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Study stabilizability and solvability for chemical kinetics of the delayed oregonator model

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Delays naturally appear in chemical reactions and they are often responsible for presence of complex behaviours, we will be take delay effects in Beulosuv-Zhabotonksiy reaction this mechanism is represented by a simple model, called the Oregonator model. Chemical kinetics of the considered Oregonator model will be taken by use of delay mass-action law and study the stabilizability and solvability by backstepping method after formally introduce the chain approximation for kinetic scheme of delayed Oregonator model. We will compar stabilizability results output between backstepping with method of steps and backstepping with chain method

Key words and phrases: Chemical reaction, DDE, Backstepping method, Mass – Action law, chain approximation, Lyapunov, Controller

Mathematics Subject Classification (2010): 34D05, 37L45.

1. Introduction

Dynamical systems' behavior may be fundamentally affected by time delay, for instance, it might have a stabilizing or destabilizing influence [2]. Consequently, state is directly represented mathematically. In many applications, Delay in actuation or measurement may be required to provide a suitable a model mathematics for a particular objective, for instance control, prediction, or simulation. Additional well-known instances include delayed reactions of the drivers when simulating traffic jams [13], models for delayed epidemics [10], or the impact of delays on the vibrations of machine tools. Delays are not part of the basic physical framework underlying networks of deterministic reactions with mass action kinetics. Yet, in biological models, the existence of delay may be a crucial component in explaining of complex dynamical actions (such as particular types of oscillatory) [12]. Additionally,

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the addition of delay can help a simpler explanation for chemical systems have fewer concentration variables than intricate mass action mechanisms [14]. Chemistry and chemical engineering use mass-action kinetics to represent the dynamics of networks of chemical reactions or systems of chemical processes. These models, which contain mass- and energy-balancing modeling relations, are a particular type of compartmental system [1].

Another crucial finding is that nonlinear nonnegative models frequently belong to the category of reaction network-architected kinetic systems. This implies that numerous procedures that aren't directly connected to chemistry and may have delayed effects, such certain traffic networks, compartmental systems, epidemic processes, and population models, can be officially classified as chemical reaction networks. [1,9]

There are various reasons why we would want to add delayed variable formulations to models that are accessible to chemists in this category. While building a DDE model of a chemical system, the focus is now on characterizing the dynamic interactions among the concentrations of important species rather than listing intermediates and their reactions. Hence, less concentration-related variables will often manifest than in a traditional mass action mechanism.

Since delay-differential equations (DDEs) have richer mathematical framework compared to ordinary differential equations (ODE), modeling systems in the field biology and chemistry by using DDEs may introduce certain benefits from the numerical point of view, DDEs may lead to describe chemical reactions by a fewer concentration variables than the classical mass-action law [15].

2. Delayed Mass – Action System

The concentrations are the dependent variables in models of othermal chemical kinetics. Both the initial functions and the permitted types of DDEs must be restricted in order to ensure that the solutions of the DDEs stay nonnegative. Let the chemical concentrations in the system being studied at time t be represented by the vector $x(t) \in R^n$; x_i is a symbol for an elements of x. The evolution equation has the following form: $\dot{x}(t) = f(x(\tau), t)$ where f is a vector-valued functional with elements f_i. An initial vector-valued function $\varnothing(t)$ with elements $\varnothing_i(t)$ defined in the interval $\tau \in \left[-\tau_{max}, 0\right]$ is part of the whole model [6].

When delays occur, as they inevitably do in the kinetics of chemical reactions because of propagation phenomena, mass-action laws might not be adequate. Chemical reaction delays can be disregarded since they are frequently minor enough. However, delay presence in system's dynamics is often responsible for unexpected behaviours [15]. In addition, as mentioned in [5] delays induced behaviours in complex chemical reactions should be considered to better understand the dynamics of such reactions. Belousov-Zhabotinsky reaction, called BZ reaction, is a very complex chemical reaction involving dozens of elementary steps and exhibiting various type of behaviours such as steady-state, periodic, and chaotic. In summary, the reaction is comprised of two distinct processes, denoted as Process A and Process B, within a single system. The dominance of one process over the other is contingent upon the concentration of bromide ions within the reaction at any given time. Process A dominates the reaction when the Bromide ion concentration is sufficiently high, and the Bromide ion is consumed monotonically during this process. Then, whenever the concentration reduces below some certain level, Process B starts to dominate the reaction. The oscillatory mechanism origins that Bromide ion is indirectly produced in Process B, hence, control of the reaction is returned to Process A. It should be noted that despite the advantages of use of DDEs in the field of bioscience, mathematical solution of a DDE is more difficult compared to ODE. Solution of a DDE requires an infinite set of initial history function, therefore, it is not trivial to ensure positivity of the solution, which is required for DDEs describing the chemical reactions [15]. Non-linear dynamical systems are controlled and stabilized using a technique from control theory known as the "backstepping method". Stabilizing the system origin iteratively is necessary to do this, as this is where the control process finishes once the last external control is assessed [5,11]. Additionally, employing a Lyapunov function and virtually error variables to guarantee system stability, creating control rules for each subsystem of the original

system of ordinary differential equations (ODEs) is a common task that is accomplished through the backstepping approach, Details are available at [4].

In Section 2 chain method will be given, in addition introduce for kinetic formal of delayed Oregonator model, by use chain method to approximate his. while Section 3 contains 4 application backstepping method for stabilizing and solving for kinetic scheme of delayed Oregonator

3. Chain method

The theory of delayed differential equations in mathematics and mathematical modeling in chemical engineering both independently introduced the chain method as an approximation technique for temporal delays [10]. The chain technique is important because it may be used to extend the results of control theory of ordinary differential equations to delay differential equations. [7] Because the phase-space has infinite dimensions, nonlinear delayed differential equations are often difficult to analyze. In order to solve this problem, delayed terms were approximated using a series of first-order differential equations, which led to the approximate of delayed differential equations by a set of ODEs [10].

In order to generalize the approximation of the chain method for kinetic scheme of delayed Oregonator model, we can introduce

$$\dot{x}(t) = Af(y(t-\tau)) - g(x(t)y(t)) + Au(x(t)) - Hh(x(t)), \forall t \geq 0 \tag{1}$$

where $x(t) \in R^n$ is the state vector, $\tau > 0$, f: $R^n \to R^n$, h: $R^n \to R^n$ are continuous functions, A, $H \in R^n$ is a constant vector, and $x(t) = \theta(t)$ for $-\tau \leq t \leq 0$ is the continuous initial function. The approximating set of ODEs with a chain containing N new state variables ('compartments') denoted by v_i for $i = 1,...,N$ is the following for each $n \in N$, $n \geq 2$, is the following:

$$\begin{aligned}
\dot{z}(t) &= Au(x(t)) - H\,h(x(t)) - g(x(t)y(t)) + \tfrac{N}{t}(t) \\
\dot{v}_1(t) &= A\,u(x(t)) - \tfrac{N}{t}V_1(t) \\
\dot{v}_2(t) &= -H\,h(x(t)) - \tfrac{N}{t}V_2(t) \\
\dot{v}_i(t) &= \tfrac{N}{t}V_{i-1}(t) - \tfrac{N}{t}V_i(t)
\end{aligned} \tag{2}$$

$$\text{with } z(0) = (0), \text{ and } v_i(0) = \int_{-i\frac{N}{\tau}}^{-(i-1)\frac{N}{\tau}} h(\theta(s))ds,\ 1 \leq i \leq N$$

$$\text{Note that } z(t) \in R^n \text{ and } v_i(t) \in R,\ i = 1,...,N$$

On compact subintervals of $[-\tau, \infty)$ the approximation is always uniform. Furthermore, the bounded solutions of (1) are uniformly approximated over the entire unbounded interval $[0, \infty]$ with certain limitations on the parameters.

4. Application Backstepping Method for Stabilizing and Solving for kinetic scheme of delayed Oregonator

The reaction steps of the irreversible Oregonator model is as given below

$$A + y \xrightarrow{k_1} x + o \tag{$R1$}$$

$$x + y \xrightarrow{k_2} 2o \tag{$R2$}$$

$$B + y \xrightarrow{k_a} 2x + h \tag{$R3$}$$

$$2x \xrightarrow{k_4} A + o \tag{$R4$}$$

$$h \xrightarrow{k_5} fy \tag{$R5$}$$

where A, B are assumed to be constant chemical species, y (Bromide ion), x (Bromous acid), and h (Cerium (IV)) are chemical variables, o is some chemical product, k_i ,i = 1,...,5, is the positive rate constant, and, f is a stoichiometric factor

The reactants in a chemical reaction are not necessarily to react whenever the activation energy is provided. In addition to such a time lag, it is not physically possible to maintain the required activation energy instantaneously. In order to take into account these type of delays in the BZ reaction, the Oregonator model can be expressed, by replacing (R1) with

$$A + y \xrightarrow{k_1} x(t+\tau) + o(t+\tau), \qquad (dR1)$$

which implies that x and o appear after τ units. The recent mechanism, which corresponds to replacing reaction step R1 by dR1, is more realistic compared to the Oregonator model from the physical point of view. since the reaction starts whenever the present Bromide ion starts to decrease, taking into account the existing time lag in R1 is more meaningful compared to elementary steps, for more details the reaction mechanism which five elementary steps see [15] .

By delayed mass-action laws, the kinetic scheme of delayed Oregonator model, which consists of the steps (dR1), $R2 - R5$, can be described by the following DDEs

$$\begin{aligned}
\dot{x}(t) &= k_1 A y(t-\tau) - k_2 x(t) y(t) + k_3 A x(t) - 2k_4 x(t)^2 \\
\dot{y}(t) &= -k_1 A y(t) - k_2 x(t) y(t) + k_5 f h(t) \\
\dot{h}(t) &= k_3 B x(t) - k_5 h(t)
\end{aligned} \qquad (3)$$

where $k_1 = 1.34M^{-1}\,sec^{-1}$, $k_2 = 1.6 \times 109M^{-1}\,sec^{-1}$, $k_3 = 8 \times 103M^{-1}\,sec^{-1}$, $k_4 = 4 \times 107M^{-1}\,sec^{-1}$, $k_5 = 1\,sec^{-1}$, $f = 1$, and $A = B = 0.06$. [15]

To stabilize and find the solution for the time-step interval [0,5], apply the approximate chain, then the first equation of system (3) will be approximated by the following the system ODE with non-constant coefficients up to the first order:

$$\begin{aligned}
\dot{x}(t) &= k_1 A y(t-\tau) - k_2 x(t) y(t) + k_3 A x(t) - 2k_4 x(t)^2 \\
\dot{z}(t) &= k_3 A z(t) - 2k_4 z(t)^2 - k_2 z(t) y(t) + \tfrac{N}{\tau} v_4(t) \\
\dot{v}_1(t) &= k_3 A x(t) - \tfrac{N}{\tau} v_1(t) \\
\dot{v}_2(t) &= -2k_4 x(t)^2 - \tfrac{N}{\tau} v_2(t) \\
\dot{v}_3(t) &= \tfrac{N}{\tau} v_2(t) - \tfrac{N}{\tau} v_3(t) \\
\dot{v}_4(t) &= \tfrac{N}{\tau} v_3(t) - \tfrac{N}{\tau} v_4(t) 2 \le i \le N, \text{ s.t } i = N = 4 \\
\dot{y}(t) &= -k_1 A y(t) - k_2 x(t) y(t) + k_5 f h(t) \\
\dot{h}(t) &= k_3 B x(t) - k_5 h(t)
\end{aligned} \qquad (4)$$

with the initial conditions:

$$\begin{aligned}
&z(0) = \varphi(0), \varphi_{10}(t) = t^2 + \tfrac{1}{4}, \\
&v_1(0) = \varphi_{20}(t) = k_3 \int_{\frac{\tau}{N}}^{-(0)\frac{\tau}{N}} (t^2 + \tfrac{1}{2}) dt, \\
&v_2(0) = \varphi_{30}(t) = k_3 \int_{-2\frac{\tau}{N}}^{-(1)\frac{\tau}{N}} (t^2 + \tfrac{1}{2}) dt, \\
&v_3(0) = \varphi_{40}(t) = k_3 \int_{-3\frac{\tau}{N}}^{-(2)\frac{\tau}{N}} (t^2 + \tfrac{1}{2}) dt, \\
&v_4(0) = \varphi_{50}(t) = k_3 \int_{-4\frac{\tau}{N}}^{-(3)\frac{\tau}{N}} (t^2 + \tfrac{1}{2}) dt \\
&\varphi_{60}(t) = t + \tfrac{1}{2}, \varphi_{70}(t) = 2t + \tfrac{1}{3}
\end{aligned}$$

where $v_i(0) = \int_{-i\frac{\tau}{N}}^{-(i-1)\frac{\tau}{N}} h((s))ds, 1 \le i \le N$

Note that $z(t) \in R^n$ and $v_i(t) \in R, i = 1,...,N$

The resulting differential equations system (4) will now be analyzed using the backstepping method as shown in the following steps:

Step (1): Consider about the system (4) first equation's stability. and let $z_1(t) = x(t)$. Therefore, $z_1(t)$ derivative is:

$$\begin{aligned}\dot{z}_1(t) &= \dot{x}(t)\\ &= k_3 A z_1(t) - 2k_4 z_1(t)^2 - k_2 z_1(t) y(t) + \frac{N}{\tau} v_4(t) + u_1(t)\end{aligned}$$

Where the term "virtual controller" refers to $x_2(t)$, and the quadratic form of the Lyapunov function is as follows:

$$V_1(z_1) = \frac{1}{2} z_1^2(t)$$

Consequently, the time derivative of its becomes:

$$\begin{aligned}\dot{V}_1(z_1) &= \frac{\partial V_1}{\partial t} = \frac{\partial V_1}{\partial z_1}\frac{\partial z_1}{\partial t}\\ &= z_1(t)\dot{z}_1(t)\\ &= z_1(t)(k_3 A z_1(t) - 2k_4 z_1(t)^2 - k_2 z_1(t) y(t) + \tfrac{N}{\tau} v_4(t) + u_1(t))\end{aligned}$$

Assume the controller $x_2(t) = \alpha_1(z_1)$.

If $u_1(t) = -2k_3 A z_1(t) + 2k_4 z_1(t)^2 + k_2 z_1(t) y(t) - \frac{N}{\tau} v_4(t)$

if $\alpha_1(z_1) = 0$, then

$\dot{V}_1(z_1) = -k_3 A z_1^2(t)$ for all $t \in [0, 4]$, which is a definite negative function.

The first equation of system (4) is asymptotically stable given the recursive feedback control $u_1(t)$ and $\alpha_1(z_1)$, where $\alpha_1(z_1)$ is an estimating function when $x_2(t)$ is regarded as a controller.

Step (2): from step (1) it is necessary to determine the error between $v_1(t)$ and $\alpha_1(z_1)$ where $z_2(t) = v_1(t) - \alpha_1(z_1)$ and $\alpha_1(z_1) = 0$. Following that

$$\dot{z}_2(t) = \dot{v}_1(t) = k_3 A z_1(t) - \frac{N}{\tau} z_2(t) + u_2(t)$$

The control Lyapunov function is consider

$$V_2(z_1, z_2) = V_1 + \tfrac{1}{2} z_2^2(t)$$

Hence, the derivative of V_2 is :

$$\begin{aligned}\dot{V}_2(z_1, z_2) &= \dot{V}_1 + z_2(t)\dot{z}_2(t)\\ &= -k_3 A z_1^2(t) + z_2(t)(k_3 A z_1(t) - \tfrac{N}{\tau} z_2(t) + u_2(t))\end{aligned}$$

If $u_2(t) = -k_3 A z_1(t)$ then:

$$\dot{V}_2(z_1, z_2) = -k_3 A z_1^2(t) - \frac{N}{\tau} z_2^2(t)$$

for all $t \in [0,4]$, which is a definite negative function. The second equation of system (4) is asymptotically stable, according to the recursive feedback control $u_2(t)$ with $\alpha_1(z_1, z_2) = 0$.

Step (3): from step (2) it is necessary to determine the error between $v_2(t)$ and $\alpha_2(z_1, z_2)$ where $z_3(t) = v_2(t)–\alpha_2(z_1, z_2)$ and $\alpha_2(z_1, z_2) = 0$. Following that

$$\dot{z}_3(t) = \dot{v}_2(t) = -2k_4 z_1(t)^2 - \frac{N}{\tau} z_3(t) + u_3(t)$$

Consider the control Lyapunov function

$$V_3(z_1, z_2, z_3) = V_2(z_1, z_2) + \frac{1}{2} z_3^2(t)$$

The V_3 derivative is:

$$\begin{aligned}\dot{V}_3(z_1, z_2, z_3) &= -k_3 A z_1^2(t) - \frac{N}{\tau} z_2^2(t) + z_3(t)\dot{z}_3(t) \\ &= -k_3 A z_1^2(t) - \frac{N}{\tau} z_2^2(t) + z_3(t)\left(-2k_4 z_1(t)^2 - \frac{N}{\tau} z_3(t) + u_3(t)\right)\end{aligned}$$

If $u_3(t) = -2k_4 z_1(t)^2$

then:

$$\dot{V}_3(z_1, z_2, z_3) = -k_3 A z_1^2(t) - \frac{N}{\tau} z_2^2(t) - \frac{N}{\tau} z_3^2(t)$$

Also for all $t \in [0,4]$, which is a definite negative function. Due to the recursive feedback control $u_3(t)$, the third equation of system (4) is asymptotically stable.

Step (4): from step (3) it is necessary to determine the error between $v_3(t)$ and $\alpha_3(z_1, z_2, z_3)$ where $z_4(t) = v_3(t)–\alpha_3(z_1, z_2, z_3)$ and $\alpha_3(z_1, z_2, z_3) = 0$. Following that

$$\dot{z}_4(t) = \dot{v}_3(t) = \frac{N}{\tau} z_3(t) - \frac{N}{\tau} z_4(t) + u_4(t)$$

Consider the control Lyapunov function

$$V_4(z_1, z_2, z_3, z_4) = V_3(z_1, z_2, z_3) + \frac{1}{2} z_4^2(t)$$

The V_4 derivative is:

$$\begin{aligned}\dot{V}_4(z_1, z_2, z_3, z_4) &= -k_3 A z_1^2(t) - \frac{N}{\tau} z_2^2(t) - \frac{N}{\tau} z_3^2(t) + z_4(t)\dot{z}_4(t) \\ &= -k_3 A z_1^2(t) - \frac{N}{\tau} z_2^2(t) - \frac{N}{\tau} z_3^2(t) + z_4(t)\left(\frac{N}{\tau} z_3(t) - \frac{N}{\tau} z_4(t) + u_4(t)\right)\end{aligned}$$

If $u_4(t) = -\frac{N}{\tau} z_3(t)$

then:

$$\dot{V}_4(z_1, z_2, z_3, z_4) = -k_3 A z_1^2(t) - \frac{N}{\tau} z_2^2(t) - \frac{N}{\tau} z_3^2(t) - \frac{N}{\tau} z_4^2(t)$$

Also for all $t \in [0,4]$, which is a definite negative function. Due to the recursive feedback control $u_4(t)$, the four equation of system (4) is asymptotically stable.

Step (5): from step (4) it is necessary to determine the error between $v_4(t)$ and $\alpha_3(z_1, z_2, z_3, z_4)$ where $z_4(t) = v_4(t) - \alpha_3(z_1, z_2, z_3, z_4)$ and $\alpha_3(z_1, z_2, z_3, z_4) = 0$. Following that

$$\dot{z}_5(t) = \dot{v}_4(t) = \frac{N}{\tau} z_4(t) - \frac{N}{\tau} z_5(t) + u_5(t)$$

Consider the control Lyapunov function

$$V_5(z_1,z_2,z_3,z_4,z_5) = V_4(z_1,z_2,z_3,z_4) + \frac{1}{2}z_5^2(t)$$

The V_5 derivative is:

$$\begin{aligned}\dot{V}_5(z_1,z_2,z_3,z_4,z_5) &= -k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) + z_5(t)\dot{z}_5(t)\\ &= -k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) + z_5(t)\left(\frac{N}{\tau}z_4(t) - \frac{N}{\tau}z_5(t) + u_5(t)\right)\end{aligned}$$

If $u_5(t) = -\frac{N}{\tau}z_4(t)$

then :

$$\begin{aligned}&V_5(z_1,z_2,z_3,z_4,z_5) =\\ &-k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) - \frac{N}{\tau}z_5^2(t)\end{aligned}$$

Also for all $t \in [0,4]$, which is a definite negative function. Due to the recursive feedback control $u_5(t)$, the five equation of system (4) is asymptotically stable.

Step (6): from step (5) it is necessary to determine the error between $y(t)$ and $\alpha_3(z_1, z_2, z_3, z_4, z_5)$ where $z_4(t) = y(t) - \alpha_4(z_1, z_2, z_3, z_4)$ and $\alpha_5(z_1, z_2, z_3, z_4, z_5) = 0$. Following that

$$\dot{z}_6(t) = \dot{y}(t) = -k_1Az_6(t) - k_2z_1(t)\ z_6(t) + k_5fh(t) + u_6(t)$$

Consider the control Lyapunov function

$$V_6(z_1,z_2,z_3,z_4,z_5,z_6) = V_5(z_1,z_2,z_3,z_4,z_5) + \frac{1}{2}z_6^2(t)$$

The V_6 derivative is:

$$\begin{aligned}\dot{V}_6(z_1,z_2,z_3,z_4,z_5,z_6) &= -z_1^2(t) - z_2^2(t) - z_3^2(t) - z_4^2(t) - z_5^2(t) + z_6(t)\dot{z}_6(t)\\ &= -k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) - \frac{N}{\tau}z_5^2(t)\\ &+z_6(t)(-k_1Az_6(t) - k_2z_1(t)\ z_6(t) + k_5fh(t) + u_6(t))\end{aligned}$$

If $u_6(t) = -(-k_2z_1(t)\ z_6(t) + k_5\ fh(t))$

then:

$$\begin{aligned}&V_6(z_1,z_2,z_3,z_4,z_5,z_6) =\\ &-k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) - \frac{N}{\tau}z_5^2(t) - k_1Az_6^2(t)\end{aligned}$$

Also for all $t \in [0,4]$, which is a definite negative function. Due to the recursive feedback control $u_6(t)$, the sixth equation of system (4) is asymptotically stable.

Step (7): from step (6) it is necessary to determine the error between $h(t)$ and $\alpha_6(z_1, z_2, z_3, z_4, z_5, z_6)$ where $z_7(t) = h(t) - \alpha_6(z_1, z_2, z_3, z_4, z_5, z_6)$ and $\alpha_6(z_1, z_2, z_3, z_4, z_5, z_6) = 0$. Following that

$$\dot{z}_7(t) = \dot{h}(t) = k_3Bz_1(t) - k_5z_7(t) + u_7(t)$$

Consider the control Lyapunov function

$$V_7(z_1,z_2,z_3,z_4,z_5,z_6,z_7) = V_6(z_1,z_2,z_3,z_4,z_5,z_6) + \frac{1}{2}z_7^2(t)$$

The V_7 derivative is:

$$\begin{aligned}
&\dot{V}_6(z_1,z_2,z_3,z_4,z_5,z_6,z_7) = \\
&-k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) - \frac{N}{\tau}z_5^2(t) - k_1Az_6^2(t) + z_7(t)\dot{z}_7(t) \\
&= -k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) - \frac{N}{\tau}z_5^2(t) - k_1Az_6^2(t) \\
&+ z_7(t)(k_3Bz_1(t) - k_5z_7(t) + u_7(t))
\end{aligned}$$

If $u_7(t) = -k_3Bz_1(t)$

then:

$$\begin{aligned}
&\dot{V}_7(z_1,z_2,z_3,z_4,z_5,z_6,z_7) = \\
&-k_3Az_1^2(t) - \frac{N}{\tau}z_2^2(t) - \frac{N}{\tau}z_3^2(t) - \frac{N}{\tau}z_4^2(t) - \frac{N}{\tau}z_5^2(t) - k_1Az_6^2(t) - k_5z_7^2(t)
\end{aligned}$$

Also for all $t \in [0,5]$, which is a definite negative function. Due to the recursive feedback control $u_7(t)$, the seventh equation of system (4) is asymptotically stable.

hence, the following result of the feedback controls are obtained:

$$\begin{aligned}
u_1(t) &= -2k_3Az_1(t) + 2k_4z_1(t)^2 + k_2z_1(t)y(t) - \frac{N}{\tau}v_4(t) \\
u_2(t) &= -k_3Az_1(t) \\
u_3(t) &= -2k_4z_1(t)^2 \\
u_4(t) &= -\frac{N}{\tau}z_3(t) \\
u_5(t) &= -\frac{N}{\tau}z_4(t) \\
u_6(t) &= -(-k_2z_1(t)z_6(t) + k_5fh(t)) \\
u_7(t) &= -k_3Bz_1(t)
\end{aligned}$$

In system (8), by substituting $u_1(t)$, $u_2(t)$ and $u_3(t)$, we obtain an ODE system with a nonconstant coefficient that has the following form:

$$\begin{aligned}
\dot{x}_1(t) &= -k_3Ax_1(t) \\
\dot{x}_2(t) &= -\frac{N}{\tau}x_2(t) \\
\dot{x}_3(t) &= -\frac{N}{\tau}x_3(t) \\
\dot{x}_4(t) &= -\frac{N}{\tau}x_4(t) \\
\dot{x}_5(t) &= -\frac{N}{\tau}x_5(t) \\
\dot{x}_6(t) &= -k_1Ax_6(t) \\
\dot{x}_7(t) &= -k_5x_7(t)
\end{aligned}$$

With initial conditions:

$$\varphi_{10}(0)=\frac{1}{4},\ \varphi_{20}(0)=4.5680e+01$$
$$\varphi_{30}(t)=1.1976e+02,\ \varphi_{40}(t)=2.6792e+02$$
$$\varphi_{50}(t)=4.9015e+02,\ \varphi_{60}(0)=t+\frac{1}{2},\ \varphi_{70}(0)=\frac{1}{3}$$

Which has the solution

$$x_1(t)=\frac{1}{4}e^{-k_3At},\qquad x_2(t)=(12.576)e^{-\frac{N}{\tau}t}$$
$$x_3(t)=(16.032)e^{-\frac{N}{\tau}t},\ x_4(t)=(22.944)e^{-\frac{N}{\tau}t}$$
$$x_5(t)=(33.312)e^{-\frac{N}{\tau}t},\quad x_6(t)=\frac{1}{2}e^{-k_1At},$$
$$x_7(t)=\frac{1}{3}e^{-k_3t}.$$

Figure (1) shows the solution of the siven equations of $x_1(t)$, $x_2(t)$, $x_3(t)$, $x_4(t)$, $x_5(t)$, $x_7(t)$ and $x_1(t)$ throughout the time step interval [0,4] using the control functions $u_1(t)$, $u_2(t)$, and $u_3(t)$ in Figure (2).

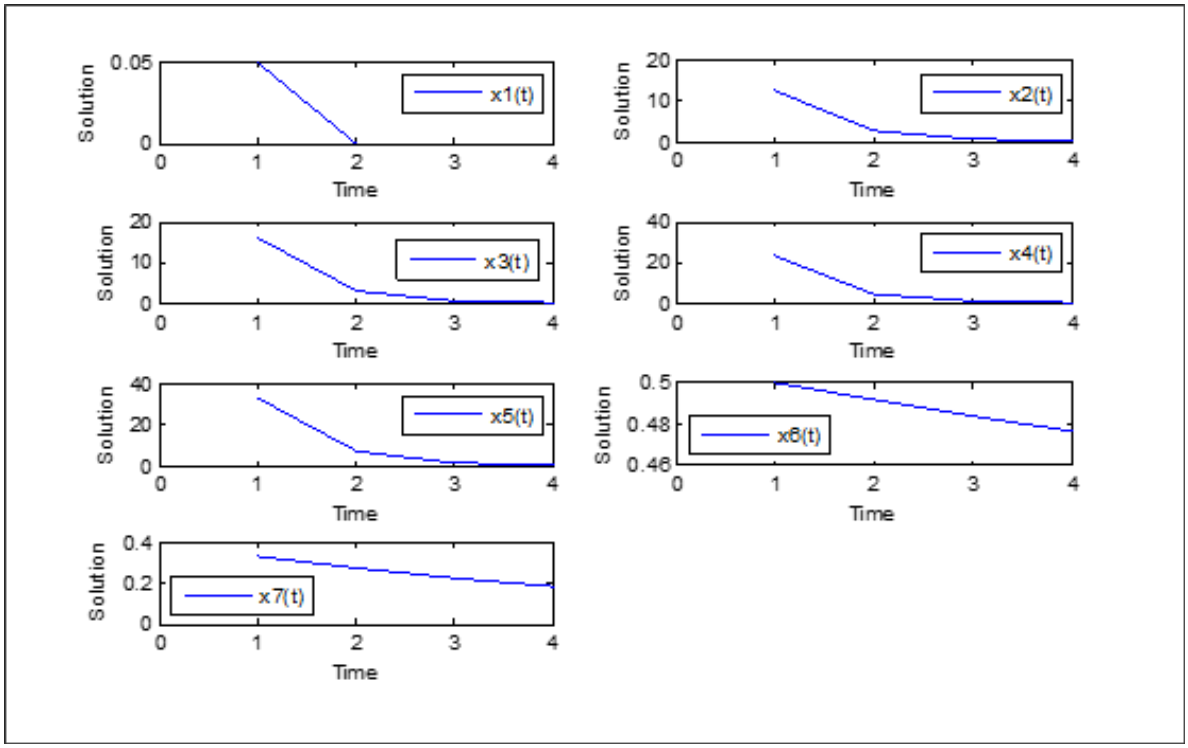

Figure 1: A first-time step [0,10] sketch of the solution to System (8).

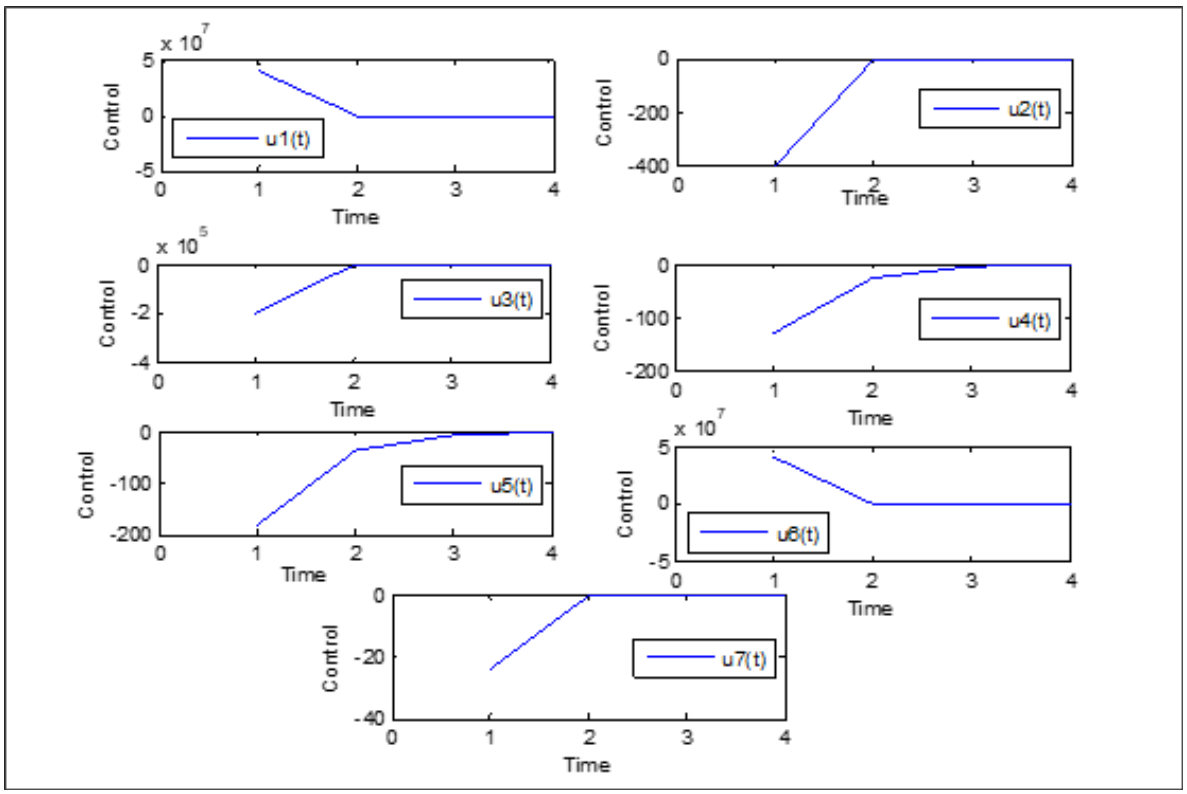

Figure 2: Sketch of control.

Conclusion

In this work, we were able approximation of the delayed-Oregonator model to ODEs by chain approximation, demonstrated its efficacy when applied this approach with examine the stability and solvability using backstepping method on chemical kinetics of the irreversible Oregonator model. For time constant DDEs, it is seen that the backstepping method based on Chain method stabilize the system faster than the backstepping method based on method of steps and also it does not require that the stabilization to be breaked down into subintervals.

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