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Solving convex monotone equations by a modified projection technique

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Optimization problems are divided into constrained optimization problems and unconstrained optimization problems. Most real-world problems fall into constrained optimization problems involving linear or non-linear constraints. To overcome these obstacles, numerical optimization is resorted to, which includes several methods and techniques to find the optimal solution. One effective technique for solving optimization problems is the projection method, and We used a new projection technique to solve large non-linear monotonic equations with convex constraints. Large-scale monotonic non-linear equations can be solved using the enhanced method., which also has the advantage of requiring less memory. It is shown that under the correct conditions, the proposed method is globally convergent, and numerical results support its effectiveness.

Keywords: constrained optimization problems, projection method, monotonic equations.

Mathematics Subject Classification (2020): 26A48, 52A41

1. Introduction

One effective technique for resolving constrained optimization issues is the projection approach. The projection approach modifies an unconstrained optimization algorithm subject to certain simplifications to address confined optimization problems since most constrained problems cannot be solved using a planned algorithm used to solve unconstrained problems. Many types of equations have been solved in this method, including linear, non-linear, monotone and convex equations. The Monotone

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equations have been used in many situations in real life Over the past ten years, The benefit of the Newton method is a reduction in the time between iterations and the Set of equation solutions. The entire series obtained converges to an equational solution despite the limited number of solutions. Numerous attempts have been made to modify and hybridize previous methods to solve monotonous and convex equations [1–4]. Take into account the non-linear monotone equations.[4]

$$F(x) = 0\,,\, x \in \Omega, \tag{1.1}$$

Where $F : \Omega \to R^n$, continuous and monotone function, and $\Omega \subseteq R^n$ Is a closed convex set, F refers to it as the monotone function,which means:

$$\left\langle F(x) - F(y),\, x - y \geq 0 \right\rangle,\, \forall\, x, y \in \Omega. \tag{1.2}$$

There are many contributions to solving optimization problems that contain non-linear monotone equations. Among these contributions, Newton's and quasi-Newton's methods are the most famous. Its global convergence was quickly proven, but working in these ways requires lengthy arithmetic operations in addition to the need for a large memory [7, 8], which led to finding more effective alternativesin solving such problems (2, ...,6).to solve unconstrained monotone equations, we used the search direction $d_K As : d_0 = -F_0, d_K = -F_K \beta_K$, Where β_K It is a parameter with different forms [9, 10]. The proposed method showed that it is suitable for solving non-linear systems of equations if they are monotone. This is done using the project technique with some updates, and Its convergence has been proven. The numerical results were also very acceptable and in a satisfactory time[11, 12]. One of the advantages of this method or this update over the projection method is that it has low memory, which makes it suitable for solving monotonous optimization problems (8, ..., 18) [13, 14]. In this paper, we will construct our changes as follows for solution (1.1); we w and

$$d_k = \begin{cases} (-F_k + \beta_k . w_k) - \vartheta_k . y_k & if\ k \geq 1 \\ -F_k & if\ k = 0 \end{cases} \tag{1.3}$$

Where:

$$F_k = F(x_k), y_k = F_{k+1} - F_k$$

$$w_k = x_{k+1} - x_k \text{ and } \vartheta_k = w_k{}^2 F_k{}^t y_k.$$

$$\beta_k = \frac{F(_{k+1})^T . \left(F_{(k+1)}\right)}{F(_k)^2}, F(_k)^2 \neq 0 \tag{1.4}$$

And the α_k must satisfy the conditions:

$$F(x_k + \alpha_k d_k)^T d_k \leq \sigma \alpha_k. \tag{1.5}$$

Where α_k is any scale.

Note that over the principle that d_k is a descent direction of the function f at the point x_k,all iterative algorithms use this important property to ensure their convergence [17, 18].

1. Proposed Methodology and Algorithms

In this section, we will explain our modification to the original Algorithm for the projection method by making simple and effective changes at the same time to: α_k, β_K And line search as the following:

Step 1: Select initial point $x_0 \in \Omega, \rho \in (0,1), \sigma \in (0,1), \varepsilon = 0.001.$ and Set $k = 0$.

Step 2: If $F(x_k) = 0$, then stop; else, calculate d_k in (1.3),

$$\alpha_k = \mu, \text{ where } \mu = \left| \frac{F(x_k)^T d_k}{d_k^T \nabla F(x_k)} \right|.$$

Step 3: find α_k which satisfy $F(x_k + \alpha_k d_k)^T d_k \le \sigma \alpha_k$, Set: $x_{k+1} = x_k + \alpha_k d_k$

Step 4: if $F(x_{k+1}) = 0$, then stop. Otherwise, compute $x_{k+1} = x_k + \alpha_k d_k$, where: $\alpha_k = \rho \alpha_k$

Step 5: Set k := k + 1. Go to Step 2.

2. Global Convergence

To prove the global convergence for the Algorithm in section 2, we take the following assumption: [19, 20].

Assumption K: the mapping F is Lipchitz continuous on Ω, i.e., ∃ a positive number L > 0, such that

$$F[x_1] - F[x_2] \le Lx_1 - x_2, \forall x_1, x_2 \in \Omega \quad (3.1)$$

Lemma 3.1. [21] Assume that the assumption K holds, and $\{x_k\}$ be the sequence created by the Algorithm (2), then for all positive numbers C > 0, we have :

$$F(x_k) \le C. \quad (3.2)$$

Proof: For all $x \in \Omega$, satisfy

$$\begin{aligned} x_{k+1} - \bar{x}^2 &= [x_k + \alpha_k d_k] - \bar{x}^2 \\ &\le x_k - \alpha_k F(x_k + \alpha_k d_k)^2 \\ &= x_k - \bar{x}^2 - 2\alpha_k F(x_k + \alpha_k d_k), x_k - \bar{x} + \alpha_k^2 F(x_k + \alpha_k d_k)^2 \\ &\le x_k - \bar{x}^2 - 2\alpha_k F(x_k + \alpha_k d_k), x_k - x_k + \alpha_k d_k + \alpha_k^2 F(z_k)^2 \\ &= x_k - \bar{x}^2 - F(x_k + \alpha_k d_k), x_k - x_k + \alpha_k d_k^2 F(x_k + \alpha_k d_k)^2 \\ &\le x_k - \bar{x}^2, \end{aligned}$$

Then $x_k - \bar{x} \le x_0 - \bar{x}$.

From (1.5), for all k, we have

$$F(x_k) = F(x_k) - F(\bar{x}) \le Lx_k - \bar{x} \le Lx_0 - \bar{x}.$$

Suppose $S = Lx_0 - \bar{x}$, then (3.2) is proved clearly. □

Theorem 3.2. Since the assumption K is satisfied and the sequence $\{x_k\}$ is generated by Algorithm (2) then: $\underset{k\to\infty}{Lim} \inf F_k = 0.$ (3.3)

Proof: Let (3.3) unverified, then for $\varepsilon > 0$

$$\| F_k \| \ge, \ \forall \ k \ge 0. \quad (3.4)$$

Let $d_k = d_k - F_k + F_k \ge -2(d_k - F_k - F_k)$

$$\begin{aligned} &\ge \psi F_k - F_k, \text{ where } 0 < \psi < 2 \\ &\ge \psi F_k \end{aligned}$$

Then $d_k \ge \psi, \ \forall k \ge 0.$ (3.5)

Since $\beta_k = \dfrac{F_{(k+1)}{}^T.\left(F_{(k+1)}\right)}{F_{(k)}{}^2}$, $F_{(k)}{}^2 \neq 0$

Then $\left|\beta_k\right| = \left|\dfrac{F_{(k+1)}{}^T.\left(F_{(k+1)}\right)}{F_{(k)}{}^2}\right|$

$$\leq \varepsilon F_k * F_{K1} / F_k^2 \tag{3.6}$$

Now $d_k \leq \beta_k.d_{k-1} + F_k$

From the above formula and Lemma 3.1, we have $\|d_k\| \leq C$ where C is a positive scalar. By (3.4), (3.5), and (3.6), there is a contradiction about inequality $d_k \leq \beta_k.d_{k-1} + F_k$. Since (3.3) clamps and Theorem are established. Finally, it must be well-defined to prove the convergence of the Algorithm used in this research. Therefore, based on assumption k, we will consider that α_k Satisfies: $F\left(x_k + \alpha_k d_k\right)^T d_k \leq \sigma\alpha_k$.

Suppose, for the sake of argument, that it is not equal. $\left|\dfrac{F\left(x_k\right)^T d_k}{d_k{}^T \nabla F\left(x_k\right)}\right|$ Then, The line search criteria arenot contentby α_k, this means:

$$F\left(x_k\right)^T d_k \geq \sigma\alpha_k d_k \tag{3.7}$$

Since $\beta_k = \dfrac{F_{(k+1)}{}^T.\left(F_{(k+1)}\right)}{F_{(k)}{}^2}$, from above and the value of σ we get the line search is will define and

$\alpha_k = \min\left\{\sigma, \left|\dfrac{F\left(x_k\right)^T d_k}{d_k{}^T \nabla F\left(x_k\right)}\right|\right\}$.

3. Numerical Results

In this sectionwill be shown. We utilized a PC with 8 GB RAM and a 2.20 GHz CPU. The MATLAB R2014 codes were created. The stop condition is $\| F.\left(x_k\right) \| \leq 10^{-8}$ to obtain excellent accuracy for all test problems. Over 500000 iterations in total. The parameters used were as follows: $\rho = 0.10, \sigma = 0.2, \varepsilon = 0.00001$. We'll compare our Algorithm to two others that addressed the same problem and employed the same parameters or a methodology for addressing the parameters in our Algorithm in their solutions. The results of [22] and [23] have been encoded as QD and MM in the tables sequentially. The following tables contain a list of the outcomes of the tested algorithms. Table 1 shows the function evaluations, the iterations, and how long each Algorithm took to complete a given task.

Table 1: Functions Evaluations (f eval), Iterations (Iter), CPU-Time (in seconds).

problem	Dim	(f eval)			(Iter)			CPU-Time (in seconds)		
		Algorithm	QD	MM	Algorithm	QD	MM	Algorithm	QD	MM
P1	500000	86	417	74	22	102	150	0.28125	1	0.687
	500000	86	572	122	22	133	964	0.304688	4.4	3081
	500000	83	606	70	21	138	142	0.28125	7.2	1.078
	500000	83	620	493	21	140	313	0.289063	9	8.156

P2	500000	8	739	72	2	92	146	0.09375	0.2	0.578
	500000	19	182	135	3	200	107	0.078125	0.8	3416
	500000	5	243	72	1	256	146	0.015625	1.7	1.078
	500000	16	287	64	2	295	132	0.0625	2.5	0.546
P3	500000	27	739	62	2	92	126	0.5	0.2	0.562
	500000	321	182	996	16	200	789	6.375	0.9	2751
	500000	187	243	64	10	256	130	3.742188	1.9	0.921
	500000	1062	287	487	50	295	312	21.375	2.6	8.031
P4	500000	152	797	63	21	169	128	0.140625	6.1	0.515
	500000	157	186	101	30	462	935	0.117188	35	3071
	500000	146	324	67	15	804	136	0.125	11	1.031
	500000	150	485	63	19	120	128	0.117188	28	0.500

Conclusion

Convex monotone equations were solved using a modification to the projection method, and the numerical results showed that our proposed method was superior for solving these equations. The suggested Algorithm's smooth global convergence has been proved. The recommended approach, distinguished by little memory usage, inexpensive calculations, and standard or adequate time, was suitable for addressing the presented problems under specified situations.

References

[1] Iusem, A. N., & Solodov, M. V. (1977). Newton-type methods with generalized distances for constrained optimization. *Optimization*, 41, 257–278.

[2] Zhang, L., & Zhou, W. J. (2006). Spectral gradient projection method for solving non-linear monotone equations. *Journal of Computational and Applied Mathematics*, 196, 478–484.

[3] Li, Q. N., & Li, D. H. (2011). A class of derivative-free methods for large-scale non-linear monotone equations. *IMA Journal of Numerical Analysis*, 31, 1625–1635.

[4] Mahdi, M. M., et al. (2022). Hybrid spectral algorithm under a convex constraint to solve non-linear equations. *Journal of Interdisciplinary Mathematics*, 25(5), 1333–1340.

[5] Dennis, J. E., & Schnabel, R. B. (1983). Numerical methods for unconstrained optimization and non-linear equations. Prentice-Hall.

[6] Grippo, L., & Sclandrone, M. (2007). Non-monotone derivative-free methods for non-linear equations. *Computational Optimization and Applications*, 37, 297–328.

[7] Cheng, W. (2009). A PRP type method for systems of monotone equations. *Mathematics and Computational Modelling*, 50, 15–20.

[8] Dwail, H. H., et al. (2022). CG method with modifying β_k for solving unconstrained optimization problems. *Journal of Interdisciplinary Mathematics*, 25(5), 1347–1355.

[9] Zhou, W. J., & Li, D. H. (2007). Limited memory BFGS method for non-linear monotone equations. *Journal of Computational Mathematics*, 25, 89–96.

[10] Logarasu, R., & Abdulgafoor, A. (2015). Bayesian saliency using the spectral form of Relaxation aid cuts. *International Journal of communication and computer Technologies*, 3(1), 37–51.

[11] Yan, Q. R., Peng, X. Z., & Li, D. H. (2010). A globally convergent derivative-free method for solving large-scale non-linear monotone equations. *Journal of Computational and Applied Mathematics*, 234, 649–657.

[12] Shiker, M. A. K., & Amini, K. (2018). A new projection-based algorithm for solving a large-scale non-linear system of monotone equations. *Croatian Operational Research Review*, 9, 63–73.

[13] Allawi, D. H., & Shiker, M. A. K. (2024). A modified technique of spectral gradient projection method for solving non-linear equation systems. *Journal of Interdisciplinary Mathematics*, 27(4), 655–665.

[14] Muthupraveen, J., & Ramakrishnaprabu, G. (2015). Improving the Grid Performance in Hybrid Renewable Energy System. *International Journal of communication and computer Technologies*, 3(1), 21–30.

[15] Habib, H. S., & Shiker, M. A. K. (2024). A modified CG method for solving non-linear systems of monotone equations. *Journal of Interdisciplinary Mathematics*, 27(4), 787–792.

[16] Surendar, A. (2024). Emerging Trends in Renewable Energy Technologies: An In-Depth Analysis. *Innovative Reviews in Engineering and Science*, 1(1), 6–10.

[17] Zabiba, M. S. M., et al. (2023). A new technique to solve the maximization of transportation problems. *AIP Conference Proceedings*, 2414, 040042.

[18] Uvarajan, K. P. (2024). Integration of Artificial Intelligence in Electronics: Enhancing Smart Devices and Systems. *Progress in Electronics and Communication Engineering*, 1(1), 7–12.

[19] Rajput, A., Kumawat, R., Sharma, J., & Srinivasulu, A. (2024). Design of Novel High Speed Energy Efficient Robust 4: 2 Compressor. *Journal of VLSI Circuits and Systems*, 6(2), 53–64.

[20] Liu, H., et al. (2015). A conjugate gradient method with sufficient descent property. *Springer*, 70, 269–286.

[21] Hashim, K. H., et al. (2021). Solving the non-linear monotone equations by using a new line search technique. *Journal of Physics: Conference Series*, 1818, 012099.

[22] Wasi, H. A., & Mushtak, S. A. K. (2021). A modified FR method to solve unconstrained optimization. *Journal of Physics: Conference Series*, 1804, 012023.

[23] Dreeb, N. K., et al. (2021). Using a new projection approach to find the optimal solution for non-linear systems of monotone equations. *Journal of Physics: Conference Series*, 1818, 012101.