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Common fixed point theorems for compatible maps in complex valued $\mathcal{G}_b$ - metric space

Vishal Gupta[1], Rajani Saini[2], Rahul Shukla[3]

[1]Department of Mathematics, MMEC, Maharishi Markandeshwar (Deemed to be University), Mullana, Haryana 133207, India; [2]Department of Mathematics, Govt. PG College Ambala Cantt, Haryana 133001, India; [3]Faculty of Natural Sciences, Department of Mathematical Sciences and Computing, Walter Sisulu University, Mthatha, 5117, South Africa

Abstract

The article investigates common fixed point theorems in the framework of complete complex-valued $\mathcal{G}_b$-metric space. It establishes such theorems for compatible mappings and employs rational inequalities to derive novel results. These findings serve to extend and generalize existing results in the literature. To illustrate the practical applicability and effectiveness of the proposed methods, the article presents several non-trivial examples.

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1. Introduction

Fixed point theory is a branch of mathematics that deals with the existence and properties of fixed points. In mathematics, fixed point theory is one of the prominent theories and remains a dynamic and evolving field, providing essential tools and insights across mathematics and applied sciences. Its rich interplay between theory and application underscores its significance and the ongoing interest in its development. Metric space theory has extensive applications, not only in mathematics, but in

Email addresses: vishal.gmn@gmail.com, vgupta@mmumullana.org (Vishal Gupta), prof.rajanisaini@gmail.com (Rajani Saini), rshukla@wsu.ac.za (Rahul Shukla)*

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the other area of quantitative sciences. [10] derived fixed point results using real-valued functions satisfying integral-type rational contractions, further enriching the literature on metric fixed point theory. These works collectively contribute to the ongoing development of fixed point theory in various generalized metric spaces and its applications. Many researchers expanded their work in the field of fixed point theory [5, 6, 7, 8, 15, 20, 21].

A number of generalizations of metric spaces are familiarized like, D-metric space, b-metric space, $\mathcal{G}$-metric space, partial metric space, $\mathcal{S}$-metric space and many more, see [9, 16]. Shukla et al. [19] explored fixed point theorems in graphical cone metric spaces and applied their findings to systems of initial value problems, demonstrating the utility of their theoretical work. Merging the concept of b -metric space and $\mathcal{G}$-metric space, [1] bring together the idea of generalized $\mathcal{G}b$-metric space. In recent years, several researchers have contributed to the advancement of fixed point theory in generalized metric spaces. [18] established coupled fixed point theorems in $\mathcal{G}b$-metric spaces, extending previous results in this framework. [23] investigated coupled coincidence point results for (ψ -ϕ)-weakly contractive mappings in partially ordered $\mathcal{G}b$-metric spaces, broadening the scope of fixed point analysis under weak contraction conditions. In a related study, [24] examined the existence of tripled coincidence points in ordered $\mathcal{G}b$-metric spaces and provided applications to systems of integral equations, linking abstract fixed point theory with practical problems. In 2011, the new notion of complex valued metric spaces introduced by [2] and many others obtained common fixed point theorems in complex valued metric spaces. [22] made significant contributions by establishing generalized common fixed point theorems in complex-valued metric spaces along with their practical applications. [4] presented the complex valued b-metric space. [17] gave some contractive results in complex valued $\mathcal{G}$-metric spaces. Combining the concept of complex valued metric spaces and $\mathcal{G}b$-metric spaces, [11] familiarized the idea of complex valued $\mathcal{G}b$-metric spaces and he obtained several elementary properties of complex valued $\mathcal{G}b$-metric spaces, see [12, 13, 14]. This area continues to grow, with ongoing research focusing on deeper properties, more general fixed point theorems, and broader applications.

2. Preliminaries

First, we recollect some basic results of complex valued metric spaces. Take $\mathbb{C}$ to be the set of complex numbers, $a,b \in \mathbb{C}$. Take a partial order $\precsim$ on $\mathbb{C}$ as below:

$$a \precsim b \text{ if and only if } Re(a) \le Re(b)$$
$$\text{and } Im(a) \le Im(b)$$

It concludes that $a \precsim b$ if any one of the next conditions is fulfilled:

1. $Re(a) = Re(b)$ and $Im(a) = Im(b)$,
2. $Re(a) < Re(b)$ and $Im(a) = Im(b)$,
3. $Re(a) = Re(b)$ and $Im(a) < Im(b)$,
4. $Re(a) < Re(b)$ and $Im(a) < Im(b)$.

We write $a \underset{\neq}{\prec} b$ if $a \neq b$ and any one of (2), (3) and (4) is fulfilled. Also, we will write $a \prec b$ if only (4) is satisfied. Also, the following statements folds:

1. If $x, y \in \mathbb{R}$ with $x \le y$ then $xa \prec ya$, for all $a \in \mathbb{C}$,
2. If $0 \precsim a \underset{\neq}{\prec} b$, then $|a| < |b|$,
3. If $a \precsim b$ and $b \precsim c$, then $a \precsim c$.

Definition 2.1: [2] *A mapping $d_c : \Upsilon \times \Upsilon \to \mathbb{C}$ satisfies the following properties, for every $h, j \in \Upsilon$:*

1. $d_c(h, j) \succeq 0$,
2. $d_c(h, j) = 0$ if and only if $h = j$,

3. $d_c(h,j) = d_c(j,h)$,
4. $d_c(h,j) \precsim d_c(h,x) + d_c(x,j)$.

The pair (Υ, d_c) is known as complex valued metric space.

Definition 2.2: [11] *A mapping $\mathcal{G}_b : \Upsilon \times \Upsilon \times \Upsilon \to \mathbb{C}$ satisfies the following properties, for every $h,j,k \in \Upsilon$:*

1. $\mathcal{G}_b(h,j,k) = 0$ if $h = j = k$,
2. $0 \prec \mathcal{G}_b(h,h,j)$ with $h \neq j$,
3. $\mathcal{G}_b(h,h,k) \precsim \mathcal{G}_b(h,j,k)$ with $h \neq k$,
4. $\mathcal{G}_b(h,j,k) = \mathcal{G}_b\{\pi(h,j,k)\}$, where π is a permutation,
5. $\mathcal{G}_b(h,j,k) \precsim s\{\mathcal{G}_b(h,x,x) + \mathcal{G}_b(x,j,k)\}$.

The pair $(\Upsilon, \mathcal{G}_b)$ is known as complex valued $\mathcal{G}_b$-metric space.

Proposition 2.3: [11] *In a complex valued $\mathcal{G}$-metric space $(\Upsilon, \mathcal{G}_b)$, for each $h,j,k \in \Upsilon$, it follows that:*

1. $\mathcal{G}_b(h,j,k) \precsim s\{\mathcal{G}_b(h,h,j) + \mathcal{G}_b(h,h,k)\}$,
2. $\mathcal{G}_b(h,j,j) \precsim 2s\mathcal{G}_b(j,h,h)$.

Definition 2.4: [11] *A sequence $\{h_n\}$ in $(\Upsilon, \mathcal{G}_b)$ is known to be convergent to a point τ if for each $c \in \mathbb{C}, c \succ 0$, there exists a positive integer n_0 such that for all $m,n \geq n_0, \mathcal{G}_b(h_n, h_m, \tau) \prec c$.*

Definition 2.5: [11] *A sequence $\{h_n\}$ in $(\Upsilon, \mathcal{G}_b)$ is known to be Cauchy if for each $c \in \mathbb{C}, c > 0$, there exists a positive integer n_0 such that for all $m,n,\ell \geq n_0, \mathcal{G}_b(h_n, h_m, h_\ell) \prec c$.*

Definition 2.6: [11] *A complex valued $\mathcal{G}_b$-metric space $(\Upsilon, \mathcal{G}_b)$ is complete if every Cauchy sequence is convergent in it.*

Proposition 2.7: [11] *In a complex valued $\mathcal{G}_b$-metric space, the following are equivalent:*

1. $\{h_n\}$ is complex valued $\mathcal{G}_b$-convergent to τ,
2. $\left|\mathcal{G}_b(h_n, h, \tau)\right| \to 0$ as $n \to \infty$,
3. $\left|\mathcal{G}_b(h_n, \tau, \tau)\right| \to 0$ as $n \to \infty$,
4. $\left|\mathcal{G}_b(h_n, h_m, \tau)\right| \to 0$ as $m,n \to \infty$.

Proposition 2.8: [11] *In a complex valued $\mathcal{G}_b$-metric space, $\{h_n\}$ is a Cauchy sequence if and only if $\left|\mathcal{G}_b(h_n, h_m, h_\ell)\right| \to 0$ as $m,n,\ell \to \infty$.*

The notion of compatible maps in metric spaces is given by [3] in 1986.

Definition 2.9: [3] *Two self-mappings $\mathcal{H},\mathcal{K}$ of a metric space (Υ, d_c) are said to be compatible if $\lim_{n\to\infty} d_c(\mathcal{H}\mathcal{K}h_n, \mathcal{K}\mathcal{H}h_n) = 0$, whenever $\{h_n\}$ is a sequence in Υ such that $\lim_{n\to\infty}\mathcal{H}h_n = \varphi = \lim_{n\to\infty}\mathcal{K}h_n$, for some $\varphi \in \Upsilon$.*

The concept of compatible mapps in $\mathcal{G}_b$ metric space is given by [13]

Definition 2.10: [13] *Let $(\Upsilon, \mathcal{G}_b)$ be a complex valued $\mathcal{G}_b$-metric space and $\mathcal{H},\mathcal{K}$ be mappings from $(\Upsilon, \mathcal{G}_b)$ into itself. The mappings $\mathcal{H},\mathcal{K}$ are called compatible if there exists a sequence $\{h_n\}$ such that* $\lim_{n\to\infty}\mathcal{G}_b(\mathcal{H}\mathcal{K}h_n, \mathcal{K}\mathcal{H}h_n, \mathcal{K}\mathcal{H}h_n) = 0$ or $\lim_{n\to\infty}\mathcal{G}_b(\mathcal{K}\mathcal{H}h_n, \mathcal{H}\mathcal{K}h_n, \mathcal{H}\mathcal{K}h_n) = 0$, whenever $\{h_n\}$ is a sequence in Υ such that $\lim_{n\to\infty}\mathcal{H}h_n = \varphi = \lim_{n\to\infty}\mathcal{K}h_n$, for some $\varphi \in \Upsilon$.

Example 2.11: [13] *Let $\Upsilon = [-1,1]$ and $(\Upsilon, \mathcal{G}_b)$ a complex valued $\mathcal{G}_b$-metric space such that $\mathcal{G}_b(h,j,k) = |h-j|^2 + |j-k|^2 + |k-h|^2$, for all $h,j,k \in \Upsilon$, where $s = 2$. Define two self-mappings $\mathcal{H},\mathcal{K} : \Upsilon \to \Upsilon$ by $\mathcal{H}(h) = h$ and $\mathcal{K}(h) = \frac{h}{3}$. Consider a sequence $h_n = \frac{1}{2n}$, we get*

$$\lim_{n\to\infty}\mathcal{G}_b\left(\mathcal{KH}h_n,\mathcal{HK}h_n,\mathcal{HK}h_n\right)=\lim_{n\to\infty}\mathcal{G}_b\left(\frac{1}{6n},\frac{1}{6n},\frac{1}{6n}\right)=0,$$

and also,

$$\lim_{n\to\infty}\mathcal{H}h_n=\mathcal{H}\left(\frac{1}{2n}\right)=0\text{and}\lim_{n\to\infty}\mathcal{K}h_n=\mathcal{K}\left(\frac{1}{2n}\right)=\left(\frac{1}{6n}\right)=0.$$

Therefore, mappings $\mathcal{H},\mathcal{K}$ are compatible.

3. Main Results

Theorem 3.1 *Consider $\left(\Upsilon,\mathcal{G}_b\right)$ be a complete complex valued $\mathcal{G}_b$-metric space with a real number $s\geq 1$ and $\mathcal{H},\mathcal{K}$ are self-mappings of Υ satisfying the following conditions:*

1. $\mathcal{H}(\Upsilon)\subseteq\mathcal{K}(\Upsilon)$,
2. $\mathcal{H}$ or $\mathcal{K}$ is continuous function,
3. $\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{H}k)\precsim\Delta\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{K}k)$,

for every $h,j,k\in\Upsilon$ with $\Delta s<1$, whenever $\mathcal{H}$ and $\mathcal{K}$ are compatible mappings, they have a unique common fixed point in Υ.

Proof. Consider a sequence $\{h_n\}\in\Upsilon$ with an initial point h_0. We can choose h_1 such that $\mathcal{H}h_0=\mathcal{K}h_1$. Choose a sequence $\{h_{n+1}\}$ such that $\varphi_n=\mathcal{H}h_n=\mathcal{K}h_{n+1},n=0,1,2\ldots$

By using Condition (3), we get

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\precsim\Delta\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right)=\Delta\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_n\right).$$

Continuing like this, we can show that

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\precsim\Delta^n\mathcal{G}_b\left(\mathcal{H}h_0,\mathcal{H}h_1,\mathcal{H}h_1\right).$$

For every $n,m\in N,n<m$, we have

$$\begin{aligned}\mathcal{G}_b\left(\varphi_n,\varphi_m,\varphi_m\right)&\precsim s\mathcal{G}_b\left(\varphi_{n'},\varphi_{n+1},\varphi_{n+1}\right)+s^2\mathcal{G}_b\left(\varphi_{n+1},\varphi_{n+2},\varphi_{n+2}\right)\\&\quad+s^3\mathcal{G}_b\left(\varphi_{n+2},\varphi_{n+3},\varphi_{n+3}\right)+\cdots,\\&\precsim s\Delta^n\mathcal{G}_b\left(\varphi_0,\varphi_1,\varphi_1\right)+s^2\Delta^{n+1}\mathcal{G}_b\left(\varphi_0,\varphi_1,\varphi_1\right)\\&\quad+s^3\Delta^{n+2}\mathcal{G}_b\left(\varphi_0,\varphi_1,\varphi_1\right)+\cdots,\end{aligned}$$

we have,

$$\mathcal{G}_b\left(\varphi_n,\varphi_m,\varphi_m\right)\precsim\frac{s\Delta^n}{1-s\Delta}\mathcal{G}_b\left(\varphi_0,\varphi_1,\varphi_1\right).$$

$\mathcal{G}_b\left(\varphi_n,\varphi_m,\varphi_m\right)$ tends to zero, as n,m tends to infinity. This implies that $\{\varphi_n\}$ is a Cauchy sequence. By completeness property of complex valued $\mathcal{G}_b$-metric space, there is a sequence which is convergent to a point $\varphi\in\Upsilon$ such that $\lim_{n\to\infty}\{\varphi_n\}=\varphi$ and $\lim_{n\to\infty}\varphi_n=\lim_{n\to\infty}\mathcal{H}h_n=\lim_{n\to\infty}\mathcal{K}h_{n+1}=\varphi$. As $\mathcal{H}$ or $\mathcal{K}$ is continuous function. Suppose, $\mathcal{H}$ is continuous and

$$\lim_{n\to\infty}\mathcal{KH}h_n=\lim_{n\to\infty}\mathcal{KK}h_{n+1}=\mathcal{K}\varphi.$$

Further, $\mathcal{H}$ and $\mathcal{K}$ are compatible. Therefore, $\lim_{n\to\infty}\mathcal{G}_b\left(\mathcal{HK}h_n,\mathcal{KH}h_n,\mathcal{KH}h_n\right)=0$. This implies that $\lim_{n\to\infty}\mathcal{HK}h_n=\lim_{n\to\infty}\mathcal{KH}h_n=\mathcal{K}\varphi$.

Again,

$$\mathcal{G}_b\left(\mathcal{HK}h_n,\mathcal{H}h_n,\mathcal{H}h_n\right)\precsim\Delta\mathcal{G}_b\left(\mathcal{KK}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right).$$

Taking limit n tends to infinity, we get

$$\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi)\precsim\Delta\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi).$$

This is not possible, as $\Delta<\frac{1}{s}<1$.Therefore, $\mathcal{K}\varphi=\varphi$.

Moreover,

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}\varphi,\mathcal{H}\varphi\right)\precsim\Delta\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}\varphi,\mathcal{K}\varphi\right).$$

Taking limit n tends to infinity gives,

$$\mathcal{G}_b(\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi)\precsim\Delta\mathcal{G}_b(\varphi,\varphi,\varphi).$$

Hence, we get $\mathcal{H}\varphi=\varphi$ and $\mathcal{H}\varphi=\mathcal{K}\varphi=\varphi$. Thus, mappings $\mathcal{H},\mathcal{K}$ has φ as their common fixed point.

Uniqueness: Consider θ as another common fixed point of $\mathcal{H},\mathcal{K}$ with $\theta\neq\varphi$.

$$\mathcal{G}_b(\varphi,\theta,\theta)=\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{H}\theta,\mathcal{H}\theta)\precsim\Delta\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\theta,\mathcal{K}\theta)=\Delta\mathcal{G}_b(\varphi,\theta,\theta),$$

which is not possible. Therefore, $\varphi=\theta$.

Example 3.2: *Let* $\Upsilon=[-1,1]$ *and* $\mathcal{G}_b(h,j,k)=(|h-j|+|j-k|+|k-h|)^2$*, for every* $h,j,k\in\Upsilon$*. It is a complete complex valued* $\mathcal{G}_b$*-metric with* $s=2$.

Define $\mathcal{H}(h)=\frac{h}{4}$ and $\mathcal{K}(h)=\frac{h}{3}$. Here, we note that $\mathcal{H}$ is continuous and $\mathcal{H}(\Upsilon)\subseteq\mathcal{K}(\Upsilon)$. Also, $\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{H}k)\precsim\Delta\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{K}k)$ holds for every $h,j,k\in\Upsilon$ and $\frac{1}{4}\leq\Delta<\frac{1}{2}$. Here, mappings $\mathcal{H}$ and $\mathcal{K}$ has 0 as their common fixed point.

Theorem 3.3: *Consider* $\left(\Upsilon,\mathcal{G}_b\right)$ *be a complete complex valued* $\mathcal{G}_b$*-metric space with a real number* $s\geq1$ *and* $\mathcal{H},\mathcal{K}$ *are self-mappings on* Υ *satisfying the following conditions:*

1. $\mathcal{H}(\Upsilon)\subseteq\mathcal{K}(\Upsilon)$,
2. $\mathcal{H}$ or $\mathcal{K}$ is continuous function,
3. $\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{H}k)\precsim\Delta\mathcal{G}_b(\mathcal{H}h,\mathcal{K}j,\mathcal{K}k)+\nabla\mathcal{G}_b(\mathcal{K}h,\mathcal{H}j,\mathcal{K}k)+\Theta\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{H}k)$,

for every $h,j,k\in\Upsilon$ with $(\Delta+\nabla+\Theta)<\frac{1}{3s^2}$.

Then, $\mathcal{H}$ and $\mathcal{K}$ have a unique common fixed point, provided $\mathcal{H}$ and $\mathcal{K}$ are compatible mappings.

Proof. Consider a sequence $\left\{h_n\right\}\in\Upsilon$ with an initial point h_0. We can choose h_1 such that

$\mathcal{H}h_0=\mathcal{K}h_1$. Choose a sequence $\left\{h_{n+1}\right\}$ such that $\varphi_n=\mathcal{H}h_n=\mathcal{K}h_{n+1},n=0,1,2\ldots$

Then, by condition (3), we have

$$\begin{aligned}\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)&\precsim\Delta\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right)+\nabla\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{H}h_{n+1},\mathcal{K}h_{n+1}\right)+\Theta\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{H}h_{n+1}\right),\\&\precsim\Delta\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_n,\mathcal{H}h_n\right)+\nabla\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_{n+1},\mathcal{H}h_n\right)+\Theta\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_{n+1}\right).\end{aligned}\tag{3.1}$$

From rectangle inequality and preposition (2.3), we have

$$\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_{n+1}\right)\precsim s\left\{\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_n\right)+2s\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\right\}. \tag{3.2}$$

Using equation (3.2) in equation (3.1), we get

$$\begin{gathered}\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\precsim\\ (\nabla+\Theta)s\left[\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_n\right)+\quad 2s\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\right],\\ \left(1-2s^2\nabla-2s^2\Theta\right)\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\precsim(\nabla+\Theta)s\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_n\right),\\ \mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\precsim\tau\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_n\right),\end{gathered}$$

where $\tau=\dfrac{(\nabla+\Theta)}{\left(1-2s^2\nabla-2s^2\theta\right)}<\dfrac{1}{s}<1$.

Continuing like this, we obtain,

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)\precsim\tau^n\mathcal{G}_b\left(\mathcal{H}h_0,\mathcal{H}h_1,\mathcal{H}h_1\right).$$

By using the same steps as used in Theorem 3.1, we can show that $\{\varphi_n\}$ is a Cauchy sequence. By completeness property of complex valued $\mathcal{G}_b$-metric space, there is a sequence which is convergent to a point $\varphi\in\Upsilon$ such that $\lim_{n\to\infty}\{\varphi_n\}=\varphi$ and $\lim_{n\to\infty}\varphi_n=\lim_{n\to\infty}\mathcal{H}h_n=\lim_{n\to\infty}\mathcal{K}h_{n+1}=\varphi$. As $\mathcal{H}$ or $\mathcal{K}$ is continuous function, suppose $\mathcal{H}$ is continuous, then

$$\lim_{n\to\infty}\mathcal{K}\mathcal{H}h_n=\mathcal{K}\mathcal{K}h_{n+1}=\mathcal{K}\varphi. \tag{3.3}$$

Further, $\mathcal{H}$ and $\mathcal{K}$ are compatible. Thus, $\lim_{n\to\infty}\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}\mathcal{H}h_n,\mathcal{K}\mathcal{H}h_n\right)=0$, which implies that

$$\lim_{n\to\infty}\mathcal{H}\mathcal{K}h_n=\mathcal{K}\mathcal{H}h_n=\mathcal{K}\varphi. \tag{3.4}$$

Again, from condition (3) of this theorem, we get

$$\begin{aligned}&\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{H}h_n,\mathcal{H}h_n\right)\precsim\Delta\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right)+\nabla\mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n,\mathcal{H}h_n,\mathcal{K}h_n\right)\\&+\Theta\mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{H}h_n\right).\end{aligned}$$

Taking limit n tends to infinity, we get

$$\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi)\precsim(\Delta+\nabla+\Theta)\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi),$$

this is not possible as $\Delta+\nabla+\Theta<1$. Hence, $\mathcal{K}\varphi=\varphi$.

Again using the condition (3), we have

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}\varphi,\mathcal{H}\varphi\right)\precsim\Delta\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}\varphi,\mathcal{K}\varphi\right)+\nabla\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{H}\varphi,\mathcal{K}\varphi\right)+\Theta\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}\varphi,\mathcal{H}\varphi\right).$$

Taking limit n tends to infinity,

$$\mathcal{G}_b(\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi)\quad\precsim\Delta\mathcal{G}_b(\varphi,\varphi,\varphi)+\nabla\mathcal{G}_b(\varphi,\mathcal{H}\varphi,\varphi)+\Theta\mathcal{G}_b(\varphi,\varphi,\mathcal{H}\varphi).$$

Using Preposition 2.3, we get,

$$\mathcal{G}_b(\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi)\quad\precsim(\nabla+\Theta)\mathcal{G}_b(\mathcal{H}\varphi,\varphi,\varphi)\precsim 2s(\nabla+\Theta)\mathcal{G}_b(\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi).$$

This implies that $\mathcal{H}\varphi=\varphi$. Therefore, $\mathcal{H}\varphi=\mathcal{K}\varphi=\varphi$. Thus, mappings $\mathcal{H}$ and $\mathcal{K}$ has φ as their common fixed point.

Uniqueness: Consider θ as another common fixed point of $\mathcal{H},\mathcal{K}$ with $\theta\neq\varphi$.

$$\mathcal{G}_b(\varphi,\theta,\theta) = \mathcal{G}_b(\mathcal{H}\varphi,\mathcal{H}\theta,\mathcal{H}\theta) \precsim \Delta\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\theta,\mathcal{K}\theta) + \nabla\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{H}\theta,\mathcal{K}\theta) + \Theta\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\theta,\mathcal{H}\theta),$$
$$= (\Delta + \nabla + \Theta)\mathcal{G}_b(\varphi,\theta,\theta),$$

this is not possible. Therefore, $\varphi = \theta$. Hence, uniqueness follows.

Example 3.4 *Let* $\Upsilon = \mathbb{C}$ *and* $\mathcal{G}_b(h,j,k) = |h-j| + |j-k| + |k-h| + i|h+j+k|$, *for every* $h,j,k \in \Upsilon$. *It is a complete complex valued* $\mathcal{G}_b$*-metric with* $s = 2$. *Define* $\mathcal{H}(h) = \frac{h}{2}$ *and* $\mathcal{K}(h) = \frac{h}{3}$. *Here, we note that* $\mathcal{H}$ *is continuous with* $\mathcal{H}(\Upsilon) \subseteq \mathcal{K}(\Upsilon)$ *and hence compatible.*

$$\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{H}k) = \mathcal{G}_b\left(\frac{h}{2},\frac{j}{2},\frac{h}{2}\right) = \left|\frac{h}{2}-\frac{j}{2}\right| + \left|\frac{j}{2}-\frac{h}{2}\right| + \left|\frac{h}{2}-\frac{h}{2}\right| + i\left|\frac{h}{2}+\frac{j}{2}+\frac{h}{2}\right|,$$
$$\mathcal{G}_b(\mathcal{H}h,\mathcal{K}j,\mathcal{K}k) = \mathcal{G}_b\left(\frac{h}{2},\frac{j}{3},\frac{k}{3}\right) = \left|\frac{h}{2}-\frac{j}{3}\right| + \left|\frac{j}{3}-\frac{k}{3}\right| + \left|\frac{k}{3}-\frac{h}{2}\right| + i\left|\frac{h}{2}+\frac{j}{3}+\frac{k}{3}\right|,$$
$$\mathcal{G}_b(\mathcal{K}h,\mathcal{H}j,\mathcal{K}k) = \mathcal{G}_b\left(\frac{h}{3},\frac{j}{2},\frac{k}{3}\right) = \left|\frac{h}{3}-\frac{j}{2}\right| + \left|\frac{j}{2}-\frac{k}{3}\right| + \left|\frac{k}{3}-\frac{h}{3}\right| + i\left|\frac{h}{3}+\frac{j}{2}+\frac{k}{3}\right|,$$
$$\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{H}k) = \mathcal{G}_b\left(\frac{h}{3},\frac{j}{3},\frac{k}{2}\right) = \left|\frac{h}{3}-\frac{j}{3}\right| + \left|\frac{j}{3}-\frac{k}{2}\right| + \left|\frac{k}{2}-\frac{h}{3}\right| + i\left|\frac{h}{3}+\frac{j}{3}+\frac{k}{2}\right|.$$

We can choose Δ,∇,Θ such that $(\Delta+\nabla+\Theta) < \frac{1}{3s^2}$ and

$$\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{H}k) \precsim \Delta\mathcal{G}_b(\mathcal{H}h,\mathcal{K}j,\mathcal{K}k) + \nabla\mathcal{G}_b(\mathcal{K}h,\mathcal{H}j,\mathcal{K}k) + \Theta\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{H}k).$$

By above theorem, mappings $\mathcal{H}$ and $\mathcal{K}$ has 0 as their common fixed point.

Theorem 3.5 *Consider* $(\Upsilon,\mathcal{G}_b)$ *be a complete complex valued* $\mathcal{G}_b$*-metric space with a real number* $s \geq 1$ *and* $\mathcal{H},\mathcal{K}$ *are self-mappings on* Υ *satisfying the following conditions, for every* $h,j,k \in Y$:

1. $\mathcal{H}(\Upsilon) \subseteq \mathcal{K}(\Upsilon)$,
2. $\mathcal{H}$ or $\mathcal{K}$ is continuous function,
3. $\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{H}k) \precsim \Delta \frac{\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{H}j) + \mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{K}j)}{\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j;\mathcal{K}k) + \mathcal{G}_b(\mathcal{H}h,\mathcal{K}k,\mathcal{H}k)} \mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{K}k)$,

with $\Delta s < 1$.

Then, $\mathcal{H}$ and $\mathcal{K}$ have a unique common fixed point in Υ, provided $\mathcal{H}$ and $\mathcal{K}$ are compatible.

Proof. Consider a sequence $\{h_n\} \in \Upsilon$ with an initial point h_0. We can choose h_1 such that $\mathcal{H}h_0 = \mathcal{K}h_1$. Choose a sequence $\{h_{n+1}\}$ such that $\varphi_n = \mathcal{H}h_n = \mathcal{K}h_{n+1}, n = 0,1,2\ldots$. By condition (3) of this theorem, we have

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)$$
$$\precsim \Delta \frac{\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{H}h_{n+1}\right) + \mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1}\mathcal{K}h_{n+1}\right)}{\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right) + \mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_{n+1}\mathcal{H}h_{n+1}\right)} \mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right).$$

We get, $\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right) \precsim \Delta\mathcal{G}_b\left(\mathcal{H}h_{n-1},\mathcal{H}h_n,\mathcal{H}h_n\right)$.

Continuing like this,

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right) \precsim \Delta^n\mathcal{G}_b\left(\mathcal{H}h_0,\mathcal{H}h_1,\mathcal{H}h_1\right) \tag{3.5}$$

By using the same steps as used in Theorem 3.1, we can show that $\{\varphi_n\}$ is a Cauchy sequence. By completeness property of complex valued $\mathcal{G}_b$-metric space, there is a sequence which is convergent to a point $\varphi \in \Upsilon$ such that $\lim_{n\to\infty}\{\varphi_n\} = \varphi$ and $\lim_{n\to\infty}\varphi_n = \lim_{n\to\infty}\mathcal{H}h_n = \lim_{n\to\infty}\mathcal{K}h_{n+1} = \varphi$. As $\mathcal{H}$ or $\mathcal{K}$ is continuous function, suppose $\mathcal{H}$ is continuous, then $\lim_{n\to\infty}\mathcal{K}\mathcal{H}h_n = \lim_{n\to\infty}\mathcal{K}\mathcal{K}h_{n+1} = \mathcal{K}\varphi$.

Further, $\mathcal{H}$ and $\mathcal{K}$ are compatible. We have,

$$\lim_{n\to\infty}\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n, \mathcal{K}\mathcal{H}h_n, \mathcal{K}\mathcal{H}h_n\right) = 0, \text{ this implies that } \lim_{n\to\infty}\mathcal{H}\mathcal{K}h_n = \lim_{n\to\infty}\mathcal{K}\mathcal{H}h_n = \mathcal{K}\varphi.$$

From condition (3) of this theorem,, we have

$$\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n, \mathcal{H}h_n, \mathcal{H}h_n\right) \precsim \Delta \frac{\mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n, \mathcal{K}h_n, \mathcal{H}h_n\right) + \mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n, \mathcal{H}h_n\mathcal{K}h_n\right)}{\mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n, \mathcal{K}h_n, \mathcal{K}h_n\right) + \mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n, \mathcal{K}h_n\mathcal{H}h_n\right)} \mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n, \mathcal{K}h_n, \mathcal{K}h_n\right).$$

We get, $\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n, \mathcal{H}h_n, \mathcal{H}h_n\right) \precsim \Delta\mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n, \mathcal{K}h_n, \mathcal{K}h_n\right)$.

Taking limit n tends to infinity,

$$\mathcal{G}_b(\mathcal{K}\varphi, \varphi, \varphi) \precsim \Delta\mathcal{G}_b(\mathcal{K}\varphi, \varphi, \varphi),$$

this implies that $\mathcal{K}\varphi = \varphi$.

Again, consider

$$\mathcal{G}_b\left(\mathcal{H}h_n, \mathcal{H}\varphi, \mathcal{H}\varphi\right) \precsim \Delta \frac{\mathcal{G}_b\left(\mathcal{K}h_n, \mathcal{K}\varphi, \mathcal{H}\varphi\right) + \mathcal{G}_b\left(\mathcal{H}h_n, \mathcal{H}\varphi, \mathcal{K}\varphi\right)}{\mathcal{G}_b\left(\mathcal{K}h_n, \mathcal{K}\varphi, \mathcal{K}\varphi\right) + \mathcal{G}_b\left(\mathcal{H}h_n, \mathcal{K}\varphi, \mathcal{H}\varphi\right)} \mathcal{G}_b\left(\mathcal{K}h_n, \mathcal{K}\varphi, \mathcal{K}\varphi\right).$$

Taking limit n tends to infinity, we have $\mathcal{H}\varphi = \varphi$. Therefore, $\mathcal{H}\varphi = \mathcal{K}\varphi = \varphi$. Thus, φ is a common fixed point of the mappings $\mathcal{H}$ and $\mathcal{K}$.

Uniqueness: Consider θ as another common fixed point of $\mathcal{H}, \mathcal{K}$ with $\theta \neq \varphi$.

$$\mathcal{G}_b(\mathcal{H}\varphi, \mathcal{H}\theta, \mathcal{H}\theta) \precsim \Delta \frac{\mathcal{G}_b(\mathcal{K}\varphi, \mathcal{K}\theta, \mathcal{H}\theta) + \mathcal{G}_b(\mathcal{H}\varphi, \mathcal{H}\theta, \mathcal{K}\theta)}{\mathcal{G}_b(\mathcal{K}\varphi, \mathcal{K}\theta, \mathcal{K}\theta) + \mathcal{G}_b(\mathcal{H}\varphi, \mathcal{K}\theta, \mathcal{H}\theta)} \mathcal{G}_b(\mathcal{K}\varphi, \mathcal{K}\theta, \mathcal{K}\theta),$$

This implies $\mathcal{G}_b(\varphi, \theta, \theta) \prec \Delta\mathcal{G}_b(\varphi, \theta, \theta)$, but this is not possible. Therefore, $\varphi = \theta$. Hence, the uniqueness is proved.

Example 3.6 *Let* $\Upsilon = \mathbb{C}$ *and* $\mathcal{G}_b(h, j, k) = |h - j| + |j - k| + |k - h| + i|h + j + k|$, *for every* $h, j, k \in \Upsilon$. *It is a complete complex valued* $\mathcal{G}_b$*-metric with* $s = 2$. *Define* $\mathcal{H}(h) = \frac{h}{2}$ *and* $\mathcal{K}(h) = \frac{h}{3}$. *Here, we note that* $\mathcal{H}$ *is continuous with* $\mathcal{H}(\Upsilon) \subseteq \mathcal{K}(\Upsilon)$ *and hence compatible.*

$$\mathcal{G}_b(\mathcal{H}h, \mathcal{H}j, \mathcal{H}k) = \mathcal{G}_b\left(\frac{h}{2}, \frac{j}{2}, \frac{k}{2}\right) = \left|\frac{h}{2} - \frac{j}{2}\right| + \left|\frac{j}{2} - \frac{k}{2}\right| + \left|\frac{k}{2} - \frac{h}{2}\right| + i\left|\frac{h}{2} + \frac{j}{2} + \frac{k}{2}\right|,$$

$$\mathcal{G}_b(\mathcal{K}h, \mathcal{K}j, \mathcal{H}j) = \mathcal{G}_b\left(\frac{h}{3}, \frac{j}{3}, \frac{j}{2}\right) = \left|\frac{h}{3} - \frac{j}{3}\right| + \left|\frac{j}{3} - \frac{j}{2}\right| + \left|\frac{j}{2} - \frac{h}{3}\right| + i\left|\frac{h}{3} + \frac{j}{3} + \frac{j}{2}\right|,$$

$$\mathcal{G}_b(\mathcal{H}h, \mathcal{H}j, \mathcal{K}j) = \mathcal{G}_b\left(\frac{h}{2}, \frac{j}{2}, \frac{j}{3}\right) = \left|\frac{h}{2} - \frac{j}{2}\right| + \left|\frac{j}{2} - \frac{j}{3}\right| + \left|\frac{j}{3} - \frac{h}{2}\right| + i\left|\frac{h}{2} + \frac{j}{2} + \frac{j}{3}\right|,$$

$$\mathcal{G}_b(\mathcal{K}h, \mathcal{K}j, \mathcal{K}k) = \mathcal{G}_b\left(\frac{h}{3}, \frac{j}{3}, \frac{k}{3}\right) = \left|\frac{h}{3} - \frac{j}{3}\right| + \left|\frac{j}{3} - \frac{k}{3}\right| + \left|\frac{k}{3} - \frac{h}{3}\right| + i\left|\frac{h}{3} + \frac{j}{3} + \frac{k}{3}\right|,$$

$$\mathcal{G}_b(\mathcal{H}h, \mathcal{K}k, \mathcal{H}k) = \mathcal{G}_b\left(\frac{h}{2}, \frac{k}{3}, \frac{k}{2}\right) = \left|\frac{h}{2} - \frac{k}{3}\right| + \left|\frac{k}{3} - \frac{k}{2}\right| + \left|\frac{k}{2} - \frac{h}{2}\right| + i\left|\frac{h}{2} + \frac{k}{3} + \frac{k}{2}\right|,$$

$$\mathcal{G}_b(\mathcal{K}h, \mathcal{K}j, \mathcal{K}k) = \mathcal{G}_b\left(\frac{h}{3}, \frac{j}{3}, \frac{h}{3}\right) = \left|\frac{h}{3} - \frac{j}{3}\right| + \left|\frac{j}{3} - \frac{k}{3}\right| + \left|\frac{k}{3} - \frac{h}{3}\right| + i\left|\frac{h}{3} + \frac{j}{3} + \frac{k}{3}\right|.$$

We can choose Δ such that $\Delta s < 1$ and

$$\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{H}k) \precsim \Delta \frac{\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{H}j)+\mathcal{G}_b(\mathcal{H}h,\mathcal{H}j,\mathcal{K}j)}{\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{K}k)+\mathcal{G}_b(\mathcal{H}h,\mathcal{K}k,\mathcal{H}k)}\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{K}k).$$

By above theorem, mappings $\mathcal{H}$ and $\mathcal{K}$ has 0 as their common fixed point.

Theorem 3.7 *Consider* $(\Upsilon,\mathcal{G}_b)$ *be a complete complex valued* $\mathcal{G}_b$*-metric space with a real number* $s \geq 1$ *and* $\mathcal{H},\mathcal{K}$ *are self-mappings on* Υ *satisfying the following conditions, for every* $h,j,k \in Y$ *:*

1. $\mathcal{H}(\Upsilon) \subseteq \mathcal{K}(\Upsilon)$;
2. $\mathcal{H}$ or $\mathcal{K}$ is continuous function;
3.

$$\mathcal{G}_b(\mathcal{H}\mathcal{H},\mathcal{H}\mathcal{J},\mathcal{H}\mathcal{K}) \precsim \begin{bmatrix} \Delta \frac{\mathcal{G}_b(\mathcal{H}j,\mathcal{K}j,\mathcal{K}j)+\mathcal{G}_b(\mathcal{H}h,\mathcal{K}h,\mathcal{K}j)}{\mathcal{G}_b(\mathcal{H}j,\mathcal{K}k,\mathcal{K}k)+\mathcal{G}_b(\mathcal{H}h,\mathcal{K}h,\mathcal{K}k)}\mathcal{G}_b(\mathcal{K}j,\mathcal{K}j,\mathcal{K}k) \\ +\nabla \frac{\mathcal{G}_b(\mathcal{K}k,\mathcal{H}h,\mathcal{K}j)+G_b(\mathcal{K}j,\mathcal{K}j,\mathcal{H}h)}{\mathcal{G}_b(\mathcal{H}j,\mathcal{K}j,\mathcal{H}h)+\mathcal{G}_b(\mathcal{H}j,\mathcal{H}j,\mathcal{K}k)}\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{H}j) \\ +\Theta \frac{\mathcal{G}_b(\mathcal{H}h,\mathcal{K}k,\mathcal{K}k)+\mathcal{G}_b(\mathcal{K}k,\mathcal{K}j,\mathcal{H}k)}{\mathcal{G}_b(\mathcal{H}h,\mathcal{K}j,\mathcal{K}j)+\mathcal{G}_b(\mathcal{K}k,\mathcal{K}k,\mathcal{H}j)}\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{H}k) \\ +\Omega \frac{\mathcal{G}_b(\mathcal{K}k,\mathcal{H}h,\mathcal{K}j)+\mathcal{G}_b(\mathcal{K}j,\mathcal{K}j,\mathcal{K}k)}{\mathcal{G}_b(\mathcal{H}j,\mathcal{K}j,\mathcal{H}h)+\mathcal{G}_b(\mathcal{H}j,\mathcal{H}j,\mathcal{H}k)}\mathcal{G}_b(\mathcal{K}h,\mathcal{K}j,\mathcal{K}j) \end{bmatrix}$$

with $\Delta,\nabla,\Theta,\Omega \geq 0$ with $s^2(\Delta+\nabla+\Theta+\Omega) < \frac{1}{3}$.

Then, $\mathcal{H}$ and $\mathcal{K}$ have a unique common fixed point in Υ, provided $\mathcal{H}$ and $\mathcal{K}$ are compatible.

Proof. Consider a sequence $\{h_n\} \in \Upsilon$ with an initial point h_0. We can choose h_1 such that $\mathcal{H}h_0 = \mathcal{K}h_1$. Choose a sequence $\{h_{n+1}\}$ such that $\varphi_n = \mathcal{H}h_n = \mathcal{K}h_{n+1}, n = 0,1,2\ldots$ and using condition (3) of this theorem,

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right) \precsim \begin{bmatrix} \Delta \frac{\mathcal{G}_b\left(\mathcal{H}h_{n+1},\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right)+\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_n,\mathcal{K}h_{n+1}\right)}{\mathcal{G}_b\left(\mathcal{H}h_{n+1},\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right)+\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_n,\mathcal{K}h_{n+1}\right)}\mathcal{G}_b\left(\mathcal{K}h_{n+1},\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right) \\ +\nabla \frac{\mathcal{G}_b\left(\mathcal{K}h_{n+1},\mathcal{H}h_n,\mathcal{K}h_{n+1}\right)+\mathcal{G}_b\left(\mathcal{K}h_{n+1},\mathcal{K}h_{n+1},\mathcal{H}h_{n+1}\right)}{\mathcal{G}_b\left(\mathcal{H}h_{n+1},\mathcal{K}h_{n+1},\mathcal{H}h_n\right)+\mathcal{G}_b\left(\mathcal{H}h_{n+1},\mathcal{H}h_{n+1},\mathcal{K}h_{n+1}\right)}\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{H}h_{n+1}\right) \\ +\Theta \frac{\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right)+\mathcal{G}_b\left(\mathcal{K}h_{n+1},\mathcal{K}h_{n+1},\mathcal{H}h_n\right)}{\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right)+\mathcal{G}_b\left(\mathcal{K}h_{n+1}\cdot\mathcal{K}h_{n+1},\mathcal{H}h_{n+1}\right)}\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{H}h_{n+1}\right) \\ +\Omega \frac{\mathcal{G}_b\left(\mathcal{K}h_{n+1},\mathcal{H}h_n,\mathcal{K}h_{n+1}\right)+\mathcal{G}_b\left(\mathcal{K}h_{n+1},\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right)}{\mathcal{G}_b\left(\mathcal{H}h_{n+1},\mathcal{K}h_{n+1},\mathcal{H}h_n\right)+\mathcal{G}_b\left(\mathcal{H}h_{n+1},\mathcal{H}h_{n+1},\mathcal{H}h_{n+1}\right)}\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_{n+1}\right) \end{bmatrix},$$

So, we get

$$\mathcal{G}_b\left(\varphi_n,\varphi_{n+1},\varphi_{n+1}\right) \precsim (\nabla+\Omega)\mathcal{G}_b\left(\varphi_{n-1},\varphi_n,\varphi_{n+1}\right). \tag{3.6}$$

By using rectangular inequality and preposition (2.3), we have

$$\begin{aligned} \mathcal{G}_b\left(\varphi_{n-1},\varphi_n,\varphi_{n+1}\right) &\precsim s\left\{\mathcal{G}_b\left(\varphi_{n-1},\varphi_n,\varphi_n\right)+\mathcal{G}_b\left(\varphi_n,\varphi_n,\varphi_{n+1}\right)\right\}, \\ &\precsim s\left\{\mathcal{G}_b\left(\varphi_{n-1},\varphi_n,\varphi_n\right)+2s\mathcal{G}_b\left(\varphi_n,\varphi_{n+1},\varphi_{n+1}\right)\right\}. \end{aligned} \tag{3.7}$$

Using equation (3.7) in (3.6), we get

$$\mathcal{G}_b\left(\varphi_{n'}\varphi_{n+1},\varphi_{n+1}\right) \precsim (\nabla+\Omega)s\left\{\mathcal{G}_b\left(\varphi_{n-1},\varphi_{n'},\varphi_n\right)+2s\mathcal{G}_b\left(\varphi_{n'}\varphi_{n+1},\varphi_{n+1}\right)\right\},$$

$$\mathcal{G}_b\left(\varphi_{n'}\varphi_{n+1},\varphi_{n+1}\right) \precsim \frac{(\nabla+\Omega)s}{1-2(\nabla+\Omega)s^2}\mathcal{G}_b\left(\varphi_{n-1},\varphi_{n'},\varphi_n\right),$$

$$\mathcal{G}_b\left(\varphi_{n'},\varphi_{n+1},\varphi_{n+1}\right) \precsim \ell\mathcal{G}_b\left(\varphi_{n-1},\varphi_{n'},\varphi_n\right).$$

Continuing like this,

$$\mathcal{G}_b\left(\varphi_{n'}\varphi_{n+1},\varphi_{n+1}\right) \precsim \ell^n\mathcal{G}_b\left(\varphi_{n-1},\varphi_{n'},\varphi_n\right).$$

Using the same steps as used in Theorem 3.1, one can show that $\{\varphi_n\}$ is a Cauchy sequence. By completeness property of complex valued $\mathcal{G}_b$-metric space, there is a sequence which is convergent to a point $\varphi \in \Upsilon$ such that $\lim_{n\to\infty}\{\varphi_n\}=\varphi$ and $\lim_{n\to\infty}\varphi_n = \lim_{n\to\infty}\mathcal{H}h_n = \lim_{n\to\infty}\mathcal{K}h_{n+1}=\varphi$. As $\mathcal{H}$ or $\mathcal{K}$ is continuous function, suppose $\mathcal{H}$ is continuous, then $\lim_{n\to\infty}\mathcal{K}\mathcal{H}h_n = \lim_{n\to\infty}\mathcal{K}\mathcal{K}h_{n+1}=\mathcal{K}\varphi$.

Further, $\mathcal{H}$ and $\mathcal{K}$ are compatible. Thus,

$\lim_{n\to\infty}\mathcal{G}_b\left(\mathcal{H}\mathcal{X}h_n,\mathcal{K}\mathcal{H}h_n,\mathcal{K}\mathcal{H}h_n\right)=0$, this implies that $\lim_{n\to\infty}\mathcal{H}\mathcal{K}h_n = \lim_{n\to\infty}\mathcal{K}\mathcal{H}h_n=\mathcal{K}\varphi$.

By using condition (3) of this theorem,, we have

$$\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{H}h_n,\mathcal{H}h_n\right) \precsim \begin{bmatrix} \Delta\dfrac{\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right)+\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right)}{\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right)+\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}\mathcal{K}h_n,\mathcal{K}h_n\right)}\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right) \\ +\nabla\dfrac{\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_{n+1},\mathcal{K}h_n,\mathcal{K}h_n\right)+\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{H}h_n\right)}{\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{H}\mathcal{K}h_n,\mathcal{H}h_n\right)+\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{H}h_n\right)}\mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n,\mathcal{K}h_{n+1},\mathcal{K}h_n\right) \\ +\Theta\dfrac{\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right)+\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{H}\mathcal{K}h_n\right)}{\mathcal{G}_b\left(\mathcal{H}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right)+\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{H}h_n\right)}\mathcal{G}_b\left(\mathcal{K}\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{H}h_n\right) \\ +\Omega\dfrac{\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{H}\mathcal{K}h_n,\mathcal{K}h_n\right)+\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}h_n,\mathcal{K}h_n\right)}{\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}h_n,\mathcal{H}\mathcal{K}h_n\right)+\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}h_n,\mathcal{H}h_n\right)}\mathcal{G}_b\left(\mathcal{K}h_nh_n,\mathcal{K}h_n,\mathcal{K}h_n\right) \end{bmatrix}.$$

Taking the limit n tends to infinity,

$$\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi) \precsim \begin{bmatrix} \Delta\dfrac{\mathcal{G}_b(\varphi,\varphi,\varphi)+\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\varphi)}{\mathcal{G}_b(\varphi,\varphi,\varphi)+\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\varphi)}\mathcal{G}_b(\varphi,\varphi,\varphi) \\ +\nabla\dfrac{\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi)+\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi)}{\mathcal{G}_b(\varphi,\mathcal{K}\varphi,\varphi)+\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi)}\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi) \\ +\Theta\dfrac{\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi)+\mathcal{G}_b(\varphi,\varphi,\mathcal{K}\varphi)}{\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi)+\mathcal{G}_b(\varphi,\varphi,\varphi)}\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi) \\ +\Omega\dfrac{\mathcal{G}_b(\varphi,\mathcal{K}\varphi,\varphi)+\mathcal{G}_b(\varphi,\varphi,\varphi)}{\mathcal{G}_b(\varphi,\varphi,\mathcal{K}\varphi)+\mathcal{G}_b(\varphi,\varphi,\varphi)}\mathcal{G}_b(\mathcal{K}\varphi,\varphi,\varphi) \end{bmatrix},$$

This is not possible as $\nabla+\Theta+\Omega<\dfrac{1}{3s^2}$. Therefore, we have, $\mathcal{K}\varphi=\varphi$.

Again, from condition (3) of this theorem, we have

$$\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{H}\varphi,\mathcal{H}\varphi\right)\precsim\begin{bmatrix}\Delta\dfrac{\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)+\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}\varphi,\mathcal{K}\varphi\right)}{\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)+\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}\varphi,\mathcal{K}\varphi\right)}\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)\\+\nabla\dfrac{\mathcal{G}_b\left(\mathcal{K}\varphi,\mathcal{H}h_n,\mathcal{K}\varphi\right)+\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\mathcal{H}\varphi)}{\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\varphi,\mathcal{H}\varphi)+\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi)}\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}\varphi,\mathcal{H}\varphi\right)\\+\Theta\dfrac{\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}\varphi,\mathcal{K}\varphi\right)+\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\mathcal{H}\mathcal{K}\varphi)}{\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}\varphi,\mathcal{K}\varphi\right)+\mathcal{G}_b(\mathcal{K}\varphi\cdot\mathcal{K}\varphi,\mathcal{H}\varphi)}\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}\varphi,\mathcal{H}\varphi\right)\\+\Omega\dfrac{\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{H}\varphi,\mathcal{K}\varphi)+\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)}{\mathcal{G}_b\left(\mathcal{H}\varphi,\mathcal{K}\varphi,\mathcal{H}h_n\right)+\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi)}\mathcal{G}_b\left(\mathcal{K}h_n,\mathcal{K}\varphi,\mathcal{K}\varphi\right)\end{bmatrix}.$$

Taking the limit n tends to infinity,

$$\mathcal{G}_b(\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi)\precsim(\nabla+\Theta)\mathcal{G}_b(\varphi,\varphi,\mathcal{H}\varphi)\precsim 2s(\nabla+\Theta)\mathcal{G}_b(\varphi,\mathcal{H}\varphi,\mathcal{H}\varphi),$$

this is again contradiction as $(\nabla+\Theta)<\dfrac{1}{3s^2}$. Therefore, we have $\mathcal{H}\varphi=\mathcal{K}\varphi=\varphi$. Thus, φ is a common fixed point of $\mathcal{H}$ and $\mathcal{K}$.

Uniqueness: Consider θ as another common fixed point of $\mathcal{H},\mathcal{K}$ with $\theta\neq\varphi$.

$$\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{H}\theta,\mathcal{H}\theta)\precsim\Delta\begin{bmatrix}\Delta\dfrac{\mathcal{G}_b(\mathcal{H}\theta,\mathcal{K}\theta,\mathcal{K}\theta)+\mathcal{G}_b\left(\mathcal{H}h_n,\mathcal{K}\varphi,\mathcal{K}\theta\right)}{\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)+\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)}\mathcal{G}_b(\mathcal{K}\theta,\mathcal{K}\theta,\mathcal{K}\theta)\\+\nabla\dfrac{\mathcal{G}_b(\mathcal{K}\theta,\mathcal{H}\varphi,\mathcal{K}\theta)+\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\mathcal{H}\varphi)}{\mathcal{G}_b(\mathcal{H}\theta,\mathcal{K}\theta,\mathcal{H}\varphi)+\mathcal{G}_b(\mathcal{H}\theta,\mathcal{H}\theta,\mathcal{H}\theta)}\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\theta,\mathcal{H}\theta)\\+\Theta\dfrac{\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\theta,\mathcal{K}\theta)+\mathcal{G}_b(\mathcal{K}\theta,\mathcal{K}\theta,\mathcal{H}\mathcal{K}\theta)}{\mathcal{G}_b(\mathcal{H}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)+\mathcal{G}_b(\mathcal{K}\varphi\cdot\mathcal{K}\varphi,\mathcal{H}\varphi)}\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\theta,\mathcal{H}\theta)\\+\Omega\dfrac{\mathcal{G}_b(\mathcal{K}\theta,\mathcal{H}\varphi,\mathcal{K}\varphi)+\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\varphi,\mathcal{K}\varphi)}{\mathcal{G}_b(\mathcal{H}\theta,\mathcal{K}\theta,\mathcal{H}\varphi)+\mathcal{G}_b(\mathcal{H}\theta,\mathcal{H}\theta,\mathcal{H}\theta)}\mathcal{G}_b(\mathcal{K}\varphi,\mathcal{K}\theta,\mathcal{K}\theta)\end{bmatrix}.$$

$\mathcal{G}_b(\varphi,\theta,\theta)\precsim(\nabla+\Theta+\Omega)\mathcal{G}_b(\varphi,\theta,\theta)$, which is not possible. Therefore, $\varphi=\theta$.

Example 3.8 *Let* $\Upsilon=\mathbb{C}$ *and* $\mathcal{G}_b(h,j,k)=|h-j|+|j-k|+|k-h|$ *for every* $h,j,k\in\mathbb{C}$. *It is a complete complex valued* $\mathcal{G}_b$*-metric. Define* $\mathcal{H}(h)=\dfrac{h}{2}$ *and* $\mathcal{K}(h)=\dfrac{h}{4}$. *Here, we note that* $\mathcal{H}$ *is continuous with* $\mathcal{H}(\Upsilon)\subseteq\mathcal{K}(\Upsilon)$ *and hence compatible.*

By taking $\Delta=\nabla=\Theta=\Omega=\dfrac{1}{16}$ and $s=1$ condition $s^2(\Delta+\nabla+\Theta+\Omega)<\dfrac{1}{3}$ is satisfied. Also, condition (3) of Theorem 3.7 is satisfied. Hence, by above theorem, mappings $\mathcal{H}$ and $\mathcal{K}$ has 0 as their common fixed point.

Conclusions and Future Works

In this paper, some common fixed point theorems of compatibility by using rational inequality incomplete complex valued $\mathcal{G}_b$-metric space are proved. Our results will help new researches to obtain new results in complex valued $\mathcal{G}_b$-metric space and in other extensions of complex valued metric space.

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