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Fixed point results in neutrosophic fuzzy metric spaces for contractions based on C-class functions

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Abstract

In this study, we employ the notion of C-class functions to develop new contraction mappings within the context of neutrosophic fuzzy metric spaces. These contractions are utilized to establish fixed point theorems applicable to complete neutrosophic fuzzy metric spaces, grounded in C-class functions. Furthermore, we present a range of fixed point results pertinent to this particular framework. An illustrative example is also provided to demonstrate our primary findings. Our results serve to extend and generalize several existing outcomes in the literature.

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Key words and phrases: Fixed point, Neutrosophic set, Neutrosophic fuzzy metric, quasi-contraction, Non-linear contraction, C-class functions, Geraghty contractions.

1. Introduction and preliminaries

In 1965, Zadeh [1] proposed a novel framework known as fuzzy sets, which allowed for the assignment of varying degrees of membership to elements within a set. Initially, this concept faced skepticism from the mathematical community; however, it eventually made significant contributions to a wide array of scientific fields and practical applications. Despite its influence, fuzzy sets have not consistently provided effective solutions to numerous problems over time. In 1986, Atanassov [2] introduced

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Intuitionistic Fuzzy Sets to tackle issues that remained unresolved within the fuzzy set framework. His work emphasized both the membership and non-membership of elements in a set. Following this, Smarandache [3] advanced the theory further by developing Neutrosophic Sets, which incorporate the notions of indeterminacy alongside membership and non-membership. Neutrosophic sets have demonstrated a diverse range of applications across various domains, as highlighted in the existing literature and referenced works [4–7, 10].

Bipolar complex fuzzy soft sets and their practical applications were investigated in [8]. The algebraic structure of normal subgroups and cosets in the setting of (γ,ϑ)-fuzzy HX-subgroups was studied in [9].

In recent years, fixed point theory has undergone substantial development through the introduction of various generalized metric spaces. Among these, MR-metric spaces [28–31] have received particular attention as they provide a flexible and broad framework for the study of contraction-type mappings. These theoretical advances have led to new fixed point results with important applications to integral equations [30], neutron transport [32], uncertainty modeling [33], weighted graph analysis [34], measure theory and convergence analysis [35], and deep learning [36].

Fixed point theory holds considerable importance in mathematics as it provides assurances regarding the existence of solutions to a wide array of problems in diverse disciplines. A fixed point of a function is defined as a point that the function maps to itself. Theorems related to fixed points, such as Banach's fixed-point theorem [11], play a vital role by confirming the existence of these points under specific conditions. For example, Banach's contraction principle, introduced by Stefan Banach in 1922, establishes that every contraction mapping on a complete metric space possesses a unique fixed point. This seminal result has inspired numerous generalizations and extensions across various mathematical frameworks. Notable contributions in this direction include the study of Proinov–C_b-contractions in b-metric spaces [12], characterizations of completeness in quasi-metric and G-metric spaces via ω-distances [13], and tripled coincidence point theorems for weak Φ-contractions [14]. Further developments have introduced novel distance spaces yielding new fixed-point theorems, with applications to fractional differential equations [15, 16], alongside detailed discussions on b-metric, metric, and G-metric spaces [17]. Furthermore, new contraction conditions in extended quasi b-metric spaces were introduced in [18]. Common fixed point results in G-metric spaces via Ω-distance were established in [19], and generalized Ω-distance mappings and related fixed point theorems were studied in [20]. For more generalizations and extensions of fixed point in various distance spaces we refer the reader to [21–27].

2. Preliminary

In this context, the interval $]0-,1+[$ is identified as the non-standard unit interval, where $(1+)=1+\varepsilon$, with "1" signifying the standard component and ε representing the non-standard element. Similarly, $(0-)=0-\varepsilon$, where "0" denotes the standard component. In this manuscript, we define $\mathbb{R}^+$ as the interval $(0,\infty)$, $\mathbb{R}_0^+$ as the interval $[0,\infty)$, and I as the interval $[0,1]$.

Definition 2.1 *[1] A fuzzy set F associated with a universal set U is characterized by the notation* $F=\{<a,\mu_F(\varsigma)>: 0\le\mu_F(\varsigma)\le 1, \varsigma\in U\}$. *Here,* $\mu_F(\varsigma)$ *denotes the degree to which the element ζ belongs to the fuzzy set F.*

Definition 2.2 *[3] A neutrosophic set V associated with a universal set U is characterized as follows:*

$$V=<\varsigma,(T_N(\varsigma),I_N(\varsigma),F_N(\varsigma))>:\varsigma\in U, T_N(\varsigma),I_N(\varsigma),F_N(\varsigma)\in]0-,1+[.$$

In this framework, $T_N(\varsigma),I_N(\varsigma)$, and $F_N(\varsigma)$ denote the degrees of membership pertaining to truth, indeterminacy, and falsity for an element ζ within the neutrosophic set V, respectively. The notation]0-, 1+[indicates a non-standard unit interval.

Definition 2.3 *[39] A neutrosophic fuzzy set B within a universal set U is characterized as follows:*

$$B=\{<x,(\mu_B(\varsigma),T_B(\varsigma,\mu),I_B(\varsigma,\mu),F(\varsigma,\mu))>:\varsigma\in U,\mu_B(\varsigma)\in[0,1],T_B(\varsigma,\mu),I_B(\varsigma,\mu),F(\varsigma,\mu)\in]0-,1+[\}$$

In this framework, the membership degree $\mu_B(\varsigma)$ is represented by three distinct components: the truth membership grade $T_B(\varsigma,\mu)$, the indeterminacy membership grade $I_B(\varsigma,\mu)$, and the falsity membership grade $F(\varsigma,\mu)$. The notation $]0-,1+[$ signifies a nonstandard unit interval.

The subsequent section recalls the definitions of triangular norms and t-norms, concepts that were first introduced by Menger (see [38]). These definitions play a crucial role in the characterization of neutrosophic metric spaces.

Definition 2.4 *Consider an operation $\Diamond : I \times I \to I$. This operation is classified as continuous T-norm (CTN) if it meets the following criteria: for any elements $\sigma_1,\sigma_2,\delta_1,\delta_2 \in I$.*

1. $\sigma_1 \Diamond 1 = \sigma_1$,
2. If $\sigma_1 \le \sigma_2$ and $\delta_1 \le \delta_2$, than $\sigma_1 \Diamond \delta_1 \le \sigma_2 \Diamond \delta_2$,
3. $\Diamond$ is continuous,
4. $\Diamond$ is commutative and associate.

Definition 2.5 *Consider an operation $\bullet : I \times I \to I$. This operation is classified as continuous T co-norm (CTC) if it meets the following criteria: for all elements $\sigma_1,\sigma_2,\delta_1,\delta_2 \in I$.*

1. $\sigma_1 \bullet 0 = \sigma_1$,
2. If $\sigma_1 \le \sigma_2$ and $\delta_1 \le \delta_2$, than $\sigma_1 \bullet \delta_1 \le \sigma_2 \bullet \delta_2$,
3. $\bullet$ is continuous,
4. $\bullet$ is commutative and associate.

The definintion of neutrosophic metric spaces is defined by Kirişci and Şimşek in 2020, and defined as follows.

Definition 2.6 *[38] A 6-tuple $(\mathcal{W},A,C,D,\Diamond,\bullet)$ is referred to as a neutrophic metric space (NMS) if the set $\mathcal{W}$ is a non-empty arbitrary collection, $\Diamond$ signifies a CTN, $\bullet$ indicates a CTC, and the elements A,C, and D are fuzzy sets established on the Cartesian product $\mathcal{W}^2 \times (0,\infty)$. These components must satisfy the following specific conditions for all elements $\varsigma,\omega,c \in \mathcal{W}$ and for all positive real numbers γ,ρ.*

1. $0 \le A(\varsigma,\omega,\gamma) \le 1$, $0 \le C(\varsigma,\omega,\gamma) \le 1$, $0 \le D(\varsigma,\omega,\gamma) \le 1$,
2. $0 \le A(\varsigma,\omega,\gamma) + C(\varsigma,\omega,\gamma) + D(\varsigma,\omega,\gamma) \le 3$,
3. $A(\varsigma,\omega,\gamma) = 1$, for $\gamma > 0$ iff $\varsigma = \omega$
4. $A(\varsigma,\omega,\gamma) = H(\omega,\varsigma,\gamma)$, for $\gamma > 0$
5. $A(\varsigma,\omega,\gamma) \Diamond A(\omega,c,\rho) \le A(\varsigma,c,\gamma+\rho)$
6. $A(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
7. $\lim_{\gamma\to\infty} A(\varsigma,\omega,\gamma) = 1$
8. $C(\varsigma,\omega,\gamma) = 0$ iff $\varsigma = \omega$
9. $C(\varsigma,\omega,\gamma) = C(\omega,\varsigma,\gamma)$,
10. $C(\varsigma,\omega,\gamma) \bullet C(\omega,c,\rho) \ge C(\varsigma,c,\gamma+\rho)$,
11. $C(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
12. $\lim_{\gamma\to\infty} C(\varsigma,\omega,\gamma) = 0$
13. $D(\varsigma,\omega,\gamma) = 0$, for $\gamma > 0$ iff $\varsigma = \omega$
14. $D(\varsigma,\omega,\gamma) = D(\omega,\varsigma,\gamma)$,
15. $D(\varsigma,\omega,\gamma) \bullet D(\omega,c,\rho) \ge S(\varsigma,c,\gamma+\rho)$,
16. $D(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
17. $\lim_{\gamma\to\infty} D(\varsigma,\omega,\gamma) = 0$
18. If $\gamma \le 0$, then $A(\varsigma,\omega,\gamma) = 0$, $C(\varsigma,\omega,\gamma) = D(\varsigma,\omega,\gamma) = 1$

The functions $A(\varsigma,\omega,\gamma)$, $C(\varsigma,\omega,\gamma)$, and $D(\varsigma,\omega,\gamma)$ represent the degrees of nearness, neutralness, and non-nearness between the elements ζ and ω in relation to the parameter γ, respectively.

Recently, Ghosh et al. [41] presented the notion of neutrosophic fuzzy metric spaces and examined various topological characteristics associated with this concept.

Definition 2.7 *[41] A 7-tuple $(\mathcal{W}, A, B, C, D, \Diamond, \bullet)$ is defined as a Neutrophic Fuzzy Metric Space (NFMS) if $\mathcal{W}$ represents an arbitrary set, $\Diamond$ denotes a CTN, $\bullet$ signifies a CTC, and the elements A,B,C, and D are fuzzy sets on $\mathcal{W}^2 \times (0,\infty)$. These elements must satisfy specific conditions for all $\varsigma,\omega,c \in \mathcal{W}$ and $\gamma,\rho > 0$.*

1. $0 \le A(\varsigma,\omega,\gamma) \le 1$, $0 \le B(\varsigma,\omega,\gamma) \le 1$, $0 \le C(\varsigma,\omega,\gamma) \le 1$, $0 \le D(\varsigma,\omega,\gamma) \le 1$,
2. $0 \le A(\varsigma,\omega,\gamma) + B(\varsigma,\omega,\gamma) + C(\varsigma,\omega,\gamma) + D(\varsigma,\omega,\gamma) \le 4$,
3. $A(\varsigma,\omega,\gamma) = 1$, iff $\varsigma = \omega$
4. $A(\varsigma,\omega,\gamma) = H(\omega,\varsigma,\gamma)$,
5. $A(\varsigma,\omega,\gamma) \Diamond A(\omega,c,\rho) \le A(\varsigma,c,\gamma+\rho)$, for $\rho,\gamma > 0$
6. $A(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
7. $\lim_{\gamma\to\infty} A(\varsigma,\omega,\gamma) = 1$
8. $B(\varsigma,\omega,\gamma) = 1$, iff $\varsigma = \omega$
9. $B(\varsigma,\omega,\gamma) = B(\omega,\varsigma,\gamma)$, for $\gamma > 0$
10. $B(\varsigma,\omega,\gamma) \Diamond B(\omega,c,\rho) \le B(\varsigma,c,\gamma+\rho)$,
11. $B(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to \mathbb{R}^+ \to I$ is continuous
12. $\lim_{\gamma\to\infty} B(\varsigma,\omega,\gamma) = 1$
13. $C(\varsigma,\omega,\gamma) = 0$, iff $\varsigma = \omega$
14. $C(\varsigma,\omega,\gamma) = C(\omega,\varsigma,\gamma)$,
15. $C(\varsigma,\omega,\gamma) \bullet C(\omega,c,\rho) \ge C(\varsigma,c,\gamma+\rho)$,
16. $C(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
17. $\lim_{\gamma\to\infty} C(\varsigma,\omega,\gamma) = 0$
18. $D(\varsigma,\omega,\gamma) = 0$, iff $\varsigma = \omega$
19. $D(\varsigma,\omega,\gamma) = D(\omega,\varsigma,\gamma)$,
20. $D(\varsigma,\omega,\gamma) \bullet D(\omega,c,\rho) \ge S(\varsigma,c,\gamma+\rho)$,
21. $D(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to I$ is continuous
22. $\lim_{\gamma\to\infty} D(\varsigma,\omega,\gamma) = 0$
23. If $\gamma \le 0$, then $A(\varsigma,\omega,\gamma) = B(\varsigma,\omega,\gamma) = 0$, $C(\varsigma,\omega,\gamma) = D(\varsigma,\omega,\gamma) = 1$

In this framework, $A(\varsigma,\omega,\gamma)$ signifies the certainty that the distance separating ζ and ω is less than γ. Meanwhile, $B(\varsigma,\omega,\gamma)$ indicates the extent of nearness, $C(\varsigma,\omega,\gamma)$ refers to the level of neutrality, and $D(\varsigma,\omega,\gamma)$ represents the degree of non-proximity between ζ and ω in relation to γ, respectively.

The convergence, Cauchy-ness, completeness were given as follows.

Definition 2.8 *[41] Let (ς_n) be a sequence in a NFMS $(\mathcal{W}, A, B, C, D, \Diamond, \bullet)$. Then*

1. (ς_n) converges to $\varsigma \in \mathcal{W}$ iff for a given $\varepsilon \in (0,1), \gamma > 0$ there is $n_0 \in N$ such that for each $n \ge n_0$

$$A(\varsigma_n,\varsigma,\gamma) > 1-\varepsilon, B(\varsigma_n,\varsigma,\gamma) > 1-\varepsilon, C(\varsigma_n,\varsigma,\gamma) < \varepsilon, D(\varsigma_n,\varsigma,\gamma) < \varepsilon.$$

   i.e.,

$$\lim_{n\to\infty} A(\varsigma_n,\varsigma,\gamma) = 1, \lim_{n\to\infty} B(\varsigma_n,\varsigma,\gamma) = 1, \lim_{n\to\infty} C(\varsigma_n,\varsigma,\gamma) = 0, \lim_{n\to\infty} D(\varsigma_n,\varsigma,\gamma) = 0.$$

2. (ς_n) is called Cauchy iff for a given $\varepsilon \in (0,1), \gamma > 0$ there is $n_0 \in \mathbb{N}$ such that for each $n,m \ge n_0$

$$A(\varsigma_n,\varsigma_m,\gamma) > 1-\varepsilon, B(\varsigma_n,\varsigma_m,\gamma) > 1-\varepsilon, C(\varsigma_n,\varsigma_m,\gamma) < \varepsilon, D(\varsigma_n,\varsigma_m,\gamma) < \varepsilon.$$

   i.e.,

$$\lim_{n.m\to\infty} A(\varsigma_n,\varsigma_m,\gamma) = 1, \lim_{n.m\to\infty} B(\varsigma_n,\varsigma_m,\gamma) = 1,$$

$$\lim_{n,m\to\infty} C(\varsigma_n,\varsigma_m,\gamma) = 0, \lim_{n,m\to\infty} D(\varsigma_n,\varsigma_m,\gamma) = 0.$$

3. $(\mathcal{W}, A, B, C, D, \lozenge, \bullet)$ is called complete if each Cauchy sequence is convergent to an element in $\mathcal{W}$.

We now revisit the concept of a C-class function as defined by Ansari in [44], and further discussed in [45, 46, 47].

Definition 2.9 *[44] A collection of mappings $\mathcal{G}:\mathbb{R}_0^{+2} \to \mathbb{R}$ is called C-class function, if it is continuous and the following conditions hold:*

- $\mathcal{G}(\vartheta,t) \le \vartheta$ for all $\vartheta, t \in \mathbb{R}_0^+$,
- $\mathcal{G}(\vartheta,t) = \vartheta$ implies that either $\vartheta = 0$ or $t = 0$.

Let $\mathcal{C}$ represent the collection of functions classified as C-class.

Example 2.10 *[44] The following functions $\mathcal{G}:\mathbb{R}_0^{+2} \to \mathbb{R}$ defined for all $\vartheta, t \in \mathbb{R}_0^+$ by:*

- $\mathcal{G}(\vartheta,t) = \vartheta - t$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow t = 0$,
- $\mathcal{G}(\vartheta,t) = \eta\vartheta$, $0 < \eta < 1$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$,
- $\mathcal{G}(\vartheta,t) = \dfrac{\vartheta}{(1+t)^r}$, $r \in (0,+\infty)$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$ or $t = 0$,
- $\mathcal{G}(\vartheta,t) = \log_c\left(\dfrac{t + c^\vartheta}{1+t}\right)$, $c > 1$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$ or $t = 0$,
- $\mathcal{G}(\vartheta,t) = \ln\left(\dfrac{1+b^\vartheta}{2}\right)$, $e > b > 1$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$,
- $\mathcal{G}(\vartheta,t) = (\vartheta + l)^{\frac{1}{1+t^r}} - l$, $l > 1, r \in (0,+\infty)$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow t = 0$,
- $\mathcal{G}(\vartheta,t) = \vartheta \log_{t+c} c$, $c > 1$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow$ $\vartheta = 0$ or $t = 0$,
- $\mathcal{G}(\vartheta,t) = \vartheta - \left(\dfrac{1+\vartheta}{2+\vartheta}\right)\left(\dfrac{t}{1+t}\right)$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow t = 0$,
- $\mathcal{G}(\vartheta,t) = \vartheta\beta(\vartheta)$, $\beta:\mathbb{R}_0^+ \to 0,1)$ a continuous function, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$,
- $\mathcal{G}(\vartheta,t) = \vartheta - \dfrac{t}{k+t}$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow t = 0$,
- $\mathcal{G}(\vartheta,t) = \vartheta - h(t)$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow t = 0$, here $h:\mathbb{R}_0^+ \to \mathbb{R}_0^+$ is a continuous function such that $h(t) = 0 \Leftrightarrow t = 0$,
- $\mathcal{G}(\vartheta,t) = \vartheta h(\vartheta,t)$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$, here $h:\mathbb{R}_0^+ \times \mathbb{R}_0^+ \to \mathbb{R}_0^+$ is a continuous function such that $h(t,\vartheta) < 1$ for all $t, \vartheta > 0$,
- $\mathcal{G}(\vartheta,t) = \vartheta - \left(\dfrac{2+t}{1+t}\right)t$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow t = 0$,
- $\mathcal{G}(\vartheta,t) = \sqrt[n]{\ln(1+\vartheta^n)}$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$,
- $\mathcal{G}(\vartheta,t) = f(\vartheta)$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$, here $f:\mathbb{R}_0^+ \to \mathbb{R}_0^+$ is a continuous function such that $f(0) = 0$ and $f(t) < t$ for $t > 0$,
- $\mathcal{G}(\vartheta,t) = \dfrac{\vartheta}{(1+\vartheta)^r}$, $r \in \mathbb{R}^+$, $\mathcal{G}(\vartheta,t) = \vartheta \Rightarrow \vartheta = 0$.

Definition 2.11 *[43] A function $\psi:\mathbb{R}_0^+ \to \mathbb{R}_0^+$ is classified as an altering distance function if it fulfills the subsequent criteria:*

(*i*) The function ψ is both non-decreasing and continuous,
(ii) The condition $\psi(t) = 0$ holds true if and only if $t = 0$.

We represent the collection of altering distance functions as Φ.

Definition 2.12 *Let Φ_u represent the set of functions $\varphi:\mathbb{R}_0^+ \to \mathbb{R}_0^+$ that fulfill the subsequent criteria:*

(i) The function ϕ is continuous;
(ii) The condition $\varphi(t) > 0$ holds for every $t > 0$, and $\varphi(0) \geq 0$.

Definition 2.13 *[48] In this context, we define a real-valued function of three variables,on* $\mathcal{W}^2 \times (0,\infty)$ *where* $\mathcal{W}$ *is any non-empty set, denoted as* $\mathcal{H}$, *to possess the property (UC) if, for any sequences* (ς_n) *and* (ω_n) *in* $\mathcal{W}$, *the following equality holds:*

$$\lim_{\gamma\to\gamma_0}\lim_{n\to\infty}\mathcal{H}(\varsigma_n,\omega_n,\gamma) = \lim_{n\to\infty}\lim_{\gamma\to\gamma_0}\mathcal{H}(\varsigma_n,\omega_n,\gamma).$$

whenever the two limits are exist.

Throughout the remainder of this study, we will assume that each of the fuzzy sets A,B,C,D exhibits the UC property.

We will commence with several pertinent lemmas.

Lemma 2.14 *[48]Let* $(\mathcal{W},A,B,C,D,\Diamond,\bullet)$ *be a NFMS. Then*

1. $A(\varsigma,\omega,\cdot), B(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ is non-decreasing
2. $C(\varsigma,\omega,\cdot)\ D(\varsigma,\omega,\cdot) : \mathbb{R}^+ \to \mathbb{R}^+$ is non-increasing

Lemma 2.15 *[48] Let* $(\mathcal{W},A,B,C,D,\Diamond,\bullet)$ *be a NFMS, and let* (ς_n) *be a sequence such that for* $\gamma > 0$

$$\begin{aligned} lA(\varsigma_p,\varsigma_q,\gamma) \geq A(\varsigma_{p-1},\varsigma_{q-1},\gamma)B(\varsigma_p,\varsigma_q,\gamma) \geq A(\varsigma_{p-1},\varsigma_{q-1},\gamma)C(\varsigma_p,\varsigma_q,\gamma) \leq C(\varsigma_{p-1},\varsigma_{q-1},\gamma)D(\varsigma_p,\varsigma_q,\gamma) \\ \leq D(\varsigma_{p-1},\varsigma_{q-1},\gamma) \end{aligned} \tag{1}$$

and

$$c\lim_{n\to\infty}A(\varsigma_n,\varsigma_{n+1},\gamma) = 1, \lim_{n\to\infty}B(\varsigma_n,\varsigma_{n+1},\gamma) = 1, \lim_{n\to\infty}C(\varsigma_n,\varsigma_{n+1},\gamma) = 0, \lim_{n\to\infty}D(\varsigma_n,\varsigma_{n+1},\gamma) = 0. \tag{2}$$

If (ς_n) is not Cauchy, then there exist an $1 > \varepsilon > 0$ and $\gamma > 0$ along with two subsequences (ς_{n_k}) and (ς_{m_k}) derived from (ς_n), where (m_k) such that one at least of the following holds.

$$\lim_{k\to\infty}A(\varsigma_{n_k},\varsigma_{m_k},\gamma) = 1-\varepsilon,$$

$$\lim_{k\to\infty}B(\varsigma_{n_k},\varsigma_{m_k},\gamma) = 1-\varepsilon,$$

$$\lim_{k\to\infty}C(\varsigma_{n_k},\varsigma_{m_k},\gamma) = \varepsilon,$$

$$\lim_{k\to\infty}D(\varsigma_{n_k},\varsigma_{m_k},\gamma) = \varepsilon.$$

3. Main Result

Definition 3.1 *Let* $(\mathcal{W},A,B,C,D,\Diamond,\bullet)$ *be a NFMS,* $\varphi \in \Phi_u$, $\psi \in \Phi$ *and* $G \in C$. *A mapping* $f : \mathcal{W} \to \mathcal{W}$ *is called* (φ,ψ,G)*-neutrosophic fuzzy contraction ((*$\phi,\ \psi,G$*)-NC) if for each* $\varsigma,\omega \in \mathcal{W}$ *and each* $\gamma > 0$, *we have*

$$\psi\left(\frac{1}{A(f\varsigma,f\omega,\gamma)}-1\right) \leq G\left(\psi\left(\frac{1}{A(\varsigma,\omega,\gamma)}-1\right),\varphi\left(\frac{1}{A(\varsigma,\omega,\gamma)}-1\right)\right),$$

$$\psi\left(\frac{1}{B(f\varsigma,f\omega,\gamma)}-1\right) \leq G\left(\psi\left(\frac{1}{B(\varsigma,\omega,\gamma)}-1\right),\varphi\left(\frac{1}{B(\varsigma,\omega,\gamma)}-1\right)\right),$$

$$\psi(C(f\varsigma,f\omega,\gamma)) \leq G(\psi(C(\varsigma,\omega,\gamma)),\varphi(C(\varsigma,\omega,\gamma))),$$

and

$$\psi(D(f\varsigma, f\omega,\gamma)) \le G(\psi(D(\varsigma,\omega,\gamma)),\varphi(D(\varsigma,\omega,\gamma))).$$

Theorem 3.2 *Let $(\mathcal{W}, A, B, C, D, \Diamond, \bullet)$ be a complete NFMS, Suppose that there is $\zeta^* \in Z$ such that $f: F \to F$ is (φ,ψ,G)-neutrosophic contraction. Consequently, the function f possesses a unique fixed point.*

Proof. Let $\varsigma_0 \in F$ represent an arbitrary point. We examine the Picard sequence (ς_n) characterized by the relation $\varsigma_{n+1} = f(\varsigma_n)$ for all $n \ge 0$. By Definition 3.1 we have

$$ll\psi(\frac{1}{A(\varsigma_n,\varsigma_{n+1},\gamma)}-1) \le G\left(\psi(\frac{1}{A(\varsigma_{n-1},\varsigma_n,\gamma)}-1),\varphi(\frac{1}{A(\varsigma_{n-1},\varsigma_n,\gamma)}-1)\right) \le \psi(\frac{1}{A(\varsigma_{n-1},\varsigma_n,\gamma)}-1), \tag{3}$$

$$ll\psi(\frac{1}{B(\varsigma_n,\varsigma_{n+1},\gamma)}-1) \le G\left(\psi(\frac{1}{B(\varsigma_{n-1},\varsigma_n,\gamma)}-1),\varphi(\frac{1}{B(\varsigma_{n-1},\varsigma_n,\gamma)}-1)\right) \le \psi(\frac{1}{B(\varsigma_{n-1},\varsigma_n,\gamma)}-1),$$

$$\psi(C(\varsigma_n,\varsigma_{n+1},\gamma)) \le G(\psi(C(\varsigma_{n-1},\varsigma_n,\gamma)),\varphi(C(\varsigma_{n-1},\varsigma_n,\gamma))) \le \psi(C(\varsigma_{n-1},\varsigma_n,\gamma)),$$

and

$$\psi(D(\varsigma_n,\varsigma_{n+1},\gamma)) \le G(\psi(D(\varsigma_{n-1},\varsigma_n,\gamma)),\varphi(D(\varsigma_{n-1},\varsigma_n,\gamma))) \le \psi(D(\varsigma_{n-1},\varsigma_n,\gamma)).$$

Thus,

$$\frac{1}{A(\varsigma_n,\varsigma_{n+1},\gamma)}-1 < \frac{1}{A(\varsigma_{n-1},\varsigma_n,\gamma)}-1,$$

$$\frac{1}{B(\varsigma_n,\varsigma_{n+1},\gamma)}-1 < \frac{1}{B(\varsigma_{n-1},\varsigma_n,\gamma)}-1,$$

$$C(\varsigma_n,\varsigma_{n+1},\gamma) < (C(\varsigma_{n-1},\varsigma_n,\gamma)),$$

and

$$D(\varsigma_n,\varsigma_{n+1},\gamma) < (D(\varsigma_{n-1},\varsigma_n,\gamma)).$$

So, we have

1. the sequence $\left(A(\varsigma_n,\varsigma_{n+1},\gamma) : n \in N\right)$ is nondecreasing in [0,1], and hence, there is $r_A \le 1$ such that r_A is the limit of this sequence.
2. the sequence $\left(B(\varsigma_n,\varsigma_{n+1},\gamma) : n \in N\right)$ is nondecreasing in [0,1], and hence, there is $r_B \le 1$ such that r_B is the limit of this sequence.
3. the sequence $\left(C(\varsigma_n,\varsigma_{n+1},\gamma) : n \in N\right)$ is nonincreasing in [0,1], and hence, there is $r_C \ge 0$ such that r_C is the limit of this sequence.
   and
4. the sequence $\left(D(\varsigma_n,\varsigma_{n+1},\gamma) : n \in N\right)$ is nonincreasing in [0,1], and hence, there is $r_D \ge 0$ such that r_D is the limit of this sequence..

Case 1: If $r_A > 0$, by taking the limit in Eq 3, we get

$$\psi\left(\frac{1}{r_A}-1\right) \le G\left(\psi\left(\frac{1}{r_A}-1\right),\varphi\left(\frac{1}{r_A}-1\right)\right) \le \psi\left(\frac{1}{r_A}-1\right),$$

which implies that $\psi\left(\frac{1}{r_A}-1\right)=0$, or, $\varphi\left(\frac{1}{r_A}-1\right)=0$, that is, $r_A=1$. a contradiction. So $r_A=1$. By the same way we conclude that $r_B=1, r_C=0$ and $r_D=0$.

Now, we claim that (ς_n) i Cauchy. If not then by Lemma 2.15, then there exist an $\varepsilon>0$ and $\gamma>0$ along with two subsequences (ς_{n_k}) and (ς_{m_k}) derived from (ς_n), where (m_k) such that one of the following holds

$$\lim_{k\to\infty} A(\varsigma_n,\varsigma_m,\gamma)=1-\varepsilon,$$

$$\lim_{k\to\infty} B(\varsigma_n,\varsigma_m,\gamma)=1-\varepsilon,$$

$$\lim_{k\to\infty} C(\varsigma_n,\varsigma_m,\gamma)=\varepsilon,$$

$$\lim_{k\to\infty} D(\varsigma_n,\varsigma_m,\gamma)=\varepsilon.$$

Using Definition 3.1, we deduce that one of the following holds

$$\psi(\frac{1}{A(\varsigma_{n_k},\varsigma_{m_k},\gamma)}-1)\le G\left(\psi(\frac{1}{A(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma)}-1),\varphi(\frac{1}{A(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma)}-1)\right),$$

$$\psi(\frac{1}{B(\varsigma_{n_k},\varsigma_{m_k},\gamma)}-1)\le G\left(\psi(\frac{1}{B(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma)}-1),\varphi(\frac{1}{B(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma)}-1)\right),$$

$$\psi(C(\varsigma_{n_k},\varsigma_{m_k},\gamma))\le G(\psi(C(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma)),\varphi(C(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma))),$$

or

$$\psi(D(\varsigma_{n_k},\varsigma_{m_k},\gamma))\le G(\psi(D(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma)),\varphi(D(\varsigma_{n_k-1},\varsigma_{m_k-1},\gamma))).$$

So, by taking the limit on $k\to\infty$, we get

$$\psi\left(\frac{1}{1-\varepsilon}-1\right)\le G(\psi\left(\frac{1}{1-\varepsilon}-1\right),\varphi\left(\frac{1}{1-\varepsilon}-1\right)),$$

which implies that $\psi\left(\frac{1}{1-\varepsilon}-1\right)=0$, or, $\varphi\left(\frac{1}{1-\varepsilon}-1\right)=0$, that is, $\varepsilon=0$. a contradiction.

or

$$\psi(\varepsilon)\le G(\psi(\varepsilon),\varphi(\varepsilon))\le\psi(\varepsilon),$$

which implies that $\psi(\varepsilon)=0$, or, $\varphi(\varepsilon)=0$, that is, $\varepsilon=0$, a contradiction

Hence (ς_n) is a Cauchy sequence, thus, there is $u\in F$ such that $\varsigma_n\to u$.

Definition 3.1 gives that

$$\psi(\frac{1}{A(fu,\varsigma_{n+1},\gamma)}-1)\le G\left(\psi(\frac{1}{A(u,\varsigma_n,\gamma)}-1),\varphi(\frac{1}{A(u,\varsigma_n,\gamma)}-1)\right)\to 0 \text{ as } n\to\infty,$$

$$\psi(\frac{1}{B(fu,\varsigma_{n+1},\gamma)}-1)\le G\left(\psi(\frac{1}{B(u,\varsigma_n,\gamma)}-1),\varphi(\frac{1}{B(u,\varsigma_n,\gamma)}-1)\right)\to 0 \text{ as } n\to\infty,$$

$$\psi(C(fu,\varsigma_{n+1},\gamma)) \le G\left(\psi(C(u,\varsigma_n,\gamma)),\varphi(C(u,\varsigma_n,\gamma))\right) \to 0 \text{ as } n \to \infty,$$

and

$$\psi(D(fu,\varsigma_{n+1},\gamma)) \le G\left(\psi(D(u,\varsigma_n,\gamma)),\varphi(D(u,\varsigma_n,\gamma))\right) \to 0 \text{ as } n \to \infty,$$

Which implies that ς_{n+1} converges to fu, hence $u = fu$.
Let $\upsilon \in F$ with $\upsilon = f\upsilon$. If $u \ne \upsilon$, then from Definition 3.1, it follows that

$$\psi(\frac{1}{A(u,\upsilon,\gamma)}-1) = \psi(\frac{1}{A(fu,f\upsilon,\gamma)}-1) \le G\left(\psi(\frac{1}{A(u,\upsilon,\gamma)}-1),\varphi(\frac{1}{A(u,\upsilon,\gamma)}-1)\right),$$

$$\psi(\frac{1}{B(u,\upsilon,\gamma)}-1) = \psi(\frac{1}{B(fu,f\upsilon,\gamma)}-1) \le G\left(\psi(\frac{1}{B(u,\upsilon,\gamma)}-1),\varphi(\frac{1}{B(u,\upsilon,\gamma)}-1)\right),$$

$$\psi(C(u,\upsilon,\gamma)) = \psi(C(fu,f\upsilon,\gamma)) \le G\left(\psi(C(u,\upsilon,\gamma)),\varphi(C(u,\upsilon,\gamma))\right),$$

and

$$\psi(D(u,\upsilon,\gamma)) = \psi(D(fu,f\upsilon,\gamma)) \le G\left(\psi(D(u,\upsilon,\gamma)),\varphi(D(u,\upsilon,\gamma))\right),$$

Therefore, $\psi(\frac{1}{A(u,\upsilon,\gamma)}-1) = 0$ or $\varphi(\frac{1}{A(u,\upsilon,\gamma)}-1) = 0$. So $u = \upsilon$.
We will now present an illustrative example to demonstrate our primary finding.

Example 3.3 *Let $\mathcal{W} = I$ with the standard metric $d(x,y) = |\varsigma - \omega|$, also, Let the t-norm and t-conorm be defined as follows $\varsigma \Diamond \omega = \min\{\varsigma,\omega\}$, $\varsigma \bullet \omega = \max\{\varsigma,\omega\}$. Additionally, let the fuzzy sets be defined as follows:*

$$A(\varsigma,\omega,\gamma) = \frac{\gamma + d(\varsigma,\omega)}{\gamma + 2d(\varsigma,\omega)}, B(\varsigma,\omega,\gamma) = \frac{\gamma}{\gamma + d(\varsigma,\omega)},$$

$$C(\varsigma,\omega,\gamma) = \frac{d(\varsigma,\omega)}{\gamma}, D(\varsigma,\omega,\gamma) = \frac{d(\varsigma,\omega)}{\gamma}.$$

Then, the self map $f : \mathcal{W} \to \mathcal{W}$, where $f(\varsigma) = 0.2\,\varsigma$ has a unique fixed point.

Proof. From [41], we have $(\mathcal{W},A,B,C,D,\Diamond,\bullet)$ is a complete NFM spaces. Now, let The function G be defined by $G(s,t) = 0.9.s$, and let $\psi(t) = t, \varphi(t) = t$. Then we have

$$\begin{aligned} \frac{1}{A(f\varsigma,f\omega,\gamma)} - 1 &= \frac{\gamma + 2d(f\varsigma,f\omega)}{\gamma + d(f\varsigma,f\omega)} - 1 \\ &= \frac{d(f\varsigma,f\omega)}{\gamma + d(f\varsigma,f\omega)} \\ &= \frac{0.2|\varsigma - \omega|}{\gamma + 0.2|\varsigma - \omega|} \end{aligned}$$

Since $|\varsigma - \omega| \le 1$, then one can verify that

$$\frac{0.2|\varsigma - \omega|}{\gamma + 0.2|\varsigma - \omega|} \le 0.9\,\frac{|\varsigma - \omega|}{\gamma + |\varsigma - \omega|}.$$

Thus, we have

$$\psi(\frac{1}{B(f\varsigma,f\omega,\gamma)}-1) \le G\left(\psi(\frac{1}{B(\varsigma,\omega,\gamma)}-1),\varphi(\frac{1}{B(\varsigma,\omega,\gamma)}-1)\right).$$

The remainder can be demonstrated in an analogous manner.

By defining the function $G(s,t)=ms$, with the constant m restricted to the interval $[0,1)$, we can draw the following conclusion.

Corollary 3.4 *Let* $(\mathcal{W},A,B,C,D,\Diamond,\bullet)$ *be a complete NFMS, and let* $\psi\in\Phi$. *Suppose that* $f:\mathcal{W}\to\mathcal{W}$ *satisfies the following for each* $\varsigma,\omega\in\mathcal{W}$ *and each* $\gamma>0$, *we have:*

$$\psi(\frac{1}{A(f\varsigma,f\omega,\gamma)}-1)\le m\,\psi\left(\frac{1}{A(\varsigma,\omega,\gamma)}-1\right),$$

$$\psi(\frac{1}{B(f\varsigma,f\omega,\gamma)}-1)\le m\,\psi\left(\frac{1}{B(\varsigma,\omega,\gamma)}-1\right),$$

$$\psi(C(f\varsigma,f\omega,\gamma))\le m\,\psi(C(\varsigma,\omega,\gamma)),$$

and

$$\psi(D(f\varsigma,f\omega,\gamma))\le m\,\psi(D(\varsigma,\omega,\gamma)).$$

Consequently, the function f possesses a unique fixed point.

By defining the function $G(s,t)=s-t$, with the constant m restricted to the interval $[0,1)$, we can draw the following conclusion.

Corollary 3.5 *Let* $(\mathcal{W},A,B,C,D,\Diamond,\bullet)$ *be a complete NFMS, and let* $\psi\in\Phi$, $\varphi\in\Phi_u$. *Suppose that* $f:\mathcal{W}\to\mathcal{W}$ *satisfies the following for each* $\varsigma,\omega\in\mathcal{W}$ *and each* $\gamma>0$, *we have:*

$$\psi\left(\frac{1}{A(f\varsigma,f\omega,\gamma)}-1\right)\le\psi\left(\frac{1}{A(\varsigma,\omega,\gamma)}-1\right)-\varphi\left(\frac{1}{A(\varsigma,\omega,\gamma)}-1\right),$$

$$\psi\left(\frac{1}{B(f\varsigma,f\omega,\gamma)}-1\right)\le\psi\left(\frac{1}{B(\varsigma,\omega,\gamma)}-1\right)-\varphi\left(\frac{1}{B(\varsigma,\omega,\gamma)}-1\right),$$

$$\psi(C(f\varsigma,f\omega,\gamma))\le\psi(C(\varsigma,\omega,\gamma))-\varphi(C(\varsigma,\omega,\gamma)),$$

and

$$\psi(D(f\varsigma,f\omega,\gamma))\le\psi(D(\varsigma,\omega,\gamma))-\varphi(D(\varsigma,\omega,\gamma)).$$

Consequently, the function f possesses a unique fixed point.

By defining the function $\psi(t)=kt$, with the constant k restricted to the interval $(0,1)$, and $\varphi(t)=Lt$, where $L>0$, we can draw the following conclusion.

Corollary 3.6 *Let* $(\mathcal{W},A,B,C,D,\Diamond,\bullet)$ *be a complete NFMS. Suppose that* $f:\mathcal{W}\to\mathcal{W}$ *satisfies the following for each* $\varsigma,\omega\in\mathcal{W}$ *and each* $\gamma>0$, *we have:*

$$\frac{1}{A(f\varsigma,f\omega,\gamma)}-1\le\left(\frac{1}{A(\varsigma,\omega,\gamma)}-1\right)-\frac{L}{k}\left(\frac{1}{A(\varsigma,\omega,\gamma)}-1\right),$$

$$\frac{1}{B(f\varsigma,f\omega,\gamma)}-1\le\left(\frac{1}{B(\varsigma,\omega,\gamma)}-1\right)-\frac{L}{k}\left(\frac{1}{B(\varsigma,\omega,\gamma)}-1\right),$$

$$C(f\varsigma,f\omega,\gamma)\le C(\varsigma,\omega,\gamma)-\frac{L}{k}C(\varsigma,\omega,\gamma),$$

and

$$D(f\varsigma, f\omega, \gamma) \leq D(\varsigma, \omega, \gamma) - \frac{L}{k} D(\varsigma, \omega, \gamma).$$

Consequently, the function f possesses a unique fixed point.

Remark 3.7 *By defining the function* $\mathcal{G}(s,t) = s\beta(s)$, $\beta : \mathbb{R}_0^+ \to 0,1)$ *a continuous function with the property* $\beta(t_n) \to 1 \Rightarrow t_n \to 0$ *for each sequence* (t_n) *in* $\mathbb{R}_0^+$, $\mathcal{G}(s,t) = s \Rightarrow s = 0$, *we can draw the following conclusion. Definition 3.5 of [48] and so Theorem 3.6 [48].*

Conclusion

Fixed-point theory encompasses a variety of theorems that examine the behavior of transformations applied to points within a specific set, ensuring the existence of at least one invariant point. These theorems are crucial for demonstrating the existence of solutions to numerous equations and systems across different mathematical disciplines. A prominent example is Banach's Fixed Point Theorem, which is fundamental in analysis and states that any contraction mapping from a complete metric space to itself has a unique fixed point. Such theorems are vital in various fields, including differential equations, economics, and computer science, as they aid in identifying equilibria and solutions. In essence, fixed-point theorems are essential tools in both theoretical and applied mathematics, providing foundational insights and effective methods for tackling complex problems by confirming the existence and, in some instances, the uniqueness of solutions.

In this research, we utilized the concept of C-class functions to create new contraction mappings within the framework of neutrosophic fuzzy metric spaces. These contractions are employed to derive fixed point theorems relevant to complete neutrosophic fuzzy metric spaces, based on C-class functions. Additionally, we presented a series of fixed point results that are significant to this specific context. An example is included to illustrate our main findings. Our results aim to extend and generalize several existing outcomes found in the literature.

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