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On the novelty of “Contracting Perimeters of Triangles in Metric Space”

Erdal Karapınar

Atilim University, Incek 06836 Ankara, Turkey.

In this note, we investigate whether the newly introduced notion of “contracting perimeters of triangles” in the context of standard metric spaces is novel or equivalent to “a variant” of Banach contraction in the setting of G-metric spaces. By using the fact that G-metric spaces are equivalent to quasi-metric spaces, we reconsider our main question as whether the fixed-point theorems via “contracting perimeters of triangles” is equivalent to a fixed point of the same mapping in the context of quasi-metric spaces.

Key words and Phrases: Fixed Point, Metric space, Interpolative metric space, Contraction.

Mathematics Subject Classification 2020: 47H10, 54H25

1. Introduction

In mathematics and all quantitative sciences, it is natural to research and examine the existence of new constructions and new frameworks to improve the existing results obtained in the literature. One of the most significant instances of this motivation in mathematics is investigating the uniqueness and existence of fixed points within the framework of various extensions of the notion of a metric. Under this motivation, the notion of metric has been improved and generalized in various distinct ways. Among them, we may recall the concepts of quasi-metric, b-metric, strong b-metric, θ-metric, symmetric, fuzzy metric, probabilistic metric, partial metric, dislocated metric, metric-like, 2-metric, D-metric, G-metric, S-metric, cone metric (Banach-Valued metric), TVS-metric, complex-valued metric, quaternion metric, and so on (for more, see e.g. [1, 2, 35, 36, 37])

In this short note, we shall mainly deal with the interesting notion of G-metric which was considered first in the paper of Mustafa and Sims [5] in 2007 to repair and improve the notion of D-metric,

Email addresses: erdalkarapinar@yahoo.com, erdal.karapinar@atilim.edu.tr (Erdal KARAPINAR)

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defined by Dhage [4]. Roughly speaking, Mustafa and Sims [5] observed that all topological properties (convergence, completeness, Cauchy, etc.) could be set up in this new abstract space. In addition, there is a nice connection between the standard metric and the G-metric. For more details on G-metric and related fixed point results on such construction, we refer to e.g. [2],[5]-[42] and the related references therein.

Recently, Samet et al. [33] and Jleli-Samet [34] reported a significant observation that under certain conditions, the notions of quasi-metric and G-metric coincide.

Very recently, Petrov [78] introduced the notion of "contracting perimeters of triangles" in the context of standard metric spaces. Several authors proved certain fixed-point theorems in this style; see e.g. [78, 79, 80, 76, 77, 81].

The goal of this note is to prove that fixed-point theorems in the context of the G-metric space that were suggested in [35, 36, 37] imply the corresponding fixed point theory via the "contracting perimeters of triangles," defined by Petrov [78], in the setting of a standard metric space.

2. Preliminaries

For the sake of the self-content paper, we recall and recollect certain basic definitions together with the fundamental results that shall be considered in the main section.

First, let us fix some basic notation and definitions as follows:

$\mathbb{N}$ denotes the set of positive integers.

$\mathbb{R}$ denotes the set of reals.

$\mathbb{R}^+$ denotes the set of nonnegative reals.

In what follows, we state the notion of contraction perimeter mappings of triangles introduced by Petrov in [78]. The author called [78] claimed that it is a generalization of the renowned Banach contraction mapping principle [3].

Definition 2.1. (Petrov [78]). *Let (X,d) be a metric space with $|X| \geq 3$. We shall say that $T : X \to X$ is a mapping contracting perimeters of triangles defined on X if there exists $\alpha \in [0,1)$ such that the inequality*

$$d(Tx,Ty) + d(Ty,Tz) + d(Tz,Tx) \leq \alpha[d(x,y) + d(y,z) + d(z,x)],$$

holds for all three pairwise distinct points $x,y,z \in X$.

Petrov proved in [78] a fixed-point theorem for these kinds of mappings:

Theorem 2.1. (Petrov [78]). *Let (X,d), $|X| \geq 3$ be a complete metric space and let $T : X \to X$ be a mapping contracting perimeters of triangles on X. Then, T has a fixed point if and only if T does not possess periodic points of prime period 2. The number of fixed points is at most 2.*

The notion of mapping contracting perimeters of triangles has been studied and extended by several authors; see e.g. [78, 79, 80, 76, 77] and so on. We examine the result of Petrov [78] by involving the notion of G-metric in the discussion. Before declaring our assertion, let us give a short brief on the G-metric notion.

2.1. G-metric

The notion of G-metric spaces is defined as follows:

Definition 2.2. (See [5]). *For X a non-empty set, let $G : X \times X \times X \to \mathbb{R}^+$ be a function satisfying the following properties:*

(G1) $G(x,y,z) = 0$ *if* $x = y = z$,
(G2) $0 < G(x,x,y)$ *for all* $x,y \in X$ *with* $x \neq y$,
(G3) $G(x,x,y) \leq G(x,y,z)$ *for all* $x,y,z \in X$ *with* $y \neq z$,

(G4) $G(x,y,z) = G(x,z,y) = G(y,z,x) = \cdots$ (*symmetry in all three variables*),
(G5) $G(x,y,z) \le G(x,a,a) + G(a,y,z)$ (*rectangle inequality*) *for all* $x, y, z, a \in X$.

Here, the function G shall be called a generalized metric or, more commonly, a G-metric on X. In addition, the pair (X,G) is called a G-metric space.

Notice the fact that every G-metric on X induces a metric d_G on X defined by

$$d_G(x,y) = G(x,y,y) + G(y,x,x), \text{ for all } x,y \in X. \tag{1}$$

We shall state the following basic examples to illustrate the connection between standard metric and G-metric.

Example 2.1. *Let (X,d) be a metric space. The function* $G : X \times X \times X \to [0,+\infty)$, *defined by*

$$G(x,y,z) = \max\{d(x,y), d(y,z), d(z,x)\},$$

for all $x, y, z \in X$, *is a G-metric on X.*

Example 2.2. (*See e.g.* [5]) *Let* $X = [0,\infty)$. *The function* $G : X \times X \times X \to [0,+\infty)$, *defined by*

$$G(x,y,z) = |x-y| + |y-z| + |z-x|,$$

for all $x, y, z \in X$, *is a G-metric on X.*

In their initial paper, Mustafa and Sims [5] also defined the basic topological concepts in G-metric spaces as follows (See [5]):

Let (X,G) be a G-metric space, and let $\{x_n\}$ be a sequence of points of X. We say that the sequence $\{x_n\}$ G-converges to $x \in X$ if

$$\lim_{n,m\to+\infty} G(x,x_n,x_m) = 0,$$

that is, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x,x_n,x_m) < \varepsilon$, for all $n,m \ge N$. We call x the limit of the sequence and write $x_n \to x$ *or* $\lim_{n\to+\infty} x_n = x$. In addition, a sequence $\{x_n\}$ is called a G-Cauchy sequence if, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n,x_m,x_l) < \varepsilon$ for all $n,m,l \ge N$, that is, $G(x_n,x_m,x_l) \to 0$ *as* $n,m,l \to +\infty$. Furthermore, a G-metric space (X,G) is called G-complete if every G-Cauchy sequence is G-convergent in (X,G).

Proposition 2.1. (See [5]) *Let the pair (X,G) denote a G-metric space. The following statements are equivalent:*

(1) $\{x_n\}$ *is G-convergent to x,*
(2) $G(x_n,x_n,x) \to 0$ *as* $n \to +\infty$,
(3) $G(x_n,x,x) \to 0$ *as* $n \to +\infty$,
(4) $G(x_n,x_m,x) \to 0$ *as* $n,m \to +\infty$.

Proposition 2.2. (See [5]). *Let the pair (X,G) denote a G-metric space. Then the following statements are equivalent:*

(1) *The sequence* $\{x_n\}$ *is G-Cauchy,*
(2) *for any* $\varepsilon > 0$, *there exists* $N \in \mathbb{N}$ *such that* $G(x_n,x_m,x_m) < \varepsilon$, *for all* $m,n \ge N$.

Definition 2.3. *Let the pair (X,G) denote a G-metric space. A mapping* $F : X \times X \times X \to X$ *is said to be continuous if for any three G-convergent sequences* $\{x_n\}$, $\{y_n\}$ *and* $\{z_n\}$ *converging to x, y and z respectively,* $\{F(x_n,y_n,z_n)\}$ *is G-convergent to F(x,y,z).*

Mustafa [7] extended the well-known Banach Contraction Principle Mapping in the framework of G-metric spaces as follows:

Theorem 2.2. (See [7]) *Let (X,G) be a complete G-metric space, and* $T : X \to X$ *be a mapping satisfying one of the following conditions (equalities) for all* $x,y,z \in X$:

$$G(Tx,Ty,Tz) \le kG(x,y,z), \ or, \tag{2}$$

$$G(Tx,Ty,Ty) \le kG(x,y,y), \tag{3}$$

where $k \in [0,1)$. Then T has a unique fixed point.

Remark 2.1. *Observe that the condition (2) implies the condition (3). The converse is true only if $k \in [0,\frac{1}{2})$. For details, see [7].*

Definition 2.4. *Let X be a nonempty and let $d: X \times X \to [0,\infty)$ be a function that satisfies:*

(d1) $d(x,y) = 0$ *if and only if* $x = y$,
(d2) $d(x,y) \le d(x,z) + d(z,y)$. *Then d is called a quasi-metric, and the pair (X,d) is called a quasi-metric space.*

Remark 2.2. *Any metric space is a quasi-metric space, but the converse is not true in general.*

Now, we recollect some basic topological notions and related results on quasi-metric spaces.

Definition 2.5. (See e.g. [34]) *Let (X,d) be a quasi-metric space, $\{x_n\}$ be a sequence in X, and $x \in X$. The sequence $\{x_n\}$ converges to x if and only if*

$$\lim_{n\to\infty} d(x_n,x) = \lim_{n\to\infty} d(x,x_n) = 0 \tag{4}$$

Definition 2.6. (See e.g. [34]) *Let (X,d) be a quasi-metric space and $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ is left-Cauchy if and only if for every $\varepsilon > 0$ there exists a positive integer $N = N(\varepsilon)$ such that $d(x_n,x_m) < \varepsilon$ for all $n \ge m > N$.*

Definition 2.7. (See e.g. [34]) *Let (X,d) be a quasi-metric space and $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ is right-Cauchy if and only if for every $\varepsilon > 0$ there exists a positive integer $N = N(\varepsilon)$ such that $d(x_n,x_m) < \varepsilon$ for all $m \ge n > N$.*

Definition 2.8. (See e.g. [34]) *Let (X,d) be a quasi-metric space and $\{x_n\}$ be a sequence in X. We say that $\{x_n\}$ is Cauchy if and only if for every $\varepsilon > 0$ there exists a positive integer $N = N(\varepsilon)$ such that $d(x_n,x_m) < \varepsilon$* for all $m,n > N$.

Remark 2.3. (See e.g. [34]) *A sequence $\{x_n\}$ in a quasi-metric space is Cauchy if and only if it is left-Cauchy and right-Cauchy.*

Definition 2.9. (See e.g. [34]) *Let (X,d) be a quasi-metric space. We say that*

(1) *(X,d) is left-complete if and only if each left-Cauchy sequence in X is convergent.*
(2) *(X,d) is right-complete if and only if each right-Cauchy sequence in X is convergent.*
(3) *(X,d) is complete if and only if each Cauchy sequence in X is convergent.*

Theorem 2.3. (See e.g. [34]) *Let (X,G) be a G-metric space. Let $d: X \times X \to [0,\infty)$ be the function defined by $d(x,y) = G(x,y,y)$. Then*

1. *(X,d) is a quasi-metric space;*
2. *$\{x_n\} \subset X$ is G-convergent to $x \in X$ if and only if $\{x_n\}$ is convergent to x in the quasi-metric space (X,d);*
3. *$\{x_n\} \subset X$ is G-Cauchy if and only if $\{x_n\}$ is Cauchy in the quasi-metric space (X,d);*
4. *(X,G) is G-complete if and only if the quasi-metric space (X,d) is complete.*

Every quasi-metric induces a metric, that is, if (X,d) is a quasi-metric space, then the function $\delta: X \times X \to [0,\infty)$ defined by

$$\delta(x,y) = max\{d(x,y), d(y,x)\}$$

is a metric on X.

As an immediate consequence of the definition above, and Theorem 2.3, the following theorem is obtained:

Theorem 2.4. (See e.g. [34]) *Let the pair (X,G) denote a G-metric space. Let* $d: X \times X \to [0,\infty)$ *be the function defined by* $d(x,y) = G(x,y,y)$. *Then*

1. *(X,d) is a quasi-metric space;*
2. *a sequence* $\{x_n\} \subset X$ *is G-convergent to* $x \in X$ *if and only if the same sequence* $\{x_n\}$ *is convergent to the same point x in* (X,δ);
3. *a sequence* $\{x_n\} \subset X$ *is G-Cauchy if and only if the same sequence* $\{x_n\}$ *is Cauchy in* (X,δ);
4. *(X,G) is G-complete if and only if the corresponding metric (X,δ) is complete.*

3. Main Results

We shall state our main result as follow:

Theorem 3.1. Theorem 2.1 follows from Theorem 2.2.

Proof. Suppose that the statements of Theorem 2.1 hold. That is, there exists $\alpha \in [0,1)$ such that the inequality

$$d(Tx,Ty) + d(Ty,Tz) + d(Tz,Tx) \leq \alpha[d(x,y) + d(y,z) + d(z,x)], \tag{5}$$

holds for all three pairwise distinct points $x,y,z \in X$.

Define a G-metric over X as follows:

$$G(x,y,z) = d(x,y) + d(y,z) + d(z,x) \tag{6}$$

for all three pairwise distinct points $x,y,z \in X$. Hence, the inequality (5) of Theorem 2.1 turns into

$$\begin{aligned} G(Tx,Ty,Tz) &= d(Tx,Ty) + d(Ty,Tz) + d(Tz,Tx) \\ &\leq \alpha[d(x,y) + d(y,z) + d(z,x)] \\ &= G(x,y,z) \end{aligned} \tag{7}$$

for all three pairwise distinct points $x,y,z \in X$. Thus, the conditions of Theorem 2.2 are fulfilled. Consequently, by taking Theorem 2.2 into account, the operator T has a unique fixed point $x_* \in X$.

Remark 3.1. *It is very important to underline that in Theorem 2.2, the statement has no restriction as "all three pairwise distinct points* $x,y,z \in X$." *That means, the results of Theorem 2.1 are very weak.*

As it is mentioned in the previous section, the following Theorem 2.1 of Petrov [78], several results were given in this direction, see e.g. [78, 79, 80, 76]. In the literature, it is crystal clear that all these fixed-point results are also equivalent derived from the corresponding fixed point theorem in the context of G-metric.

We also recall the fact that G-metric can be derived from quasi-metric; see, e.g. Theorem 2.3. Apparently, the fixed point of a perimeter contraction is the same fixed point that satisfies the corresponding relation in the setting of quasi-metric spaces.

Notice also by Theorem 2.4 that topological concepts (convergence of the sequence, being Cauchy of the sequence, etc) in the quasi-metric spaces work equivalently, and they are valid in the set-up of standard metric spaces.

4. Further Results via Auxiliary Functions

In this section, we shall give a list of the fixed-point theorems in G-metric spaces and state the corresponding fixed-point result in the setting of "perimeter contraction" in standard metric spaces.

Theorem 4.1. (See 17) *Let (X,G) be a G-metric space. Let* $T: X \to X$ *be mapping such that*

$$G(Tx,Ty,Tz) \leq aG(x,y,z) + bG(x,Tx,Tx) + cG(y,Ty,Ty) + dG(z,Tz,Tz) \tag{8}$$

for all x,y,z where a,b,c,d are positive constants such that $k = a + b + c + d < 1$. *Then there is a unique* $x \in X$ *such that* $Tx = x$.

The above theorem can be transformed in the "perimeter contraction" in standard metric spaces, as follows:

Theorem 4.2. *Let (X,d) be a standard metric space. Let* $T: X \to X$ *be mapping such that*

$$d(Tx,Ty)+d(Ty,Tz)+d(Tz,Tx) \le a(d(x,y)+d(y,z)+d(z,x)) \\ + b(2d(x,Tx)+d(Tx,x)) + c(2d(y,Ty)) + d(2d(z,Tz)) \tag{9}$$

holds for all three pairwise distinct points $x,y,z \in X$, *where a,b,c,d are positive constants such that* $k = a+b+c+d < 1$. *Then there is a unique* $x \in X$ *such that* $Tx = x$.

Proof.

$$\begin{aligned} d(Tx,Ty)+d(Ty,Tz)+d(Tz,Tx) &= G(Tx,Ty,Tz)\, aG(x,y,z) + bG(x,Tx,Tx) + cG(y,Ty,Ty) + dG(z,Tz,Tz) \\ &\le a(d(x,y)+d(y,z)+d(z,x)) + b(d(x,Tx)+d(Tx,Tx)+d(Tx,x)) \\ &\quad + c(d(y,Ty)+d(Ty,Ty)+d(Ty,y)) + d((d(z,Tz)+d(Tz,Tz)+d(Tz,z))) \\ &= a(d(x,y)+d(y,z)+d(z,x)) + b(2d(x,Tx)+d(Tx,x)) \\ &\quad + c(2d(y,Ty)) + d(2d(z,Tz)) \end{aligned} \tag{10}$$

Thus, we have

$$d(Tx,Ty)+d(Ty,Tz)+d(Tz,Tx) \le a(d(x,y)+d(y,z)+d(z,x)) \\ + b(2d(x,Tx)+d(Tx,x)) + c(2d(y,Ty)) + d(2d(z,Tz)) \tag{11}$$

By employing the same techniques to the following theorems, we may get a number of fixed-point theorems in the setting of "perimeter contraction" in standard metric spaces:

Theorem 4.3. (See [16]) *Let (X,G) be a G-metric space. Let* $T: X \to X$ *be mapping such that*

$$G(Tx,Ty,Tz) \le k\left[G(x,Tx,Tx)+G(y,Ty,Ty)+G(z,Tz,Tz)\right] \tag{12}$$

for all x,y,z where $k \in [0,\frac{1}{3})$. *Then there is a unique* $x \in X$ *such that* $Tx = x$.

Theorem 4.4. (See [17]) *Let (X,G) be a G-metric space. Let* $T: X \to X$ *be mapping such that*

$$G(Tx,Ty,Tz) \le aG(x,y,z) + b\left[G(x,Tx,Tx)+G(y,Ty,Ty)+G(z,Tz,Tz)\right] \tag{13}$$

for all x,y,z where a,b are positive constants such that $k = a+b < 1$. *Then there is a unique* $x \in X$ *such that* $Tx = x$.

Theorem 4.5. (See [17]) *Let (X,G) be a G-metric space. Let* $T: X \to X$ *be mapping such that*

$$G(Tx,Ty,Tz) \le aG(x,y,z) + b\max\left\{G(x,Tx,Tx),G(y,Ty,Ty),G(z,Tz,Tz)\right\} \tag{14}$$

for all x,y,z where a,b are positive constants such that $k = a+b < 1$. *Then there is a unique* $x \in X$ *such that* $Tx = x$.

Theorem 4.6. (See [6]) *Let (X,G) be a G-metric space. Let* $T: X \to X$ *be mapping such that*

$$G(Tx,Ty,Tz) \le k\max\left\{\begin{matrix} G(x,y,z),G(x,Tx,Tx),G(y,Ty,Ty),G(z,Tz,Tz), \\ G(z,Tx,Tx),G(x,Ty,Ty),G(y,Tz,Tz) \end{matrix}\right\} \tag{15}$$

for all x,y,z where $k \in [0,\frac{1}{2})$. *Then there is a unique* $x \in X$ *such that* $Tx = x$.

Theorem 4.7. (See e.g. [34]) *Let (X,G) be a complete G-metric space, and* $T: X \to X$ *be a given mapping satisfying*

$$G(Tx,Ty,Tz) \le G(x,y,z) - \varphi\left(G(x,y,z)\right) \tag{16}$$

if for all x,y $\in X$, *where* $\varphi: [0,\infty) \to [0,\infty)$ *is continuous with* $\varphi^{-1}(\{0\}) = 0$. *Then there is a unique* $x \in X$ *such that* $Tx = x$.

Theorem 4.8. *Let (X,d) be a complete metric space and* $T: X \to X$ *be a mapping with the property:*

$$d(Tx,Ty) \leq q \max\{d(x,y), d(x,Tx), d(y,Ty), d(x,Ty), d(y,Tx)\} \tag{17}$$

for all $x \in X$, where q is a constant such that $q \in [0,1)$. Then T has a unique fixed point.

4.1. Remarks and Conclusion

It is clear that the list of fixed-point theorems in this section is not complete and it is not needed to be complete either. On the other hand, by the main theorem and Theorem 4.2, we conclude that the fixed point theorems via "perimeter contraction" in standard metric spaces are not novel. Indeed, they are weak consequences of fixed point theorems in the context of G-metric metric spaces.

Competing interests

The authors declare that there is no conflict of interests regarding the publication of this article.

Authors' contributions

All authors contributed equally and significantly in writing this article. All authors read and approved the final manuscript.

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