}{c} - \frac{i(\lambda_3^{(k)} \sigma_1^{(k)} - \lambda_1^{(k)} \sigma_3^{(k)}) \lambda_2^{(k)}}{c} - \frac{i(\lambda_1^{(k)} \sigma_2^{(k)} - \lambda_2^{(k)} \sigma_1^{(k)}) \lambda_3^{(k)}}{c} = \\ & = \frac{i}{c} \left| \begin{matrix} \sigma_2^{(k)} & \sigma_3^{(k)} \\ \lambda_2^{(k)} & \lambda_3^{(k)} \end{matrix} \right| \lambda_1^{(k)} - \frac{i}{c} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_3^{(k)} \\ \lambda_1^{(k)} & \lambda_3^{(k)} \end{matrix} \right| \lambda_2^{(k)} + \frac{i}{c} \left| \begin{matrix} \sigma_1^{(k)} & \sigma_2^{(k)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} \end{matrix} \right| \lambda_3^{(k)} = \frac{i}{c} \left| \begin{matrix} \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \\ \sigma_1^{(k)} & \sigma_2^{(k)} & \sigma_3^{(k)} \\ \lambda_1^{(k)} & \lambda_2^{(k)} & \lambda_3^{(k)} \end{matrix} \right| = 0. \end{aligned}$$

(A2) Let us consider the equations

$$\frac{d\sigma_{ij}^{(k)}}{ds_k} = \frac{e_k}{m_k c^2} \left(\left(\sum_{n=1, n \neq k}^3 F_{im}^{(kn)} + F_{im}^{(k)rad} \right) \sigma_{mj}^{(k)} - \sigma_{im}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{mj}^{(kn)} + F_{mj}^{(k)rad} \right) \right) \equiv \frac{e_k \tilde{F}_{ij}^{(k)}}{m_k c^2}.$$

We notice that $F_{im}^{(kn)} = -F_{mi}^{(kn)}$; $F_{im}^{(k)rad} = -F_{mi}^{(k)rad}$; $\sigma_{mj}^{(k)} = -\sigma_{jm}^{(k)}$.

Therefore

$$\begin{aligned} \tilde{F}_{ij}^{(k)} &= \left(\sum_{n=1, n \neq k}^3 F_{im}^{(kn)} + F_{im}^{(k)rad} \right) \sigma_{mj}^{(k)} - \sigma_{im}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{mj}^{(kn)} + F_{mj}^{(k)rad} \right) - \\ &- \left(\left(\sum_{n=1, n \neq k}^3 F_{mi}^{(kn)} + F_{mi}^{(k)rad} \right) \sigma_{jm}^{(k)} - \sigma_{mi}^{(k)} \left(\sum_{n=1, n \neq k}^3 F_{jm}^{(kn)} + F_{jm}^{(k)rad} \right) \right) = -\tilde{F}_{ji}^{(k)}. \end{aligned}$$

It follows $\tilde{F}_{ii}^{(k)} = 0$ ($i = 1, 2, 3, 4$).

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