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An induced neutrosophic using soft set (J,L) over M with parameter H

Mohanad Abdulkareem Hasan Hasab[1], Shuker Khalil[2,*], Abu Firas Muhamad Al -Musawi[3]

[1]Department of Psychological Counseling, College of Education for Human Sciences, University of Basrah, Iraq; [2]Department of Mathematics, College of Science, Basrah University, Basrah 61004, Iraq; [3]Department of Mathematics, College of Education for Pure Sciences, University of Basrah, Iraq.

Abstract

A review of the class of soft sets was recently generalized using a variety of methodologies. The neutrosophic set is one of the most significant in mathematics because it can deal with greater uncertainty than other non-classical sets such as fuzzy sets and intuitionistic sets. An innovative method depends on two operations $\underset{J(L)}{P}$ and $\underset{J(L)}{Q}$ is given in this work. As a result, we derive new classes such as like neutrosophic soft b–closed set ($NSbCS$), neutrosophic soft b–closed topological space ($NSbCT$), neutrosophic soft b–closed continuous mapping ($NSbCCM$), neutrosophic soft b–closed mapping ($NSbCM$), neutrosophic soft b–closed subcover ($NSbC$) – SC, neutrosophic soft b–closed compact ($NSbC$) – C, neutrosophic soft b–closed connected ($NSbC$) – C. Some specific cases involving these classes are discussed. Furthermore, we constructed $\underset{J(L)}{Q}$ (induced neutrosophic by soft set (J,L)), where (J,L) is a soft set over the universe of the provided set with a specified set of parameters. Furthermore, some of its fundamental features are described. A novel method for combining neutrosophic and soft sets is employed to define new structures in ($NSbCT$). The novel classes generated by this method in ($NSbCT$) are examined. The relationships between them are discussed and provided.

Key words and phrases: neutrosophic sets, soft sets, neutrosophic soft sets, absolute N-soft set.

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Email addresses: mohanad.hasan@uobasrah.edu.iq (Mohanad Abdulkareem Hasan Hasab); shuker.alsalshuker.khalil@uobasrah.edu.iq (Shuker Khalil)*; abufirasalmusawi@uobasrah.edu.iq (Abu Firas Muhamad Al -Musawi)

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1. Introduction

Neutronosophic sets (NS) are new non-classical sets proposed and defined by Smarandache [1]. They are a modification of fuzzy sets (FS) proposed by Zadeh [2]. Furthermore illustrating the consequences of the NeutrosophicSet (NS) is Atanassav [3]. After that, Salama and Alblowi[4] proposed a new idea that has just lately been investigated in this type of topological space: a neutrosophic topological space (NTS).

Rao and Srinivasa [5] then investigated the concept of a neutrosophic per-closed. In 2018, Ebenanjar M et al. [6] presented a study in (NTS) that described neutrosophic b-closed. In 2020 [7], (NTS) proposed and examined the meaning of a neutrosophic bg-closed. Some non-classical sets, such as soft set ([8–12]), fuzzy set ([13–16]), neutrosophic set ([17–29]), permutation set ([30–51]) and other sets in other domains ([52–54]), were addressed in recent years. To study our non-classical expansion, we'll employ the neutrosophic idea. The neutrosophic closur/ interior $\mathrm{cl}(\mathrm{K})/\mathrm{int}(\mathrm{K})$ of each (NS) K in (NTS) (ω,τ) is given as $\mathrm{cl}(\mathrm{K})=\cap\{\mathrm{K}\subseteq \mathrm{W};\ \mathrm{W}^c\in\tau\}, d\mathrm{int}(\mathrm{K})=\cup\{\mathrm{W}\subseteq\mathrm{K};\mathrm{W}\in\tau\}$, accordingly. The notion of the soft set is introduced by Molodtsov [55], and then their properties were studied. The purpose of this work is to introduce and investigate various novel concepts such as (NSbCT), (NSbCM), (NSbCCM), (NSbC)-SC, (NSbC)-C, and (NSbC)-C'. Furthermore, we constructed $\underset{J(L)}{Q}$ (induced neutrosophic by soft set (J,L)). Furthermore, some of its fundamental features are listed.

2. Definitions and Notations

The descriptions that follow were used to produce the results and attributes presented in this research. $M=\langle x,\gamma_M(x),\ \rho_M(x),\ r_M(x)\rangle : x\in X$

Definition 2.1. [5] Let $M\neq\emptyset,\ F=\{\langle x,A_F(x),\ B_F(x),\ C_F(x)\rangle : x\in M\}$ and $M=\{\langle x,\gamma_M(x),\rho_M(x), r_M(x)\rangle\}$ be neutrosophic sets (NSs), Then;

$$D^c=\{\langle x,C_F(x),1-B_F(x),\ A_F(x)\rangle : x\in M\}, \tag{1}$$

(2) $D\subseteq F$ iff $A_D(x)\le A_F(x), B_D(x)\ge B_F(x)$ and $C_D(x)\ge C_F(x)$,

$$D\cup F=\{\langle x,\ \max\{A_D(x),A_F(x)\},\min\{B_D(x),B_F(x)\},\ \min\{C_D(x),C_F(x)\}\rangle :\ x\in M\}, \tag{2}$$

$$D\cap F=\{\langle x,\ \min\{A_D(x),A_F(x)\},\max\{B_D(x),B_F(x)\},\ \max\{C_D(x),C_F(x)\}\rangle :\ x\in M\}. \tag{3}$$

Definition 2.2. [5] Assume that $\rho=\{F_d\mid d\in\Delta\}$ is a collection of neutrosophic sets (NSs) of M. We say (M,ρ) is a neutrosophic topological space (NTS) if ρ satisfies:

$$F_s\cap F_h\in\rho,\forall F_s,F_h\in\rho, \tag{4}$$

$$0_N=\{\langle x,(0,1,1)\rangle : x\in M\}\in\rho,\&1_N=\{\langle x,(1,0,0)\rangle : x\in M\}\in\rho, \tag{5}$$

(2) $\cup_{d\in\nabla}F_d\in\rho$, for any $\nabla\subseteq\Delta$. Also, if $F_d\in\rho$, we have F_d is called a neutrosophic open set (NOS), and $F_d{}^c$ is called neutrosophic closed set (NCS).

Definition 2.3. ([56])

Let $\emptyset\neq L\subseteq H$, where H is a set of a parameter. A pair (J,L) is said to be a soft set (SS) (over M) where J is a mapping $J:L\to P(M)$. The family of all the soft sets (SSs) over M and the parameter set L is refereed by $SS(M_L)$. We say (J,L) is a null soft set and symbolized by $\Phi=(\phi,\phi)$, if for each $J(k)=\phi,\forall k\in L$. Also, we say symbolized universal soft set as (M, H), if $J(k)=M,\forall k\in L$.

Definition 2.4. [57] Let M be an initial universe set and let H be a set of parameters. Let I^M denotes the collection of all fuzzy subsets of M and $L \subseteq H$. Then the mapping $J_L : L \to I^M$ and $J_L(e)$ is said to be fuzzy soft set (FSS) over (M,H), also $J_L(e) = \bar{0}$ if $e \in M - L$ and $J_L(e) \neq \bar{0}$ if $e \in L$, where $\bar{0}(x) = 0$ and $\bar{1}(x) = 1, \forall x \in M$. The family of all (FSSs) over (M,H) is symbolized as $FS(M,H)$. We say $J_\phi \in FS(M,H)[J_H \in FS(M,H)]$ is null [absolute] fuzzy soft set and it is denoted by $\Phi[\bar{M}]$, if $J(e) = \bar{0}[J(e) = \bar{1}]$, $\forall e \in H$.

Remark 2.5. [58] The Cardinality of $SS(M_K)$ is defined as $n(SS(M_K)) = 2^{n(M)\times n(K)}$.

Example 2.6. if $M = \{m_1, m_2, m_3\}$ and $K = \{k_1, k_2\}$, then $n(SS(M_K)) = 2^{3\times 2} = 64$.

Definition 2.7. [59] Assume that M is non–empty universe set with parameter set H and N^M refereed the family that contains any neutrosophic set of M. If $L \subseteq H$. Then (J,L) is said to be a neutrosophic soft set (NSS) over M where J is a mapping given by $J : L \to N^M$. In general, for every $k \in L$, $J(k)$ is a (NS) of M. Also, $J(K)$ Could be expressed as a (NS) satisfies $J(k) = \{(x, \mu_{J(k)}(x), I_{J(k)}(x), \upsilon_{J(k)}(x)), x \in M\}$. The set of all (NSSs) over M with parameters from H is called a neutrosophic soft class and it is denoted by $NSS(M_H)$.

Definition 2.8. [59] Assume that (J_1, L_1) and (J_2, L_2) are (NSSs) over M, then their union is defined as:

$(J_3, L_3) = (J_1, L_1)\overline{\coprod}(J_2, L_2)$, where $L_3 = L_1 \cup L_2$ and $\forall k \in L_3$, we have

$$J_3(k) = \begin{cases} J_1(k) & if\, k \in L_1 \setminus L_2 \\ J_2(k) & if\, k \in L_2 \setminus L_1, \\ J_1(k) \cup J_2(k) & if\, k \in L_1 \cap L_2. \end{cases}$$

Also, their intersection is defined as: $(J_3, L_3) = (J_1, L_1)\overline{\prod}(J_2, L_2)$, where $L_3 = L_1 \cap L_2$ and $J_3(k) = J_1(k) \cap J_2(k), \forall k \in L_3$. We say (J_1,L_1) is neutrosophic soft subset of (J_2,L_2) over M if $L_1 \subseteq L_2$ and $J_1(k) \subseteq J_2(k), \forall k \in L_1$. The complement of (NSS) (J,L) is defined by $(J,L)^c$ and it is realized as $(J,L)^c = (J^c, L)$, and $J^c : L \to N^U$ is mapping given by $J^c(k) = [J(k)]^c, \forall k \in L$. Therefore, we consider that, if $J(k) = \{(x, \mu_{J(k)}(x), I_{J(k)}(x), \upsilon_{J(k)}(x)), x \in M\}$, then $\forall k \in L$,

$$J^c(k) = [J(k)]^c = J(k) = \left\{(x, \upsilon_{J(k)}(x), 1 - I_{J(k)}(x), \mu_{J(k)}(x)), x \in M\right\}.$$

A (NSS) (J,L) over U is called a absolute [null] neutrosophic soft set and is denoted by $\bar{M}_L[\Phi_L]$ if $\forall k \in L, J(k)$ is the absolute [null] neutrosophic set $\bar{1}[\bar{0}]$, where $\bar{1}(x) = 1[\bar{0}(x) = 0]; \forall x \in M$.

Definition 2.9. [59] Let $\rho \subseteq NSS(M_H)$, then ρ is said to be a neutrosophic soft topology (NST) on M, if ρ such that:

1. $\bar{M}_H, \Phi_H$ belong to ρ.
2. The intersection of any (NSSs) in ρ belongs to ρ.
3. The union of any number of (NSSs) in ρ belongs to ρ.

We say (M_H,ρ) is neutrosophic soft topological space (NSTS). Moreover, if $(J,L) \in \rho[(J,L)^c \in \rho]$, then $(J,L)[(J,L)^c]$ is called a neutrosophic soft open [closed] set, for short (NSOS) [(NSCS)] in M.

Definition 2.10. [59] Assume that (J_1,L_1) and (J_2,L_2) are (NSSs) over M. We define the difference between them by $(J_1, L_1) - (J_2, L_2) = (J_3, L_3)$ where $L_3 = L_1 \cap L_2$ and $\forall k \in L_3, \forall x \in M, \mu_{J_3(k)}(x) = \min\{\mu_{J_1(k)}(x), \upsilon_{J_2(k)}(x)\}$, $I_{J_3(k)}(x) = \max\{I_{J_1(k)}(x), 1 - I_{J_2(k)}(x)\}$, and $\upsilon_{J_3(k)}(x) = \max\{\upsilon_{J_1(k)}(x), \mu_{J_2(k)}(x)\}$. That means $(J,L)^c = \bar{M}_H - (J,L), {\phi_H}^c = \bar{M}_H$ and $(\bar{M}_H)^c = \phi_H$.

Definition 2.11. [59] Let (J,L) be (NSS). We say (J,L) is a neutrosophic soft point (NSP), denoted by k_J, if for the element $k \in L, J(k) \neq \bar{0}$ and $J(t) = \bar{0}, \forall t \in L - \{k\}$.

Definition 2.12. [59] The complement of a (NSP) k_J is a (NSP) k_{J^c} with $J^c(k) = (J(k))^c$.

3. Neutrosophic Soft b-Closed Sets

In the following paragraphs, we present several novel notions, such as (NSbCT), (NSbC)-continuous, (NSbC)-mapping, (NSbC-SC), (NSbC)-C, and (NSbC). Also, we investigated and discussed these notions. Furthermore, other examples provide support for the present work.

Definition 3.1. Let (M_H,ρ) be a (NSTS) and $(J,L) \in NSS(M_H)$, we say (*J*,*L*) is neutrosophic soft *b*–closed set (NSbCS) if $cl(int(J,L)) \cap int(cl(J,L)) \subseteq (J,L)$. Also, we denote to the complement of (NSbCS) by (NSbOS).

Definition 3.2. Let (M_H,ρ) be a (NSTS). We say it is a neutrosophic soft b-closed topological space (NSbCT), if every $(J,L) \in \rho - \{\Phi_H, M_H\}$ is (NSbCS).

Example 3.3. Let *M* be a set of students whose need to buy mathematical books from the stationery store under consideration, say $M = \{m_1, m_2, m_3\}$. Let *H* be the set of some attributes of such students, say H = $\{h_1, h_2, h_3, h_4\}$, where h_1, h_2, h_3, h_4 stand for the attributes “cheap ”, “english book”, “with pictures”, “with big pages”, respectively.

Define $R, Y, J, N : H \to N^M$ as:

$$
\begin{aligned}
&R(h_1) = (m_1, 0.3, 0.6, 0.7), R(h_1) = (m_2, 0.8, 0.5, 0.6), R(h_1) = (m_3, 0.2, 0.8, 0.9)\\
&R(h_2) = (m_1, 0.4, 0.5, 0, 6), R(h_2) = (m_2, 0.2, 0.3, 0.4), R(h_2) = (m_3, 0.9, 0.4, 0.5)\\
&R(h_3) = (m_1, 0.3, 0.5, 0.6), R(h_3) = (m_2, 0.7, 0.3, 0.4), R(h_3) = (m_3, 0.6, 0.22, 0.32)\\
&R(h_4) = (m_1, 0.8, 0.1, 0.2), R(h_4) = (m_2, 0.3, 0.4, 0.5), R(h_4) = (m_3, 0.6, 0.8, 0.9)\\
&Y(h_1) = (m_1, 0.7, 0.2, 0.3), Y(h_1) = (m_2, 0.6, 0.7, 0.8), Y(h_1) = (m_3, 0.9, 0.1, 0.2)\\
&Y(h_2) = (m_1, 0.6, 0.3, 0.4), Y(h_2) = (m_2, 0.4, 0.1, 0.2), Y(h_2) = (m_3, 0.5, 0.9, 0.9)\\
&Y(h_3) = (m_1, 0.6, 0.2, 0.3), Y(h_3) = (m_2, 0.4, 0.6, 0.7), Y(h_3) = (m_3, 0.32, 0.5, 0.6)\\
&Y(h_4) = (m_1, 0.1, 0.7, 0.8), Y(h_4) = (m_2, 0.5, 0.2, 0.3), Y(h_4) = (m_3, 0.9, 0.5, 0.6)\\
&J(h_1) = (m_1, 0.3, 0.6, 0.7), J(h_1) = (m_2, 0.6, 0.7, 0.8), J(h_1) = (m_3, 0.2, 0.8, 0.9)\\
&J(h_2) = (m_1, 0.4, 0.5, 0.6), J(h_2) = (m_2, 0.2, 0.3, 0.4), J(h_2) = (m_3, 0.5, 0.9, 0.9)\\
&J(h_3) = (m_1, 0.3, 0.5, 0.6), J(h_3) = (m_2, 0.4, 0.6, 0.7), J(h_3) = (m_3, 0.32, 0.5, 0.6)\\
&J(h_4) = (m_1, 0.2, 0.7, 0.8), J(h_4) = (m_2, 0.5, 0.2, 0.3), J(h_4) = (m_3, 0.9, 0.5, 0.6)\\
&N(h_1) = (m_1, 0.7, 0.2, 0.3), N(h_1) = (m_2, 0.8, 0.5, 0.6), N(h_1) = (m_3, 0.9, 0.1, 0.2)\\
&N(h_2) = (m_1, 0.4, 0.5, 0.6), N(h_2) = (m_2, 0.4, 0.1, 0.2), K(h_2) = (m_3, 0.5, 0.9, 0.9)\\
&N(e_3) = (x_1, 0.6, 0.2, 0.3), N(e_3) = (x_2, 0.7, 0.3, 0.4), N(e_3) = (x_3, 0.6, 0.22, 0.32)\\
&N(h_4) = (m_1, 0.8, 0.1, 0.2), N(h_4) = (m_2, 0.5, 0.2, 0.3), N(h_4) = (m_3, 0.9, 0.5, 0.6)
\end{aligned}
$$

Then $\rho = \{\bar{M}_H, \phi_H, (R,H), (Y,H), (J,H), (N,H)\}$ is a (*NSTS*) over *M*. Moreover, any member in $(R,H),(Y,H),(J,H),(N,H)$ is (*NSbCS*). Hence (M_H,ρ) is (*NSbCT*).

Definition 3.4. Assume that (M_H,ρ_1) and (W_D, ρ_2) are two (*NSTSs*). Let $\mathrm{T} : (M_H, \rho_1) \to (W_D, \rho_2)$ be a mapping. We say λ is a neutrosophic soft *b*–closed continuous mapping (*NSbCCM*) if $\mathrm{T}^{-1}(J,L) \in \rho_1$ with $\mathrm{T}^{-1}(J,L)$ is a (*NSbCS*) in M_H for every (*NSbCS*) $(J,L) \in \rho_2$ in W_D.

Definition 3.5. Assume that (M_H,ρ_1) and (W_D, ρ_2) are two (*NSTSs*). Let $\lambda : (M_H, \rho_1) \to (W_D, \rho_2)$ be a mapping. We say λ is a neutrosophic soft *b*–closed mapping (*NSbCM*), if $\lambda(J,L) \in \rho_2$ and is a (*NSbCS*) in W_D for each (*NSbCS*) $(J,L) \in \rho_1$ in M_H.

Theorem 3.6. Assume that (M_H,ρ_1) and (W_D, ρ_2) are two (NSTSs). Let $\lambda:(M_H,\rho_1)\to(W_D,\rho_2)$ be a mapping. Then the statements below are equivalent.

1. λ is a (NSbCCM).
2. $\lambda^{-1}(J,L)\in\rho_1$ is a (NSbOS) in M_H for any (NSbOS) $(J,L)\in\rho_2$ in W_D.

Proof: Let λ be a (*NSbCCM*) and (*J,L*) be a $(NSbOS)\in\rho_2$ in W_D. Then $(J,L)^c$ is (*NSbCS*) in W_D, however λ is a (NSbCCM). Thus $\lambda^{-1}((J,L)^c)$ is (NSbCS) and $\lambda^{-1}((J,L)^c)\in\rho_1$. But, $\lambda^{-1}((J,L)^c)=[\lambda^{-1}(J,L)]^c$, thus $\lambda^{-1}(J,L)$ is a (*NSbOS*) in M_H. Conversely, let $\lambda^{-1}(J,L)$ be a (*NSbOS*) in M_H for any (*NSbOS*) (*H,C*) in W_D. Now, for each (*NSbCS*) (*J,L*) in W_D we get $(J,L)^c$ is (*NSbOS*) in W_D. Therefore $\lambda^{-1}((J,L)^c)$ is (*NSbOS*) in M_H. But, $\lambda^{-1}((J,L)^c)=[\lambda^{-1}(J,L)]^c$, thus $\lambda^{-1}(J,L)$ is a (*NSbCS*) in M_H. Hence λ is a (*NSbCCM*).

Definition 3.7. Let β be a collection of the neutrosophic soft open sets. We say it is an open cover [(*NSO*)−*C*, in brevity] of a (*NSS*) (*J,L*) if $(J,L)\subseteq\coprod\{(J_t,L)\mid(J_t,L)\in\beta,t\in\Omega\}$. Let $\tilde{\lambda}\subseteq\beta$ be subcover of (*J,L*). Then $\tilde{\lambda}$ is said to be a neutrosophic soft *b*–closed subcover [(*NSbC*) − *SC*] if (J_t,L) is a $(NSbCS),\forall(J_t,L)\in\tilde{\lambda}$. Also, if $\tilde{\lambda}$ is finite set, then it is denoted as $(NSbC)_f - SC$.

Definition 3.8. Let (M_H,ρ) be (*NSTS*) and $(J,L)\in NSS(M_H)$. A (*NSS*) (*J,L*) is said to be neutrosophic soft *b*–closed compact [(*NSbC*) − *C*], if every (*NSO*) − *C* of (*J,L*) has a $(NSbC)_f - SC$. Also, we say (M_H,ρ) is a neutrosophic soft *b*–closed compact [(*NSbC*) − *C*], if every (*NSO*) − *C* of M_H has a $(NSbC)_f - SC$.

Example 3.9. Assume that (M_H,ρ) is a (*NSTS*) and *M* is a finite universe set, then (M_H,ρ) is (*NSbC*) − *C*.

Proposition 3.10. Assume that (*J,L*) is a (*NSCS*) in (M_H,ρ). If (M_H,ρ) is (*NSbC*) − *C*. Then (*J,L*) is (*NSbC*) − *C* too.

Proof: Suppose that $\Psi=\{(J_j,L)\mid(J_j,L)\in\rho,j\in\Omega\}$ is a (*NSO*) − *C* of (*J,L*). But ψ with $(J,L)^c$ is a (*NSO*) − *C* of M_H, since $(J,L)^c$ is a (NSOS), say $\tilde{\Psi}=\{\Psi,(J,L)^c\}$ is a (*NSO*) − *C* of M_H. That means $M_H\subseteq\left\{\coprod_{j\in\Omega}(J_j,L)\right\}\coprod(J,L)^c$. But, (M_H,ρ) is (NSbC) – C, thus M_H has a $(NSbC)_f$ − *SC*, say Γ' of $\tilde{\Psi}$ satisfies:

$$\Gamma'=\begin{cases}\{(J_j,L);j=1,2,\dots,d\}, & if\,(J,L)^c\; is\;\; not\,(NSbCS),\\ \{\{(J_j,L);j=1,2,\dots,d\},(J,L)^c\}, & if\,(J,L)^c\; is\,(NSbCS).\end{cases}$$

Furthermore, if $(J,L)^c$ is not (*NSbCS*). Hence (*J,L*) is (*NSbC*) – *C*. Also, if $(J,L)^c$ is (*NSbCS*). Then $M_H\subseteq\left\{\coprod_{j=1}^{d}(J_j,L)\right\}\coprod(J,L)^c$. This implies that $(J,L)\subseteq\left\{\coprod_{j=1}^{d}(J_j,L)\right\}\coprod(J,L)^c$. Now, for each neutrosophic soft point $k_J\in(J,L)^c$ we get $k_J\notin(J,L)^c$. That means $(J,L)^c$ does not cover its complement in any neutrosophic soft point. Then $\{(J_j,L)\mid j=1,\dots,d\}$ is $(NSbC)_f - SC$ of ψ with $(J,L)\subseteq\left\{\coprod_{j=1}^{d}(J_j,L)\right\}$. Hence (*J,L*) is (*NSbC*) – *C*.

Theorem 3.11. Assume that (M_H,ρ_1) and (W_D, ρ_2) are two (*NSTSs*). Let $\lambda:(M_H,\rho_1)\to(W_D,\rho_2)$. If (M_H,ρ_1) is (*NSbC*) – *C*, then (W_D, ρ_2) is (*NSbC*) – *C*.

Proof: Suppose that $\beta=\{(J_j,L)\mid(J_j,L)\in\rho_2,j\in\Omega\}$ is a (*NSO*) − *C* of (W_D, ρ_2). i.e $W_D\subseteq\coprod_{j\in\Omega}(J_j,L)$. Thus $\lambda^{-1}(W_D)\subseteq\lambda^{-1}(\coprod_{j\in\Omega}(J_j,L))$, then $M_H\subseteq\coprod_{j\in\Omega}\lambda^{-1}((J_j,L))$. So $\lambda^{-1}((J_j,L))\in\rho_1,\forall j\in\Omega$ (since λ is (*NSbCCM*)). But, (M_H,ρ_1) is (*NSbC*) – *C*, therefore (M_H,ρ_1) has a $(NSbC)_f$ − *SC* say $\{\lambda^{-1}((J_j,L))\mid j=1,\dots,d\}$ of

$\left\{\lambda^{-1}((J_j,L)) \mid j\in\Omega\right\}$. As $M_H \subseteq \coprod_{j=1}^{d}\lambda^{-1}((J_j,L))$, this implies that $W_D=\lambda(M_H)\subseteq\lambda(\coprod_{j=1}^{d}\lambda^{-1}(J_j,L))=\coprod_{j=1}^{d}\lambda\lambda^{-1}$ $((J_j,L))=\coprod_{j=1}^{d}(J_j,L)$ (since λ is onto). So we consider that (W_D, ρ_2) has a $(NSbC)_f - SC$ of β. Therefore (W_D, ρ_2) is (*NSbC*) – *C*.

Definition 3.12. Assume that (M_H,ρ) is a (*NSTS*) over *M*. We say (M_H,ρ) is neutrosophic soft b-closed disconnected $[(NSbC)-C']$, if $\bar{M}_H=(J,L)\overline{\coprod}(D,V),(J,L)\overline{\prod}(D,V)=\Phi_H$, for some no-null (*NSbCS*) (*J,L*)∈ρ and (*D,V*)∈ρ.

Example 3.13. Consider (M_H,ρ) in Example (3.3), we not have any pair (*J,L*), (*D,V*) of no-null (*NSbCS*) each one of them belongs to ρ with $\bar{M}_H=(J,L)\overline{\coprod}(D,V)$ and $(J,L)\overline{\prod}(D,V)=\Phi_H$. Then ($M_H$,ρ) is (*NSbC*) – *C*.

Remark 3.14. Assume that (M_H,ρ) is a (*NSTS*) over *M* and $(J,L)\in NS(M_H)$, then (*J,L*) is (*NSbCS*) and (*NSbOS*) if (*J,L*)∈ρ and $(J,L)^c$∈ρ [i.e, (*J,L*) is (*NSO*) and (*NSC*)].

Theorem 3.15. Assume that (M_H,ρ) is a (*NSTS*) over *M*. Then (M_H,ρ) is (*NSbC*) – *C* if there is no proper (*NSbCS*) (*J,L*) with (*J,L*)∈ρ and $(J,L)^c$∈ρ.

Proof: Suppose that (M_H,ρ) is a (*NSbC*) – *C* and (*J,L*) be a proper (NSbCS) with (*J,L*)∈ρ and $(J,L)^c$∈ρ. But, $(J,L)^c$ is a (*NSbCS*) and $\Phi_H\neq(J,L)^c\neq\bar{M}_H$. Furthermore, $\bar{J}_L=(J,L)\overline{\coprod}(J,L)^c,(J,L)\overline{\prod}(J,L)^c=\Phi_H$. Hence we get ($M_H$,ρ) is a (*NSbC*) –*C′*. But this is a contradiction. Therefore Φ_H and $\bar{M}_H$ that only (*NSbCS*) with they are (*NSO*) and (*NSC*).

Conversely, let (M_H,ρ) be a (*NSbC*) –*C′*, then $\bar{M}_H=(J,L)\overline{\coprod}(D,V),(J,L)\overline{\prod}(D,V)=\Phi_H$, for some no-null (*NSbCS*) (*J,L*)∈ρ and (*D,V*)∈ρ. Let $(J,L)=\bar{M}_H$, thus $(D,V)=\Phi_H$, but this is a contradiction. Then (*J,L*) ≠ $\bar{M}_H$. So, (*J,L*) = $(D,V)^c$. So, (*J,L*)∈ρ and (*D,V*) = $(J,L)^c$∈ρ with $\Phi_H\neq(J,L)\neq\bar{M}_H$. That is a contradiction. Then (M_H,ρ) is (*NSbC*) – *C*.

Definition 3.16. Assume that (*J,L*) is a soft set over *M* and define $P:SS(M_H)\to FS(M,H)$ by $P(J,L)=\underset{J(L)}{P},\forall(J,L)\in SS(M_H)$, where $\underset{J(L)}{P}(k)$ is fuzzy subset of *M* defended by:

$$(\underset{J(L)}{P}(k))(x)=\begin{cases}1, & if\ \ x\neq J(k)\&x\in J(k)\\ 1/n(SS(M_L)), & if\ x=J(k)\\ n(SS(M_L))/n(SS(M_H)), & if\ x\neq J(k)\&x\notin J(k)\neq\phi\\ 0, & if\ \ J(k)=\phi\end{cases},\forall x\in M,\text{ if } k\in L \text{ and } (\underset{J(L)}{P}(k))(x)$$
$$=\bar{0}(x),\forall x\in M \text{ if } k\in H-L.$$

We say $\underset{J(L)}{Q}$ is an induced neutrosophic by soft set (*J,L*) over *M*, where $Q:FS(M,H)\to NS(M_H)$ is a mapping and for $\underset{J(L)}{P}\in FS(M,H)$ the image of $\underset{J(L)}{P}$ under *Q* denoted by $\underset{J(L)}{Q}$, is defended by:

$$\underset{J(L)}{Q}(e)=\left\{(x,(\underset{J(L)}{P})(e))(x),(\underset{J(L)}{P})(e))(x)/2,\,1-(\underset{J(L)}{P})(e))(x);x\in M\right\},\forall e\in L.$$

Example 3.17. Let *M* = $\{m_1,m_2,m_3\}$ and *H* = $\{e_1,e_2,e_3,e_4\}$, and let (*J,L*) be a (SS) over *M* where $L=\{e_1,e_3,e_4\},J(e_1)=\{m_1\},J(e_3)=\{m_1,m_2\},J(e_4)=\phi$. Then $n(SS(M_L))=2^9\ \&\ n(SS(M_H))=2^{12}$. Hence $\underset{J(L)}{P}=\left\{\underset{J(e_1)}{P},\underset{J(e_2)}{P},\underset{J(e_3)}{P},\underset{J(e_4)}{P}\right\}$, where $\underset{J(e_1)}{P}=(\underset{J(e_1)}{P}(m_1),\underset{J(e_1)}{P}(m_2),\underset{J(e_1)}{P}(m_3))=(1/n(SS(M_L)),n(SS(M_L))/n(SS(M_H)),$ $n(SS(M_L))/n(SS(M_H)))=(0.00195125,0{,}125,0{,}125)$, $\underset{J(e_2)}{P}=(0,0,0)$, $\underset{J(e_3)}{P}=(1,1,n(SS(M_L))/n(SS(M_H)))=$ $(1,1,0.125)$, $\underset{J(e_4)}{P}=(0,0,0)\}$. See table (1).

Table 1: An Induced neutrosophic by soft set (*J,L*).

Q (*e*)(*x*) *J*(*L*)	e_1	e_2	e_3	e_4
m_1	(0,0,1)	(0.00195125, 0.000975625, 0.99804875)	(1,0.5,0.875)	(0,0,1)
m_2	(0,0,1)	(0.125, 0.0625, 0.875)	(1,0.5,0.875)	(0,0,1)
m_3	(0,0,1)	(0.125, 0.0625, 0.875)	(0.125, 0.0625, 0.875)	(0,0,1)

$$\underset{J(L)}{Q}(e)=\left\{(m,(\underset{J(L)}{P})(e))(m),(\underset{J(L)}{P})(e))(m)/2,\ 1-(\underset{J(L)}{P})(e))(m);m\in M\right\}.$$

Remarks 3.18. For each $(M,H)\in SS(M_H)$ we consider the following:

1. There is a (*NSS*) can be generated by using (*M,H*) under the composition of $P: SS(M_H)\to FS(M,H)$ and $Q: FS(M,H)\to NS(M_H)$.
2. $(Q\circ P)(\Phi)=\Phi_H$ is Null (*NSS*), where $\Phi=(\phi,\phi)$ is Null (*SS*).
3. $(Q\circ P)(M,H)=\bar{M}_H$ is Absolute (*NSS*), where (M,H) is Absolute (*SS*).

Definition 3.19. Suppose that (M,H,τ) is a soft topological space (*STS*) over *M* and (M_H,ρ) is a (*NSTS*). We say (M_H,ρ) is an induced neutrosophic soft topological space (*INSTS*) by (M,H,τ) if and only if $Q\circ P(\tau)=\rho$.

Remarks 3.20.

(1) If (M,H,τ) is (*STS*) with $P(\tau)=\psi$, then (M,H,ψ) is fuzzy soft topological space (*FSTS*) induced by (M,H,τ).

(2) If (M,H,ψ) is (FS??S) with $Q(\psi)=\rho$, then (M_H,ρ) is (*NSTS*) induced by (M,H,ψ).

Proposition 3.21. Let $(J,L),(R,V)\in SS(M_H)$, the following terms are valid:

(1) If $(J,L)\subseteq(R,V)$, then $\underset{J(L)}{P}\subseteq\underset{R(V)}{P}$,

(2) $P(int(J,L))=int(\underset{J(L)}{P})$,

(3) $P(cl(J,L))=cl(\underset{J(L)}{P})$.

Proof: (1) Since $(J,L)\subseteq(R,V)$, then $L\subseteq V$ and $J(c)\subseteq R(c),\forall c\in L$. Thus $\underset{J(L)}{P}\subseteq\underset{R(V)}{P}$ whenever $c\in H\setminus L$ (since $(\underset{J(L)}{P}(c))(m)=\bar{0}(m)=(\underset{R(V)}{P}(c))(m),\forall m\in M$). If $c\in L$, then we have 4 situations, as follows:

State (1) $(\underset{J(L)}{P}(c))(m)=1$, if $m\neq J(c)\&m\in J(c)$. But, $J(c)\subseteq R(c)$ and hence *J*(*c*) = *R*(*c*). Then $(\underset{J(L)}{P}(c))(m)=1=(\underset{R(V)}{P}(c)(m)$, if $m\neq J(c)\&m\in J(c)$.

State (2) $(\underset{J(L)}{P}(c))(m)=1/n(SS(M_L))$, if $m=J(c)$. However, from $J(c)\subseteq R(c)$ we have either *J*(*c*) = *R*(*c*) or $J(c)\subset R(c)$, if *J*(*c*)=*R*(*c*) we get $(\underset{J(L)}{P}(c))(m)=1/n(SS(M_L))=(\underset{R(V)}{P}(c))(m)$. Moreover, if $J(c)\subset R(c)$ then we obtain $m\neq R(c)\&m\in R(c)$. Hence $(\underset{R(V)}{P}(c))(m)=1$, then $(\underset{J(L)}{P}(c))(m)<(\underset{R(V)}{P}(c))(m)$, if $m = J(c)$.

State (3) $(\underset{J(L)}{P}(c))(m)=n(SS(M_L))/n(SS(M_H))$ if $m\neq J(c)\&m\notin J(c)\neq\phi$. Furthermore, from $J(c)\subseteq R(c)$ we have $m\neq R(c)\&R(c)\neq\phi$. Now, if $m\in R(c)$, then $(\underset{J(L)}{P}(c))(m)<(\underset{R(V)}{P}(c))(m)=1$ and if $m\notin R(c)$ we get $(\underset{J(L)}{P}(c))(m)=n(SS(M_L))/n(SS(M_H))\leq(n(SS(M_V))/n(SS(M_H))=(\underset{R(V)}{P}(c))(m)$. Hence, we consider that $(\underset{J(L)}{P}(c))(m)\leq(\underset{R(V)}{P}(c))(m)$, if $m\neq J(c)\&m\notin J(c)\neq\phi$.

State (4) $(\underset{J(L)}{P}(c))(m)=0$, if $J(c)$ = ϕ. Thus $(\underset{J(L)}{P}(c))(m) \le (\underset{R(V)}{P}(c))(m), \forall m \in M$. Finally, we obtain $(\underset{J(L)}{P}(c))(m) \le (\underset{R(V)}{P}(c))(m), \forall c \in L$ and $m \in M$. Then $\underset{J(L)}{P} \subseteq \underset{R(V)}{P}$.

Proof: (2) Since $int(J,L) = \bigcup\{(R,V) \mid (R,V) \in \tau \wedge (R,V) \subseteq (J,L)\}$. Hence, we get $P(int(J,L)) = P(\bigcup\{(R,V) \mid (R,V) \in \tau \wedge (R,V) \subseteq (J,L)\}) = \coprod\{\underset{R(V)}{P} \mid \underset{R(V)}{P} \in \psi \wedge \underset{R(V)}{P} \subseteq \underset{J(L)}{P}\}$.

Where (M,H,ψ) is (*FSTS*) induced by (M,H,τ). Moreover, $int(\underset{J(L)}{P}) = \coprod\left\{\underset{R(V)}{P} \mid \underset{R(V)}{P} \in \psi \wedge \underset{R(V)}{P} \subseteq \underset{J(L)}{P}\right\}$. Then $P(int(L,L)) = int(\underset{J(L)}{P})$.

Proof: (3) Since $cl(J,L) = \bigcap\left\{(R,V) \mid (R,V)^c \in \tau \wedge (J,L) \subseteq (R,V)\right\}$. Hence $P(cl(J,L)) = P\left(\bigcap\left\{(R,V) \mid (R,V)^c \in \tau \wedge (J,L) \subseteq (R,V)\right\}\right) = \prod\left\{\underset{R(V)}{P} \mid \underset{R(V)}{P}^c \in \psi \wedge \underset{J(L)}{P} \subseteq \underset{R(V)}{P}\right\}$.

Where (M,H,ψ) is (FSTS). However, $cl\left(\underset{J(L)}{P}\right) = \prod\left\{\underset{R(V)}{P} \mid \underset{R(V)}{P}^c \in \psi \wedge \underset{J(L)}{P} \subseteq \underset{R(V)}{P}\right\}$. Then $P(cl(J,L)) = cl\left(\underset{J(L)}{P}\right)$.

Proposition 3.22. Let (M,H,τ) be a soft *b*–closed topological space (*SbCTS*), then the (*FSTS*) induced by (M,H,τ) is a fuzzy soft *b*–closed topological space (*FSbCTS*).

Proof: Assume that (M,H,ψ) is (*FSTS*) induced by (M,H,τ). Then $P(\tau) = \psi$. Hence, for any (*SS*) $(J,L) \in \tau$, we get $\underset{J(L)}{P} \in P(\tau) = \psi$. Also, for any (*FSS*)$J_L \in \psi$, there exists (*SS*) $(J,L) \in \tau$ with $P((J,L)) = \underset{J(L)}{P} = J_L$. Now, we need to show that $cl(int(J_L)) \prod int(cl(J_L)) \subseteq J_L, \forall J_L \in \psi$. Moreover, $J_L \in \psi$ and hence we get $cl(int(J_L)) \prod int(cl(J_L)) = cl(J_L) \prod int(cl(J_L)) = int(cl(J_L))$. Since (M,H,τ) is a (SbCTS), then $(J,L) \in \tau$ with $cl(int(J,L)) \prod int(cl(J,L)) = int(cl(J,L)) \subseteq (J,L)$. Thus $P(int(cl(J,L))) = int(cl(\underset{J(L)}{P})) \subseteq \underset{J(L)}{P}$ [From Proposition (3.21)], however $\underset{J(L)}{P} = J_L$. Therefore, we get $cl(int(J_L)) \prod int(cl(J_L)) \subseteq J_L, \forall J_L \in \psi$. Then the (*FSTS*) induced by (M,H,τ) is a (*FSbCTS*).

Proposition 3.23. Let $J_L, R_V \in FS(M,H)$, the following terms are valid:

(1) If $J_L \subseteq R_V$, then $\underset{J(L)}{Q} \subseteq \underset{R(V)}{Q}$,

(2) $Q(int(J_L)) = int(\underset{J(L)}{Q})$,

(3) $Q(cl(J_L)) = cl(\underset{J(L)}{Q})$.

Proof: (1) Since $J_L \subseteq R_V$, then $(\underset{J(L)}{Q}(c))(m) \le (\underset{R(V)}{Q}(c))(m), \forall m \in M$ and $\forall c \in H$, and this implies that $1-(\underset{J(L)}{Q}(c))(m) \ge 1-(\underset{R(V)}{Q}(c))(m), \forall m \in M$ and $\forall c \in H$. Then $\underset{J(L)}{Q} \subseteq \underset{R(V)}{Q}$.

Proof: (2) Since $int(J_L) = \coprod\{R_V \mid R_V \in \psi \wedge R_V \subseteq J_L\}$. Hence $Q(int(J_L)) = Q\left(\coprod\{(R_V \mid R_V \in \psi \wedge R_V \subseteq F_A\}\right) = \overline{\coprod}\left\{\underset{R(V)}{Q} \mid \underset{R(V)}{Q} \in \rho \wedge \underset{R(V)}{Q} \subseteq \underset{J(L)}{Q}\right\}$.

Where (M_H, ρ) is an (*INSTS*) by (M,H,ψ). Moreover, $int(\underset{J(L)}{Q}) = \overline{\coprod}\left\{\underset{R(V)}{Q} \mid \underset{R(V)}{Q} \in \rho \wedge \underset{R(V)}{Q} \subseteq \underset{J(L)}{Q}\right\}$. Then $Q(int(J_L)) = int(\underset{J(L)}{Q})$.

Proof: (3) Since $cl(J_L) = \prod\left\{R_V \mid R_V{}^c \in \psi \wedge J_L \subseteq R_V\right\}$. Hence $Q(cl(J_L)) = Q\left(\prod\left\{(R_V \mid R_V{}^c \in \psi \wedge J_L \subseteq R_V\right\}\right) = \overline{\prod}\left\{\underset{R(V)}{Q} \mid \underset{R(V)}{Q}^c \in \rho \wedge \underset{J(L)}{Q} \subseteq \underset{R(V)}{Q}\right\}$.

Where (M_H,ρ) is an (*INSTS*) by (M,H,ψ). Moreover, $cl(\underset{J(L)}{Q}) = \overline{\prod}\left\{\underset{R(V)}{Q} \mid \underset{R(V)}{Q}^{c} \in \rho \wedge \underset{J(L)}{Q} \subseteq \underset{R(V)}{Q}\right\}$. Hence $Q(cl(J_L)) = cl\left(\underset{J(L)}{Q}\right)$.

Proposition 3.24. Let (M,H,ψ) be a (*FSbCTS*), then the (INSTS) by (M,H,ψ) is a (*NSbCT*).

Proof: Assume that (M_H,ρ) is an (*INSTS*) by (M,H,ψ). Then $Q(\psi)$ = ρ. So, for any (*FSS*) J_L ∈ ρ we get $\underset{J(L)}{Q} \in Q(\psi) = \rho$, Furthermore, for any (*NSS*) *(J,L)* ∈ ρ, there exists (*FSS*) $J_L \in \psi$ with $Q(J_L) = \underset{J(L)}{Q} = (J,L)$. Now, we need to show that $cl(int(J,L))\overline{\prod}int(cl(J,L)) \subseteq (J,L), \forall (J,L) \in \rho$. However, (*J,L*) ∈ ρ this implies that $cl(int(J,L))\overline{\prod}int(cl(J,L)) = cl(J,L)\overline{\prod}int(cl(J,L)) = int(cl(J,L))$. Since (M,H,ψ) is a (FSbCTS). So, $J_L \in \psi$ satisfies $cl(int(J_L)) \prod int(cl(J_L)) = int(cl(J_L)) \subseteq J_L$. Therefore $Q\left(int(cl(J_L))\right) = int\left(cl(\underset{J(L)}{Q})\right) \subseteq \underset{J(L)}{Q}$ [From Proposition (3,23)], however $\underset{J(L)}{Q} = (J,L)$. Then for any (*J,L*) ∈ ρ with $cl(int(J,L))\overline{\prod}int(cl(J,L)) \subseteq (J,L)$. Hence the (*INSTS*) by (M,H,ψ) is a (NSbCT).

Proposition 3.25. Assume that $(M_H,\rho_1),(W_D,\rho_2)$, and ($Z_G$, ρ_3) are three (*NSTSs*). Let $\lambda_1 : (M_H,\rho_1) \to (W_D,\rho_2)$ be a (*NSbCM*) and $\lambda_2 : (M_H,\rho_1) \to (Z_G,\rho_3)$ be (*NSbCCM*). If (Z_G, ρ_3) is induced by (*FSbCS*) (*Z,G,ψ*). Then for any (*FSS*)$J_L \in \psi$, there exists (*NSbCS*) (*R,V*) in W_D with (*R,V*) ∈ ρ_2.

Proof: Let $J_L \in \psi$. Then J_L is a (*FSbCS*) [since (*Z,G,ψ*) is (*FSbCTS*)]. But, $\underset{J(L)}{Q} \in Q(\psi) = \rho_3$ (since (Z_G, ρ_3) is an induced by (*Z,G,ψ*)). Also, $\underset{F(A)}{\eta}$ is a (*NSbCS*) in Z_G [From Proposition (3,24)]. Moreover, $\lambda_2^{-1}(\underset{J(L)}{Q}) \in \rho_1$ and $\lambda_2^{-1}(\underset{J(L)}{Q})$ is a (*NSbCS*) in M_H [since λ_2 is a (*NSbCCM*)]. This implies that $\lambda_1\left(\lambda_2^{-1}\left(\underset{J(l)}{Q}\right)\right) \in \rho_1$ and $\lambda_1\left(\lambda_2^{-1}\left(\underset{J(l)}{Q}\right)\right)$ is a (*NSbCS*) in W_D [since λ_1 is a (*NSbCM*)]. Let $\lambda_1\left(\lambda_2^{-1}\left(\underset{J(l)}{Q}\right)\right) = (R,V)$. Then for any (*FSS*)$J_L \in \psi$ there exists (*NSbCS*) (*R,V*) in W_D with $(R,V) \in \rho_2$.

4. Conclusion

The goal of this work is to introduce and investigate several innovative concepts, such as (*NSbCT*), (*NSbCM*), (*NSbCCM*), $(NSbC)_f$*–SC*, (*NSbC*)*–C*, (*NSbC*)*–C′*, $\underset{H(A)}{\eta}$. We have provided some basic features for these ideas. Furthermore, it is fascinating to investigate the compositions of soft and Neutrosophic sets. In addition, the composition of two mappings *P* and *Q* can be tested using additional types of soft sets, such as soft *b*–closed (*J,L*), to ensure that $Q \circ P(J,L)$ is (NSbCS). Assuming (M,H,τ) is a soft topological space over *M*, the (*INSTS*) by (M,H,τ) must be (*NSbCS*).

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