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Finding the least time and lowest cost to build residential complexes using the simplex method

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Abstract

This paper explores the dual optimization of time and cost objectives in the construction of residential complexes. By employing multiple objective functions, this study aims to determine the most efficient construction schedule that simultaneously minimizes time and cost. By applying mathematical optimization techniques, such as linear programming and multi-objective optimization algorithms, we seek to develop a systematic approach to balancing these competing objectives while considering constraints such as resource availability, project scope, and quality standards. By evaluating different trade-offs between cost and time, we found insights into optimal scheduling strategiesFor residential complex projects, which contributed to enhancing project management practices and improving project outcomes in the constructionindustry. Due to the complexities of residential projects, they involve many interconnected activities that must be implementedcarefully, planned, and coordinated. This paper aims to reduce the time required to complete the project at the lowest possible cost because delay leads to substantial financial losses.

Keywords and Phrases: Residential Complexes, Construction Project Management Optimization, Simplex Method, Multiple Objective Functions, Linear Programming.

Mathematical Subject Classification: 46N10, 47N10, 90C30

1. Introduction

Project completion requires careful planning and resource management for timely completion while minimizing project costs [1, 2]. To ensure project success and stakeholder satisfaction, achieving Time

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and cost minimization and balancing these goals is critical. At the same time, ensuring that quality standards are maintained [3]. The trade-off between time and cost is a challenge, and in response to this challenge, cost optimization techniques have emerged in construction projects [4, 5].

The concept of objective programming was created and developed. By Lee [6] and Ignisio [7]. As a result, goal programming has become an invention for solution and decision-making. Among these techniques, the simple method, a basic algorithm in linear programming, provides a systematic framework for optimizing project schedules considering multiple objective functions [8]. Yano and Sakawa [9] proposed an interactive fuzzy method for decision-making for multiple problemobjectives: Stochastic fuzzy linear programming andimproving the fragility criteria to obtain an acceptable solution. Nirma and Khan [10] presented a simple approach to solving the multi-objective linear programming (MOLP) problem, where all objectives are optimized simultaneously.

This paper aims to explore the extent of the application of the simple method to solve a multi-objective problem in terms of time and cost. The focus was on finding solutions with the least time and lowest price for building residential complexes. By formulating the construction project as a linear programming problem with multiple objective functions, using the weighted exaggeration method. Comparison between objectives was achieved to find the optimal solution to reduce time and cost and, simultaneously, to balance these competing objectives, considering constraints such as the availability of resources, the scope of the project, and quality standards.

2. The Problem of Multi-Objective Linear Programming

A multi-objective linear programming problem has objective functions k of n variables Subject to m constraints where the objective functions and constraints are linear [11]. Standard form It can be written as follows:

Minimize $(f_1(x) = c_1x, f_2(x) = c_2x, f_3(x) = c_3x, \ldots, f_k(x) = c_k(x)$ (1)

Subjects to

$$0 \le gj(x) \le 0, j = 1, 2, \ldots, m$$

$$hi(x) = 0, i = 1, 2, \ldots, n$$

Where $c_1 = (c_{11}, c_{12}, c_{13}, \ldots, c_{1n}), c_2 = (c_{21}, c_{22}, c_{23}, \ldots, c_{2n}), \ldots, c_k = (c_{k1}, c_{k2}, c_{k3}, \ldots, c_{kn}), x = (x_1, x_2, \ldots, x_n)$

3. Weighted-Sums Method

The weighted sum method is an effective method for solving multi-objective optimization problems. We use this method to find the shortest time and lowest cost to build an apartment complex. This method combines multiple objectives into one objective function, as in Equation (1), and uses weights that reflect the relative importance of each objective. A weight is assigned to each objective function, and then the functions are combined to build one objective function, such that the sum of the weights assigned to the functions equals one, where these weights represent the importance of each goal to decision-makers [12, 13]. Therefore, the above optimization model will be in the following form:

Minimize $W_1f_1 + W_2f_2 + W_3f_3 + \cdots . + W_kf_k$

Subjects to:

$$\sum_{k=1}^{p} w_k = 1,\ 0 \le w_k \le 1$$
$$0 \le g_i(x) \le 0,\ j = 1,2,3,\ldots,m$$
$$h_i(X) = 0,\ i = 1,2,3,\ldots,n$$
$$X \in R_n$$

4. Simplex Algorithm for Solving Multi-Objective Linear Programming Problems

1- Set of objective functions: The combined objective function can be generated by weighting the cost and time functions.

1.a. Define the Objectives:

- Objective 1: Minimize Cost
- Objective 2: Minimize Time

1.b. Assign Weights to Objectives:

Assign weights w_k such that $\sum_{k=1}^{p} w_k = 1$

1.c. Formulate the Combined Objective Function:

Using the weights, combine the cost and time objectives into a single objective function.

2-Constraints: Constraints are prepared according to the requirements of the problem. Where all necessary restrictions related to work, materials, and non-negativity are specified.

3- Optimization: Using the Simplex method, the liner program function is used to solve the optimal solution of the combined objective function. This is done by repeatedly applying the Simplex method to different weight combinations to find optimal Pareto solutions.

4-Pareto Solutions: These solutions provide the optimal values of decision variables, cost, and time, allowing decision-makers to select the best trade-off between cost and time.

5. Mathematical formulation

We define the cost and time objective functions:

Cost Function:

Estimate the proposed cost: Minimize cost $z_c = 10x_1 + 35x_2 + 70x_3 + 105x_4 + 60x_5$

Time Function:

Suggested time estimate: Minimize time $z_t = 7x_1 + 28x_2 + 40x_3 + 28x_4 + 60x_5$

Where x_1: Site preparation, x_2: Basic floor structure works, x_3: Structure construction, x_4: Floor and wall painting works, x_5: Electrical and finishing works. The coefficients of the proposed time function are the number of weeks to complete the task, while the cost function coefficients are the cost in millions during one week.

Using the weighted vulnerability sums method (interactive method), a weight is assigned to each objective function, where the functions are combined to build one objective function, noting that the sum of the weights equals one. Therefore, the proposed optimal model will be converted to the following optimization formula:

Minimize $Z = w_1 z_c + w_2 z_t$

$Z = w_1(10x_1 + 35x_2 + 70x_3 + 105x_4 + 60x_5) + w_2(7x_1 + 28x_2 + 40x_3 + 28x_4 + 60x_5)$

By suggesting a weight for each function by diet specialists where W1 = 0.4, W2 = 0.6 so, the mathematical model will be as follows:

$Z = 0.4(10x_1 + 35x_2 + 70x_3 + 105x_4 + 60x_5) + 0.6(7x_1 + 28x_2 + 40x_3 + 28x_4 + 60x_5)$

Minimize $Z = 8.2x_1 + 30.8x_2 + 52x_3 + 58.8x_4 + 60x_5$

Constraints

$$5x_1 + 20x_2 + 25x_3 + 20x_4 + 15x_5 \geq 35$$
$$2x_1 + 6x_2 + 4x_3 + 6x_5 \leq 20$$
$$6x_2 + 3x_3 + 3x_4 + 5x_5 \geq 10$$
$$x_1 + 3x_2 + 2x_3 + 2x_4 \leq 6$$
$$2x_1 + 2x_2 + 2x_5 \geq 6$$
$$x_1, x_2, x_3, x_4, x_5 \geq 0$$

Where x_1: Site preparation, x_2 : Foundation floor structure works, x_3: Building the structure, x_4: Floor and wall coating work, x_5: Electric work and finishing. To obtain the initial basic solution possible using Big-M, the problem is converted to canonical form by adding slack, redundant, and artificial variables, where S_1, S_2, S_3, S_4, $S_5 \geq 0$, as a redundant variable and A_1, A_2, $A_3 \geq 0$ as the artificial and suitable variable.

Iteration 1		C_j	8.2	30.8	52	58.8	60	0	0	0	0	0	M	M	M	$\frac{X_B}{X_2}$
B	C_B	x_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	A_1	A_2	A_3	
A_1	M	35	5	20	25	20	15	-1	0	0	0	0	1	0	0	$\frac{35}{20}=1.75$
S_2	0	20	2	6	4	0	5	0	1	0	0	0	0	0	0	$\frac{20}{6}=3.33$
A_2	M	10	0	(6)	3	3	5	0	0	-1	0	0	0	1	0	$\frac{10}{6}=1.66$
S_4	0	6	1	3	2	2	0	0	0	0	1	0	0	0	0	$\frac{6}{3}=2$
A_3	M	6	2	2	0	0	2	0	0	0	0	-1	0	0	1	$\frac{6}{2}=3$
Z = 51M		Z_j	7M	28M	28M	23M	22M	-M	0	-M	0	-M	M	M	M	
		Z_j-C_j	7M-8.2	28M-30.8 ↑	28M-52	23M-58.8	22M-60	-M	0	-M	0	-M	0	0	0	

The positive maximum $Z_j - C_j$ is 28M-30.8, so the input variable is x_2, the minimum ratio is 1.6667, and the departure basis variable is A_2.

Iteration 2		C_j	8.2	30.8	52	58.8	60	0	0	0	0	0	M	M	$\frac{X_B}{X_3}$
B	C_B	x_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	A_1	A_3	
A_1	M	1.66	5	0	(15)	10	-1.66	-1	0	3.33	0	0	1	0	$\frac{1.66}{1.5}=0.11$
S_2	0	10	2	0	1	-3	1	0	1	1	0	0	0	0	$\frac{10}{1}=10$
X_2	30.8	1.66	0	1	0.5	0.5	0.83	0	0	-0.16	0	0	0	0	$\frac{1.66}{0.5}=3.33$
S_4	0	1	1	0	0.5	0.5	-2.5	0	0	0.5	1	0	0	0	$\frac{1}{0.5}=2$
A_3	M	2.66	2	0	-1	-1	0.33	0	0	0.33	0	-1	0	1	—
Z = 4.33M + 51.33		Z_j	7M	30.8	14M+15.4	9M+15.4	-1.33M+25.66	-M	0	3.66M-5.13	0	-M	M	M	
		Z_j-C_j	7M-8.2	0	14M-36.6↑	9M-43.4	-1.33M-34.33	-M	0	3.66M-5.13	0	-M	0	0	

The positive maximum Z_j–C_j is 14M-36.6, so the input variable is X_3, the minimum ratio is 0.11, and the departure basis variable is A_1.

Iteration 3	C_j		8.2	30.8	52	58.8	60	0	0	0	0	0	M	$\frac{X_B}{X_1}$
B	C_B	x_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	A_1	
x_3	52	0.11	(0.33)	0	1	0.66	-0.11	-0.06	0	0.22	0	0	0	$\frac{0.11}{0.33}=0.33$
S_2	0	9.88	1.66	0	0	-3.66	-0.11	0.06	1	0.77	0	0	0	$\frac{9.88}{1.66}=5.93$
X_2	30.8	1.61	-0.16	1	0	0.166	0.88	0.03	0	-0.27	0	0	0	—
S_4	0	0.94	0.83	0	0	0.166	-2.44	0.03	0	0.38	1	0	0	$\frac{0.94}{0.83}=1.13$
A_3	M	2.77	2.33	0	0	-0.33	0.22	-0.06	0	0.55	0	-1	1	$\frac{2.77}{2.33}=1.19$
Z = 2.77M + 55.4	Z_j		2.33M+ 12.2	30.8	52	-0.33M+ 39.8	0.22M+ 21.6	-0.66M- 2.44	0	0.55M+ 3	0	-M	M	
	Z_j–C_j		2.33M+ 4↑	0	0	-0.33M-19	0.22M- 38.4	-0.06M- 2.44	0	0.55M+ 3	0	-M	0	

The positive maximum Z_j– C_j is 2.33M+4, so the input variable is x_1, the minimum ratio is 1 so the departure basis variable is x_3

Iteration 4	C_j		8.2	30.8	52	58.8	60	0	0	0	0	0	$\frac{X_B}{X_5}$
B	C_B	x_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	
x_1	8.2	0.33	1	0	3	2	-0.33	-0.2	0	0.66	0	0	—
S_2	0	9.33	0	0	-5	-7	0.66	0.4	1	-0.33	0	0	$\frac{9.33}{0.66}=14$
X_2	30.8	1.66	0	1	0.5	0.5	0.83	0	0	-0.16	0	0	$\frac{0.66}{0.83}=2$
S_4	0	0.66	0	0	-2.5	-1.5	-2.16	0.2	0	-1.66	1	0	—
A_3	M	2	0	0	-7	-5	(1)	0.4	0	-1	0	1	$\frac{2}{1}=2$
Z = 2M + 54.06	Z_j		8.2	30.8	-7M+40	-5M+ 31.8	M+22.93	0.4M-1.64	0	-M+0.33	0	-M	
	Z_j–C_j		0	0	-7M-12	-5M-27	M-37.06↑	0.4M-1.64	0	-M+0.33	0	-M	

The positive maximum Z_j– C_j is M-37.06, so the input variable is X_5, the minimum ratio is five, and the departure basis variable is A_3.

Iteration 5	C_j		8.2	30.8	52	58.8	60	0	0	0	0	0	$\frac{X_B}{X_5}$
B	C_B	x_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5	
x_1	8.2	1	1	0	0.66	0.33	0	-0.06	0	0.33	0	-0.33	—
S_2	0	8	0	0	2-.66	3-.66	0	0.13	1	0.33	0	0.66	$\frac{8}{0.13}=60$
X_2	30.8	0	0	1	6.33	4.66	0	-0.33	0	0.66	0	0.83	—

S_4	0	5	0	0	-17.66	-12.33	0	(1.06)	0	-2.33	1	-2.16	$\frac{5}{1.06} = 4.68$
X_5	60	2	0	0	-7	-5	1	0.4	0	-1	0	-1	$\frac{2}{0.4} = 5$
Z = 128.2		Z_j	8.2	30.8	-219.46	-153.53	60	13.18	0	-36.73	0	-37.06	
		$Z_j – C_j$	0	0	-27.4667	-212.33	0	13.18↑	0	-36.73	0	-37.06	

The positive maximum $Z_j – C_j$ is 13.18, so the input variable is S_1, the minimum ratio is 4.68, and the departure basis variable is S_4.

Iteration 6	C_j		8.2	30.8	52	58.8	60	0	0	0	0	0
B	C_B	x_B	x_1	x_2	x_3	x_4	x_5	S_1	S_2	S_3	S_4	S_5
X_1	8.2	1.31	1	0	-0.43	-0.43	0	0	0	0.18	0.06	-0.46
S_2	0	7.25	0	0	1.37	-2.12	0	0	1	0.62	-0.12	0.93
X_2	30.8	1.56	0	1	0.81	0.81	0	0	0	-0.06	0.31	0.15
S_1	0	4.68	0	0	-16.56	-11.56	0	1	0	-2.18	0.93	-2.03
X_5	60	0.12	0	0	-0.37	-0.37	1	0	0	-0.12	-0.37	-0.18
Z = 66.38		Z_j	8.2	30.8	-1.06	-1.06	60	0	0	-7.88	-12.36	-10.28
		$Z_j – C_j$	0	0	-53.06	-59.06	0	0	0	-7.88	-12.36	-10.28

Since all $Z_j – C_j \leq 0$ Hence, the optimal solution is arrived with value of variables as

$x_1 = 1.312$, $x_2 = 1.562$, $x_3 = 0$, $x_4 = 0$, $x_5 = 0.125$, Min $Z = 66.387$

6. Results and Discussion

The above model has five variables and five constraints. Using the Simplex method, a solution to this problem was obtained. The Simplex method produced a set of Pareto optimal solutions, each representing a different balance between cost and time. Where Minimize $Z = 66.387$, $x_1 = 1.312$, $x_2 = 1.562$, $x_3 = 0$, $x_4 = 0$, $x_5 = 0.125$, Minimize Cost = 63.5 million per week, Minimize Time = 60 weeks. The lowest possible cost was obtained within one week, with the least time required to complete the project after prioritizing cost over time. Critical activities requiring resource allocation are prioritized to minimize delays and costs. Resource constraints such as labor availability and physical limits were strictly adhered to, ensuring realistic and practical solutions. These solutions have provided various options that enable stakeholders to choose based on their priorities.

7. Conclusion

Using the weighted sum method, we can effectively find the optimal trade-offs between minimizing the cost and time of building an apartment complex. The project was completed in 60 weeks. And at a weekly cost of 63.5 million per week. Therefore, this approach helped us make informed decisions that balanced the financial and time constraints of the project, leading to better project management. The Simplex method has proven to be highly effective in identifying optimal solutions for both cost and time. Its mathematical accuracy and ability to handle multiple constraints make it a suitable tool for complex construction projects. The method's ability to generate Pareto optimal solutions provided decision-making flexibility, allowing stakeholders to choose solutions that best fit their strategic objectives.

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