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Controlling the movement of hexacopter along the intended route

Gulush Nabadova

Department of Mathematics, Baku Engineering University, Baku, Azerbaijan; Modeling and management of continuous processes, İnstitute of Control Systems, Baku, Azerbaijan.

The article examines the issue of controlling the movement of a hexacopter-type unmanned aerial vehicle along the intended route. The movement of the hexcopter is assumed as the movement of a solid body, gravity and aerodynamic drag forces are taken into account. It is assumed that the feedback data during control (accelerometer and gyroscope data) ise obtained from MPU6050 type sensors. MPU6050 type sensors do not measure orientation angles, but their rate of change, therefore, quaternions were used as orientation parameters in the mathematical model of hexocopter movement. In addition, quaternions do not have any singularities in the attitude expression. Moreover, quaternions are known to be more computationally efficient than Euler angles in attitude representations. In this article, the movement route is described as a trajectory consisting of straight sections, and an algorithm for calculating the base values of the control parameters is given, which ensures stable flight in each straight section of the trajectory, when all the engines of the hexocopter are working normally. During the study, the issue of ensuring straight-line movement of hexacopters when any of the hexacopter engines is faulty (out of order) was also considered, in this case, the optimal control parameters ensuring straight-line flight of the hexacopter were determined.

Key words and phrases: Quaternion, Unmanned Aerial Vehicle, Faulty engine, Hexacopter

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Email address: nabadova.g@gmail.com (Gulush Nabadova)

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1. Introduction

The development of control methodologies for unmanned aerial vehicles (UAVs) has led to a wide range of applications, making advanced UAV control methods an active area of research. UAVs have proven to be beneficial in fields such as agriculture, logistics, disaster relief, surveying, and biological animal research, among others. As the demand for UAVs increases, there is a growing need for more sophisticated control methods to allow UAVs to safely operate over populated areas, particularly in urban settings. However, UAVs face risks such as crashes and collisions, exemplified by an incident in Japan where a drone crash injured six people. This highlights the necessity for improved control methods to mitigate failure risks.

One critical failure that could lead to severe accidents is rotor failure, necessitating the development of fault-tolerant control systems. UAV flight is vulnerable to rotor thrust loss; if a rotor fails, the stability of the UAV may be compromised due to insufficient thrust from the remaining rotors. Various fault-tolerant control strategies have been suggested for UAVs, including sliding mode control, gain-scheduled PID control, PD control, model predictive control, LQ control, and adaptive control. Comparisons of these fault-tolerant control methods have been presented by researchers, providing insights into their effectiveness [1–3]. These types include, but are not limited to, tricopters, quadcopters, hexacopters, octocopters, and others. Among these, quadcopter-type UAVs have been more extensively researched, leading to the production of various models and increased manufacturing. However, when flaws arise in the construction or operation of quadcopters, issues with their control can emerge. Therefore, hexacopter-type UAVs with six-rotor propulsion systems are considered more reliable in terms of speed, durability, and maneuverability due to certain design features A hexacopter is an unmanned aerial vehicle with six arms, each equipped with a rotor symmetrically parallel to the symmetry axis passing through the center of the hexacopter. As a result of rotor operation, thrust forces and torque moments are generated parallel to the symmetry axis.

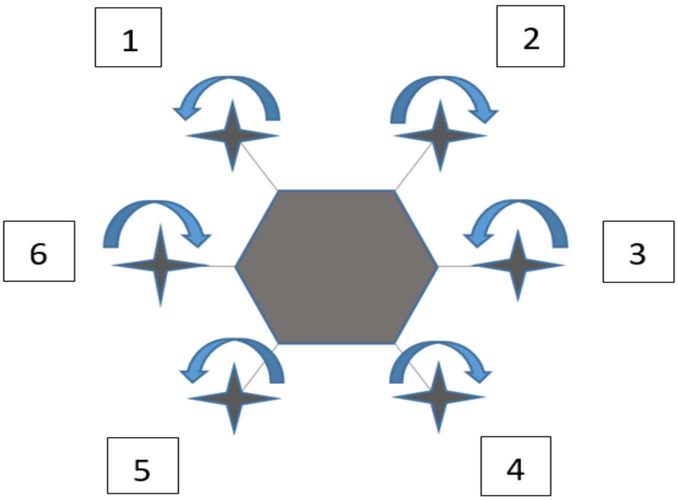

Figure 1. Direction of the rotors of hexacopter

In scientific and technical literature, various simulation models of hexacopter motion can be encountered. Depending on the characteristics of sensors used during the resolution of automatic control issues, the flight models of hexacopters differ from each other. During research, the use of MPU6050 type sensors in the UAV has made it more appropriate to use quaternions as orientation parameters in its mathematical model. MPU6050 type sensors measure the rate of change of orientation rather than orientation angles. Several articles by various authors have been dedicated to the model expressed with quaternions [5, 6, 7, 8]. Here, the model considered is fundamentally consistent with the one proposed in [6]. This article investigates the control problem of automatic flight of a hexacopter-type UAV along the required route. The flight route is viewed as a trajectory consisting of straight sections, and an algorithm for calculating the base values of control parameters ensuring the stable flight of the hexacopter in each straight section is provided. During the use of hexacopters, particular attention is paid to the occurrence of defects in their rotors. Therefore, in research work, parameters controlling the flight of the hexacopter in the examined straight section are determined even if one of the rotors is not functioning.

2. Problem statement

The implementation of automatic flight of the UAV along the required route primarily considers the provision of the route. Typically, the flight route is expressed as a sequence of straight sections and is given by the coordinates of its end points. Therefore, the flight along the route considers straight flight on each segment of the trajectory separately. The UAV's flight along the straight sections of the route differs practically depending on the direction and how the flight altitude changes along the considered straight section. Naturally, the UAV's stable flight along this trajectory can be ensured by maintaining a certain fixed orientation. In other words, the control parameters should be such that the thrust forces and moments generated by the rotors do not cause deviations during flight. Thus, the issue of controlling the hexacopter's movement along the route boils down to controlling it along a straight line segment. Below, the mathematical formalization of the problem and its solution are provided.

3. Coordinate Systems

The mathematical model of the hexacopter is expressed in terms of the relationship between quantities calculated in local and global coordinate systems. *OXYZ* is the reference coordinate system fixed to the ground, while is the local coordinate system associated with the hexacopter used to determine its orientation in space. For clarity, let's assume that the origin *O* of the *cYZ* system is a designated point on the surface. The *OXYZ* coordinate system has the *OY* axis pointing north, the *OX* axis pointing east, and the *OZ* axis pointing upwards perpendicular to the *OX* and *OY* axes. Let's assume that the *Ox* axis aligns with the first arm of the hexacopter, the *Oy* axis is perpendicular to the surface and parallel to the *Oz* axis, and the *Oz* axis is perpendicular to the plane formed by the *OX* and *OY* axes and is oriented along the symmetry axis of the hexacopter. In a horizontal equilibrium state, the *Oz* axis points upwards, and the *Oxy* plane is considered a positively oriented system.

4. Orienting with quaternions

In research, the orientation of the UAV is expressed with quaternions. Let's provide a brief overview of quaternions. A quaternion is a 4-dimensional hypercomplex number represented in the general form as $q = q_0 + q_1 i + q_2 j + q_3 k$ where q_0, q_1, q_2, q_3 are real numbers, and i, j, k are imaginary units. The quaternion's properties allow for comfortable and adequate use in expressing rotations. Using quaternions, the orientation of the aircraft is represented as follows:

$$q = \begin{pmatrix} cos\dfrac{\varphi}{2} \\ u_1 sin\dfrac{\varphi}{2} \\ u_2 sin\dfrac{\varphi}{2} \\ u_3 sin\dfrac{\varphi}{2} \end{pmatrix}$$

Here $u = (u_1, u_2, u_3)$ is the principal rotation vector in order to determine the current attitude of the aircraft, φ is the principal rotation angle [11]. In a special case, if the attitude vector aligns with the *Oy* axis and φ is the principal attitude angle, then $u = (0, 1, 0)$ and

$$q = \begin{pmatrix} \cos\frac{\varphi}{2} \\ 0 \\ \sin\frac{\varphi}{2} \\ 0 \end{pmatrix} \quad (1)$$

Suppose that, the local coordinate system *oxyz* is obtained from rotation of the inertial coordinate system *OXYZ* by the *q* vector. Then, the following transformation matrix can be applied to calculate the coordinates of the given vector in the inertial coordinate system from the local coordinate system [11]:

$$Q = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 + q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix} \quad (2)$$

It is obvious that, to find the coordinates of a vector given in *OXYZ* coordinate system relative to *oxyz* coordinate system, the inverse of the transformation matrix should be applied. Let's denote the elements of the inverse matrix as follows:

$$Q^{-1} = \begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix}$$

5. Mathematical model of the hexacopter

As mentioned above, for simplicity, the weight force and aerodynamic forces will be considered in the mathematical model of the UAV. At the given time $t \geq 0$ the coordinates of the center of gravity of the hexacopter in the coordinate system *OXYZ* is denoted by $X(t)$, $Y(t)$, $Z(t)$, the quaternion of the related local coordinate system with respect to the inertial coordinate system $q_0(t) + q_1(t)i + q_2(t)j + q_3(t)k$, and the rotation frequency of the blades of the *i–th* engine as ω_i. Then the movement of the hexacopter can be expressed by a system of equations that relate its $x(t)$, $y(t)$, $z(t)$ coordinates, $\omega_1, \omega_2, \ldots, \omega_6$ frequencies and q_0, q_1, q_2, q_3 components and written as follows. If we denote the velocity of the hexacopter relative to its local coordinate system *Oxyz* as $(v_x(t), v_y(t), v_z(t))$, then the motion equations expressed with quaternions for the hexacopter's velocity coordinates can be written as follows [11]:

$$\begin{cases} mv_x' = -p_{13}mg - c_A |v| v_x, \\ mv_y' = -p_{23}mg - c_A |v| v_y, \\ mv_z' = -p_{33}mg + \sum_{i=1}^{6} \omega_i^2 - c_A |v| v_z. \end{cases} \quad (3)$$

Lets denote the angular velocity of hexacopter as $w(t) = (w_x(t), w_y(t), w_z(t))$. In that case, considering the resultant moment $M = (M_1, M_2, M_3)$ created by the forces acting on the hexacopter, we can write the following equations with respect to the angular velocity [8,9]:

$$\begin{cases} J_1 w_1' + (J_3 - J_2) w_2 w_3 = M_1, \\ J_2 w_2' + (J_1 - J_3) w_1 w_3 = M_2, \\ J_3 w_3' + (J_2 - J_1) w_1 w_2 = M_3. \end{cases} \tag{4}$$

The dependence of the moment M on the freguencies $\omega_1, \omega_2, ..., \omega_6$ can be expressed as [6]:

$$M = \begin{bmatrix} \frac{\sqrt{3}}{2} k l_0 cos\alpha(-\omega_2^2 + \omega_3^2 + \omega_5^2 - \omega_6^2 \\ k l_0 cos\alpha(\omega_1^2 - \frac{1}{2}\omega_2^2 + \omega_3^2 - \omega_4^2 + \omega_5^2 - \omega_6^2 \\ b(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 + \omega_5^2 - \omega_6^2 \end{bmatrix} \tag{5}$$

By subtracting the angular velocity $w(t)$ obtained from equation (4), we can formulate the following ordinary differential equations system (Poisson kinematic equations) to solve for the quaternion representing the current orientation of the hexacopter [9]:

$$\begin{cases} q_0' = \frac{1}{2}(-q_1 w_1 - q_2 w_2 - q_3 w_3), \\ q_1' = \frac{1}{2}(q_0 w_1 + q_2 w_3 - q_3 w_2), \\ q_2' = \frac{1}{2}(q_3 w_1 + q_0 w_2 - q_1 w_3), \\ q_3' = \frac{1}{2}(-q_2 w_1 + q_1 w_2 + q_0 w_3). \end{cases} \tag{6}$$

The key conditions for controlling flight along a route

As mentioned earlier, flight along the straight segments of the route involves variations not only in direction but also in altitude along each respective straight segment. Therefore, to determine the solution principle for finding the control parameters along the route, it's feasible to consider each segment separately. Thus, without loss of generality, it can be assumed that the hexacopter's flight is intended along a straight line connecting specific points $A(x_a, 0, z_a)$ and $B(x_b, 0, z_b)$ in the Oxz plane at a given velocity V_0.

It is clear that if the hexacopter had roll, then it wouldn't fly along the straight line. In such a case, the deviation of the roll angle from zero during flight along the line would necessitate a moment that would force it to deviate from this trajectory. Therefore, during flight along the straight trajectory, the hexacopter's roll must be zero.

When it comes to the hexacopter's yaw, it should be such that the thrust force of the propeller compensates for the sum of the weight force and aerodynamic resistance forces along the Oz axis. On the other hand, the thrust should also be such that the component of the weight force directed in the flight direction compensates for the appropriate aerodynamic resistance force. Thus, it is required to find rotation frequencies $\omega_1, \omega_2, ..., \omega_6$ so that the UAV flies from point $A(x_a, 0, z_a)$ to the point $B(x_b, 0, z_b)$ with the required speed V_0. Let's split velocity vector V_0 into components along the vector $\overrightarrow{AB}$. Considering that unit vector along $\overrightarrow{AB}$ is

$$n = \frac{1}{p}(x_b - x_a, 0, z_b - z_a)$$

$$p = \sqrt{(x_b - x_a)^2 + (z_b - z_a)^2}$$

then we get

$$v = \begin{pmatrix} v_x(t) \\ v_y(t) \\ v_z(t) \end{pmatrix} = V_0$$

$$n = \begin{pmatrix} V_0 n_x \\ 0 \\ V_0 n_z \end{pmatrix}$$

Let's assume that the movement of the UAV with the required speed V_0 is ensured by the pitch angle φ. Then, the quaternion representing the appropriate orientation will be of the form of (2). Therefore, $p_{13} = sin\varphi, p_{23} = 0, p_{33} = cos\varphi$. We take into account these values in the 1st equation of (3) system

$$mgsin\varphi + c_A \mid v \mid v_x = 0,$$

$$\varphi = -arcsin \frac{c_A}{mg} \mid v \mid v_z$$

From 3rd equation

$$-mgcos\varphi + \sum_{i=1}^{6} \omega_i^2 - c_A \mid v \mid v_z = 0$$

$$\sum_{i=1}^{6} \omega_i^2 = f_0$$

here $f_0 = c_A \mid v \mid v_z + mgcos\varphi$. Since the torques of hexacopter are equal to zero,

$$\begin{cases} -\omega_1^2 + \omega_3^2 + \omega_5^2 - \omega_6^2 = 0, \\ \omega_1^2 - \frac{1}{2}\omega_2^2 + \omega_3^2 - \omega_4^2 + \frac{1}{2}\omega_5^2 - \omega_6^2 = 0, \\ \omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2 + \omega_5^2 - \omega_6^2 = 0. \end{cases}$$

Thus, the quantities $\omega_1, \omega_2, ..., \omega_6$ satisfying the stated requirements must satisfy the system of equations (7)-(8).

Ensuring control in normal operation mode

Let's assume that all the engines of hexacopter work normally. In this case, let's examine the issue of determining the quantities $\omega_1, \omega_2, ..., \omega_6$ that satisfy the system (7)-(8). For each k let's denote ω_k^2 with ξ_k, $k = 1, 2, ..., 6$. Then system (7)-(8) can be written as follows:

$$\begin{cases} -\xi_1 + \xi_3 + \xi_5 - \xi_6 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 - 2\xi_6 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 - \xi_6 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6 = f_0. \end{cases} \tag{9}$$

As can be seen, (9) is a system of linear equations written in terms of 6 unknowns, and its rank is equal to four. Therefore, this system has an infinite number of unique solutions. In order to choose the most suitable one from set of possible solutions according to the nature of problem,let's introduce the following optimality criterion:

$$\Im \equiv \sum_{i \neq 1} (\xi_i - \xi_j)^2 \to min,\ i, j = 1, 2, ..., 6. \tag{10}$$

Minimization of functional $\Im$ is essentially a requirement that the quantities ξ_i and ultimately rotational frequencies ω_k^2 be as close to each other as posssible. This requirement is justified by that, when controlling UAV in rectilinear motion, its engines are loaded as equally as possible. From mathematical point of view, the problem (9)–(10) is a conditional extremum problem with respect to ξ_k variables, to find its solution the method of Lagrange multipliers can be used [10]. For this purpose, let's denote the expressions included in the left side of equations (9) as $\mu_1, \mu_2, \mu_3, \mu_4$. Then, including $\lambda_1, \lambda_2, \lambda_3\, \lambda_4$ multipliers the Lagrange function for (9)–(10) is written as follows:

$$\Lambda \equiv \Im + \lambda_1\mu_1 + \lambda_2\mu_2 + \lambda_3\mu_3 + \lambda_4\mu_4. \tag{11}$$

Thus, the conditional extremum problem (9)-(10) comes to the problem of finding unconditional minimum of the functional (11). To find the minimum of functional Λ, let's calculate its special derivatives with respect to variables $\xi_1,...,\xi_6$ and $\lambda_1,...,\lambda_4$ and make them equal to zero. Then the following system of equations is obtained:

$$\begin{cases} 10\xi_1 - 2\xi_2 - 2\xi_3 - 2\xi_4 - 2\xi_5 - 2\xi_6 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\ -4\xi_1 + 20\xi_2 - 4\xi_3 - 4\xi_4 - 4\xi_5 - 4\xi_6 - 2\lambda_1 - \lambda_2 - 2\lambda_3 + 2\lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 + 10\xi_3 - 2\xi_4 - 2\xi_5 - 2\xi_6 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 - 2\xi_3 + 10\xi_4 - 2\xi_5 - 2\xi_6 - \lambda_2 - \lambda_3 + \lambda_4 = 0, \\ -4\xi_1 - 4\xi_2 - 4\xi_3 - 4\xi_4 + 20\xi_5 - 4\xi_6 + 2\lambda_1 + \lambda_2 + 2\lambda_3 + 2\lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 - 2\xi_3 - 2\xi_4 - 2\xi_5 + 10\xi_6 - \lambda_1 - \lambda_2 - \lambda_3 + \lambda_4 = 0, \\ -\xi_2 + \xi_3 + \xi_5 - \xi_6 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 - 2\xi_6 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 - \xi_6 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6 = f_0. \end{cases}$$

If we solve this system of equations by Cramer's rule, we get:

$$\xi_1 = \xi_2 = \xi_3 = \xi_4 = \xi_5 = \xi_6 \approx 0{,}166 f_0, (\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 0)$$

Accoring to the found values of quantities $\xi_1,...,\xi_6$ we get the following values for the rotational frequencies of propeller:

$$\omega_1 = \omega_2 = ... = \omega_6 \approx 0{,}4\sqrt{c_A\,|\,v\,|\,v_z + mgcos\varphi} \tag{12}$$

Thus, for hexacopter in order to fly in rectilinear motion, it is first brought to the appropriate orientation by changing the rotational frequencies of propellers and a suitable pitch is achived (note that, the issue of calculating the rotational frequencies of the propellers for changing the orientation of UAV is not considered in this article). Then it is controlled along the trajectory corresponding to rotational frequencies.

Providing control in case of failure of one of the engines

As can be seen from equations (12) optimal control of flight in rectilinear motion when all engines are working normally is ensured by rotation of all propellers at the same frequency. Now suppouse that one of the engines of hexacopter has failed. Then, without distrupting the overall system, it can be considered that the damaged one for example, 6th engine. This means that when solving system (6)–(7), it is necessary to take $\omega_6 = 0$. Thus, the system (6)–(7) becomes a system consisting of four equations written in terms of five unknowns. After the substitution $\xi_k = \omega_k^2$ $(k = 1, 2, ..., 5)$, the analog of system (8) can be written as follows:

$$\begin{cases} -\xi_1 + \xi_3 + \xi_5 - \xi_6 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 - 2\xi_6 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 - \xi_6 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 + \xi_6 = f_0. \end{cases} \tag{13}$$

In order to choose the most suitable one from the set of possible solutions according to the nature of the problem, we can take the following optimality criterion as an analogue of the functional (9):

$$\Im \equiv \sum_{i \neq 1} (\xi_i - \xi_j)^2 \to min,\ i, j = 1, 2, ..., 5.$$

Analogously, we can write the appropriate Lagrangian function Λ by marking the expressions included in the left side of equations (13) as $\varphi_1, \varphi_2, \varphi_3, \varphi_4$ and applying $\lambda_1, \lambda_2, \lambda_3 \lambda_4$ multipliers. By equating its specific derivatives to zero, we get the following system of linear-algebraic equations:

$$\begin{cases} 8\xi_1 - 2\xi_2 - 2\xi_3 - 2\xi_4 - 2\xi_5 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\ -4\xi_1 + 16\xi_2 - 4\xi_3 - 4\xi_4 - 4\xi_5 - 2\lambda_1 - \lambda_2 - 2\lambda_3 + 2\lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 + 8\xi_3 - 2\xi_4 - 2\xi_5 + \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 = 0, \\ -2\xi_1 - 2\xi_2 - 2\xi_3 + 8\xi_4 - 2\xi_5 - \lambda_2 - \lambda_3 + \lambda_4 = 0, \\ -4\xi_1 - 4\xi_2 - 4\xi_3 - 4\xi_4 + 16\xi_5 + 2\lambda_1 + \lambda_2 + 2\lambda_3 + 2\lambda_4 = 0, \\ -\xi_2 + \xi_3 + \xi_5 = 0, \\ 2\xi_1 - \xi_2 + 2\xi_3 - 2\xi_4 + \xi_5 = 0, \\ \xi_1 - \xi_2 + \xi_3 - \xi_4 + \xi_5 = 0, \\ \xi_1 + \xi_2 + \xi_3 + \xi_4 + \xi_5 = f_0. \end{cases}$$

When solving the system of equations with Cramer's formulas, the following answers are obtained:

$$\begin{cases} \xi_1 = \xi_2 = \xi_4 = \xi_5 == \approx 0,25 f_0, \\ (\xi_3 = 0, (\lambda_1 = 2,5 f_0, \lambda_2 = 5 f_0, \lambda_3 = -5 f_0, \lambda_4 = -0,5 f_0) \end{cases}$$

According to the found values of the quantities , we get the following values for the rotational frequencies of the propellers:

$$\begin{cases} \omega_1 = \omega_2 = \omega_4 = \omega_5 == \approx 0,5\sqrt{c_A \mid v \mid v_z + mgcos\varphi}, \\ \omega_3 = 0. \end{cases}$$

Discussion

The obtained results were conditioned by the choice of functional $\Im$. As mentioned above, the extreme value of the $\Im$ functional lays down condition that the UAV's engines to be approximately equally loaded during the rectilinear motion that covers most of the UAV's flight. Such loading serves to ensure that its various engines do not wear out more than others during the operation of the UAV. From this perspective, when all engines are tuned to the same level, it is natural for controlling rotational frequencies for its rectilinear flight to be the same, in accordance with formula (12). When one of the UAV's engines fails, the functional (11) obtained by discarding the thresholds involved in the rotation frequency of the failed engine was minimized for the calculation of the control parameters during its control on the route. The results expressed by the formulas (15) show that in case of failure of one of the engines, the control of the hexocopter along a straight trajectory takes place in the order of control of the quadcopter. At the same time, this case shows that the hexocopter can be controlled normally on a straight trajectory even if two symmetrically located engines fail. At the next stage of the research, it is planned to consider the case of failure of two engines of the UAV, which are not located symmetrically at the same time. However, it should be noted that in these cases, the proposed criterion for finding the optimal rotation frequencies of the engines (similar to the functional I) can be more complicated. So, unlike the cases discussed above, depending on which two engines fail, the additional load assigned to the other engines may exceed their technical capabilities. Therefore, in the case of 2 or more engine failures, the function of optimal determination of rectilinear flight control parameters should also include restrictions on the rotation frequencies of the engines.

Conclusion

Thus, the issue of determining the optimal control parameters for the hexacopter's straight flight along the route, expressed with quaternions in the motion equations, has been studied. A procedure has been developed for calculating the propeller speeds in a regime where the propellers are approximately equally loaded. It has been shown that when all propellers of the hexacopter are functioning properly, straight motion is ensured with the same propeller speeds. However, when one propeller malfunctions, to ensure straight flight, the propeller speed of the malfunctioning propeller parallel to the non-working propeller should be set to 0, while the propeller speeds of the other propellers should be the same. This also means that the hexacopter can still be controlled normally along the straight trajectory even when two symmetrically located propellers are out of order.

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