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Solution techniques for time dependent availability in complex and repairable system: a comparative study

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Abstract

The paper elucidates two distinct techniques for conducting a comparative study aimed at ascertaining the transient availability of a repairable system. The use of Markov modeling forms the basis of this investigation strategy. The set of differential equations in the current study is formulated, and they are then solved using the Laplace transform method and the matrix method, respectively. To help with a clearer understanding of the system, these approaches are subjected to a thorough evaluation and comparative study. Availability guarantees that critical processes, failure analysis and services continue to run smoothly, which is essential for business continuity. In sectors like finance, public health, and telecommunications, this is especially crucial. Productivity and availability are directly related. Systems that are continuously available allow workers and resources to be used to their greatest potential, increasing productivity and efficiency.

Key words and phrases: Transient availability, Differential equations, Markov modeling, Failure analysis, Public health

Mathematics Subject Classification (2010): 97M10

1. Introduction and Literature Review

Engineering systems are a complicated structure that is closely tied to the growth and advancement of modern civilization. Systems for engineering are carefully created to guarantee that they will function consistently for the duration of their expected service life. These systems include a broad range

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of areas, such as advanced machinery, vital infrastructure networks, and industrial processes. Given the inherent danger of system degradation, it is critical to carefully monitor how system availability changes over time. Their efficacy and efficiency have a significant impact on people's everyday lives and the economy. However, it is crucial to recognize that engineering systems are prone to deterioration over time due to the influence of external influences and operating demands. This deterioration manifests as a gradual decline in their overall performance, or, in more severe instances, as complete system failure. In practice, a system or unit can traverse an array of transitional phases, each marked by a distinctive probability of occurrence, lying on the spectrum between optimal functionality and complete failure.

It is important to remember that severe breakdowns in production systems can have disastrous effects that extend well beyond the economy and pose serious risks to human welfare and the environment. Consequently, it is essential to proactively eliminate significant failure modes as soon as possible, ideally in the design stage, as doing so is far less expensive than dealing with these problems in the manufacturing or testing phases.

The concepts of availability and reliability are crucial in the domains of technology, engineering, and research. Researchers frequently examine and measure these factors in order to determine how well systems, products, or services operate and are reliable. Researchers in these domains strive to comprehend the constraints and possible modes of failure of systems, devise tactics to augment reliability and availability, and use these conclusions to enhance the comprehensive efficacy and user contentment with the commodities or amenities under evaluation. The concept of redundancy is introduced for improving the reliability in the systems. The reliability of a standby system with repair has been shown by Jacob Cherian et al. [1]. The machine repair problem with spares, reneging, additional repairman, and two types of failure was examined by Jain et al. [2]. Lisnianski et al. [3] introduced a method for reliability optimization for system with time redundancy. Csenki [4] investigated performance measures for semi-markov dependability models and mentioned how these measures can be determined as solutions of certain systems using continuous time parameter. Jain and Maheshwari [5] provided a transient study of a redundant repairable system with additional repair. Reliability research in both series and parallel configurations has been widely recognized as a standard approach that facilitates the planning and organization of complex repairable systems' operating and maintenance operations (Barabady et al. [6]). Li and Ni [7] developed a maximum likelihood estimation technique that takes into account the distinction between flawless and imperfect maintenance work-orders in order to discover estimated parameters based on the operational data of manufacturing systems. A comprehensive approach was presented by Doguc and Marquez [8], which makes use of historical information about the system that is to be represented as a BN and offers effective methods for automating the creation of the BN model and estimating the reliability of the system. El-Damcese and Temraz [9] proposed a technique for determining repairable parallel system availability using Markov models. Chybowski [10] used a Boolean function approach to study the reliability behavior of a complicated engineering system that links three subsystems in parallel redundancy. A computed and implemented approach based on the Disjoint Sum of Product (DSOP) algorithm was given by Bourezg and Meglouli [11]. In order to assess reliability indices and the costs of various substation configurations, the method was used to estimate the reliability expression of a substation. Earth pressure balance tunnel boring machines (EPB-TBMs) used in urban tunneling projects were covered by Khoshalan et al. [12]. It has been demonstrated that the mechanical subsystem with the highest failure frequency has the lowest reliability and maintainability after the reliability and maintainability functions for each subsystem were calculated. James Li [13] compared active redundancy against standby redundancy using a markov modeling from a reliability point of view. A repairable system with M major components, S spare components, and a repairman was studied by Yang and Tsao [14]. They examined the mean-time-to-failure (MTTF) and reliability function based on the Laplace transform approach, and they utilized the matrix-analytic method to compute the steady state availability. The mining sector uses equipment including haul trucks, loaders, dozers, shovel-dumpers, and draglines. These are more complicated, repairable systems that are employed in very difficult working conditions, according to Saurabh Vashistha et al. [15]. RAM study of the material hauling system in an earth pressure balance tunnel boring machine (EPB-TBM) was conducted by Ahmadi et al. [16]. However, increasing the system's availability might contribute to its increased reliability and maintainability. Ensuring the availability of complex systems at an elevated level should need the design of systems with better availability, system components, or both. When using these tactics to increase the system's availability or reliability, decision-makers must carefully take into account both the real business needs and the quality criteria. Therefore, while keeping in mind the competitive environment, the behavior of such systems may be examined in terms of their availability, reliability, and maintainability.

Additionally, earth pressure balance tunnel boring machines were explored by Agrawal et al. [17]. His research aims to produce a markov diagram showing the different EPBTBM subsystems. It is possible to generate a steady state availability expression with a constant failure and repair rate. In order to increase the system's overall availability and reliability and that of its subsystems in particular—a preventative maintenance (PM) strategy has been established. In order to assess the Steady State Availability of the system, Patil et al. [18] presented a methodology for Time-Between-Failure (TBF) and Time-To-Repair (TTR) data analysis, combined with Markov chains. They also identified the critical sub-systems from reliability, maintainability, and availability point of view. In a case study, Koohsari et al. [19] provided the findings of a critical examination of the RAM of Earth pressure balance machines (EPBMs). Their research also showed that overall availability might be improved by anticipating the right maintenance and making plans that are appropriate. Antosz et al. [20] focused on the availability and reliability of engineering systems for contemporary businesses, particularly in light of industry demands and difficulties. RAM engineering is a well-established topic of study that has expanded into several other engineering specialties in addition to software and mechanical engineering. In this context, RAM analysis is one of the top methods for increasing project utilization in oil and mining engineering because of its machinery-based character. A thorough analysis of the many statistical methods that have been used for fault prediction and reliability from both theoretical and practical viewpoints was proposed by Odeyar et al. [21]. Eliwa et al. [22] analyzed the reliability of constant, partially accelerated life tests utilizing progressive first failure type-II censored data based on the Lomax distribution. In contrast, Alburaikan et al. [23] examined mathematical models for evaluating reliability.

High availability is desirable in many applications, especially essential ones where downtime might have major effects, such as data centers, power plants, medical equipment, and transportation systems. Reliability engineering solutions aim to improve and maintain high availability levels in such systems by minimizing downtime and addressing errors effectively. Two different methods are used, namely in the field of reliability engineering, to evaluate and ascertain the availability of systems: the matrix technique and the Laplace transform method. The study and solution of dynamic systems, such as continuously changing mechanical and electrical circuits, are the primary applications of the Laplace transform approach. It is a mathematical technique that may be applied to systems that are defined by differential equations to ascertain how systems respond over time. However, the matrix technique is used to look at and figure out a system's steady-state availability, especially when it comes to reliability study. It focuses mainly on static or quasi-static systems, where the primary issue is the system's tendency to operate at a specific moment. In this case, the Laplace transform approach is contrasted with the matrix method to determine the availability of the complex system in a time dependent condition.

2. Mathematical Understanding of Availability

Availability is a measure used in reliability engineering to assess the operational performance and reliability of a system, part, or piece of machinery. It gauges how well a system is functioning and ready to perform its assigned function when called upon. It is the likelihood that the system will function successfully at any given moment in time when operational, active repair, logistical, and administrative time are all included in the overall period.

$$Availability = \frac{Operable\ time}{Mission\ time}$$

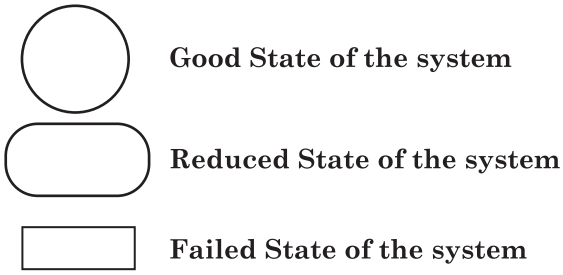
$$Availability = \frac{MTBF}{MTBF + MTTR}$$

MTBF is mean time between failures and MTTR is mean time to repair.

Availability $Av(t)$ expresses the probability of being completely operable at any given time.

The interval availability $Av_i(t)$ for the interval $[0, T]$ is defined by

$$Av_i(t) = \frac{1}{T}\int_0^T Av(t)\mathrm{d}x$$



3. System Description

3.1. The System

The complex system consists of following units in series.

Q: consists of one unit system subjected to major failures only.
R: consists of one unit system subjected to major failures only.
S_i: ($i = 1, 2, 3$) these are three different units.
U: consists of one unit system subjected to major failures only.
V: consists of one unit system subjected to major failures only.
W: consists of one unit system and works in reduced capacity.

The following notations and assumptions are employed for the purpose of mathematical analysis of performance of the complex system.

3.2. Notations

Q, R, S_i, U, V, W: represent good working states of the complex system. ($i = 1, 2, 3$).
q, r, s_i, u, v: represent failed states of the complex system. ($i = 1, 2, 3$).
w: represent reduced state of the complex system.
α_i: respective mean constant failure rates of units S_i, ($i = 1, 2, 3$).
α_4: mean constant failure rate of unit W.
α_j: respective mean constant failure rates of units $Q, R, U,$ and V, ($j = 5, 6, 7, 8$).
β_i: respective mean constant repair rates of units S_i, ($i = 1, 2, 3$).
β_4: mean constant repair rate of unit W.
β_j: respective mean constant repair rates of units $Q, R, U,$ and V, ($j = 5, 6, 7, 8$).
$P_i(t)$: state probabilities that the system is in i^{th} state at time t.
s: Laplace-transform parameter.

3.3. Assumptions

- All the units are initially operating and are in good state.
- Each unit has two states: good and failed state. Unit W works at reduced capacity on transit to degraded state.

- Each unit is as good as new after repair.
- The failure rates and repair rates of all units are taken constant.
- Failure and repair events are statistically independent.
- Whenever a unit fails its repair begins immediately.

4. Formulation of Transition Diagram

Notations, symbols and assumptions are employed for representing the states of the subsystems. Figure 1 shows the transition diagram of the complex system consisting of six components with one component working with reduced capacity. The transition diagram defines the transitions of subsystem's one state to another.

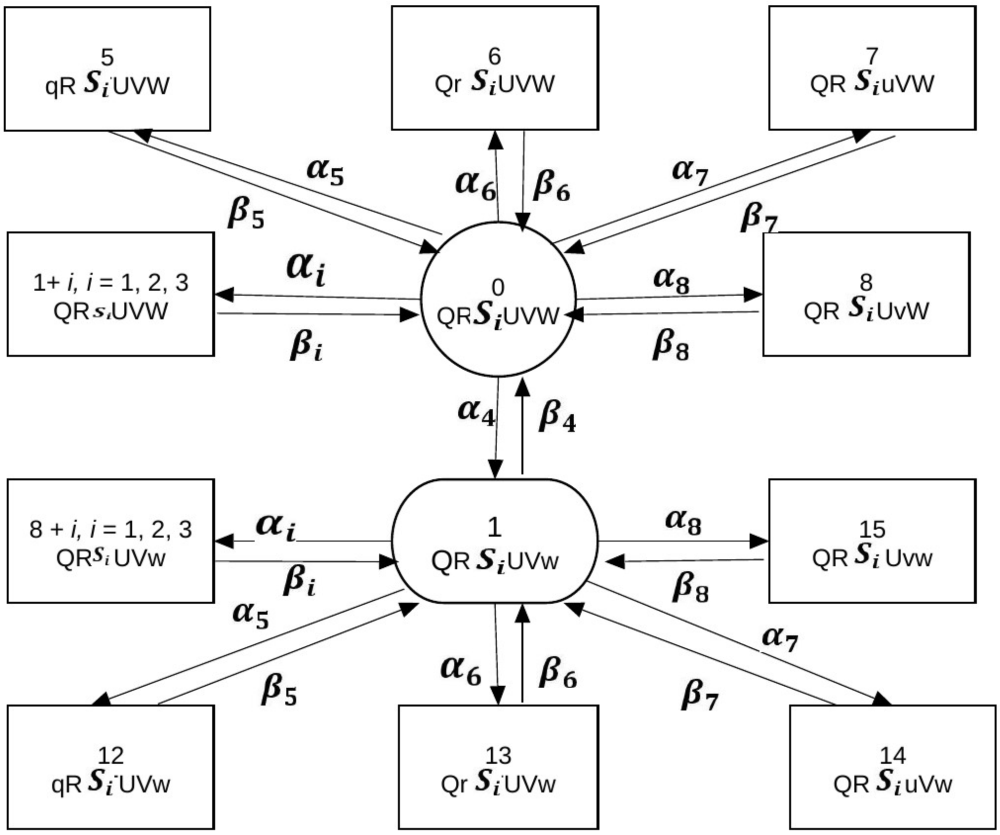


FIGURE 1: TRANSITION DIAGRAM

Figure 1: Transition Diagram.

5. Mathematical Model Using Markov Approach

5.1. Markov Approach

The system's transient availability is estimated using the Markov technique. This method is used in situations where the rates of failure and repair remain constant. A process known as a Markov process characterizes the behavior of the system state if the probability rule of its future state of existence depends solely on the state it is in and not on how the system obtained there. The future in a Markov approach is not affected by the past. It is an effective method for determining the availability ($Av(t)$) of the repairable system, considering constant failure and repair rates. Hence, the basis of Markov analysis is mathematical modeling, where the failure states depend only on the current state at that particular moment.

5.2. Transient State

The laws of probability and transition diagram are used, and equations (1)–(16) are developed. The differential-difference equations obtained from the state transition diagram using Mnemonic rule at time $(t + \Delta t)$ are:

$$\begin{aligned} P_0(t+\Delta t) = {} & [1-\alpha_1\Delta t-\alpha_2\Delta t-\alpha_3\Delta t-\alpha_4\Delta t-\alpha_5\Delta t-\alpha_6\Delta t-\alpha_7\Delta t-\alpha_8\Delta t]P_0(t) \\ & + P_1(t)\beta_4\Delta t + P_2(t)\beta_1\Delta t + P_3(t)\beta_2\Delta t + P_4(t)\beta_3\Delta t + P_5(t)\beta_5\Delta t \\ & + P_6(t)\beta_6\Delta t + P_7(t)\beta_7\Delta t + P_8(t)\beta_8\Delta t \end{aligned}$$

$$\begin{aligned} P_0(t+\Delta t) - P_0(t) = {} & [-\alpha_1\Delta t-\alpha_2\Delta t-\alpha_3\Delta t-\alpha_4\Delta t-\alpha_5\Delta t-\alpha_6\Delta t-\alpha_7\Delta t-\alpha_8\Delta t]P_0(t) \\ & + P_1(t)\beta_4\Delta t + P_2(t)\beta_1\Delta t + P_3(t)\beta_2\Delta t + P_4(t)\beta_3\Delta t + P_5(t)\beta_5\Delta t \\ & + P_6(t)\beta_6\Delta t + P_7(t)\beta_7\Delta t + P_8(t)\beta_8\Delta t \end{aligned}$$

Dividing both sides by Δt, we get:

$$\begin{aligned} \frac{P_0(t+\Delta t)-P_0(t)}{\Delta t} = {} & [-\alpha_1-\alpha_2-\alpha_3-\alpha_4-\alpha_5-\alpha_6-\alpha_7-\alpha_8]P_0(t) + P_1(t)\beta_4 + P_2(t)\beta_1 \\ & + P_3(t)\beta_2 + P_4(t)\beta_3 + P_5(t)\beta_5 + P_6(t)\beta_6 + P_7(t)\beta_7 + P_8(t)\beta_8 \end{aligned}$$

Taking $\Delta t \to 0$

$$\begin{aligned} P_0'(t) = {} & -(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)P_0(t) + P_1(t)\beta_4 + P_2(t)\beta_1 + P_3(t)\beta_2 + P_4(t)\beta_3 \\ & + P_5(t)\beta_5 + P_6(t)\beta_6 + P_7(t)\beta_7 + P_8(t)\beta_8 \end{aligned}$$

$$\begin{aligned} P_0'(t) + (\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8)P_0(t) = {} & \beta_4P_1(t) + \beta_1P_2(t) + \beta_2P_3(t) + \beta_3P_4(t) \\ & + \beta_5P_5(t) + \beta_6P_6(t) + \beta_7P_7(t) + \beta_8P_8(t) \end{aligned}$$

and letting $(\alpha_1+\alpha_2+\alpha_3+\alpha_4+\alpha_5+\alpha_6+\alpha_7+\alpha_8) = C_0$, to obtain the equation:

$$P_0'(t) + C_0P_0(t) = \beta_4P_1(t) + \beta_1P_2(t) + \beta_2P_3(t) + \beta_3P_4(t) + \beta_5P_5(t) + \beta_6P_6(t) + \beta_7P_7(t) + \beta_8P_8(t) \tag{1}$$

Similarly,

$$\begin{aligned} P_1'(t) + (\alpha_1+\alpha_2+\alpha_3+\alpha_5+\alpha_6+\alpha_7+\alpha_8+\beta_4)P_1(t) = {} & \alpha_4P_0(t) + \beta_1P_9(t) + \beta_2P_{10}(t) + \beta_3P_{11}(t) + \beta_5P_{12}(t) \\ & + \beta_6P_{13}(t) + \beta_7P_{14}(t) + \beta_8P_{15}(t) \end{aligned}$$

and letting $(\alpha_1+\alpha_2+\alpha_3+\alpha_5+\alpha_6+\alpha_7+\alpha_8+\beta_4) = C_1$, to obtain the equation:

$$P_1'(t) + C_1P_1(t) = \alpha_4P_0(t) + \beta_1P_9(t) + \beta_2P_{10}(t) + \beta_3P_{11}(t) + \beta_5P_{12}(t) + \beta_6P_{13}(t) + \beta_7P_{14}(t) + \beta_8P_{15}(t) \tag{2}$$

$$P_2'(t) + \beta_1P_2(t) = \alpha_1P_0(t) \tag{3}$$

$$P_3'(t) + \beta_2P_3(t) = \alpha_2P_0(t) \tag{4}$$

$$P_4'(t) + \beta_3P_4(t) = \alpha_3P_0(t) \tag{5}$$

$$P_5'(t) + \beta_5P_5(t) = \alpha_5P_0(t) \tag{6}$$

$$P_6'(t) + \beta_6P_6(t) = \alpha_6P_0(t) \tag{7}$$

$$P_7'(t) + \beta_7P_7(t) = \alpha_7P_0(t) \tag{8}$$

$$P_8'(t) + \beta_8P_8(t) = \alpha_8P_0(t) \tag{9}$$

$$P_9'(t) + \beta_1P_9(t) = \alpha_1P_1(t) \tag{10}$$

$$P_{10}'(t) + \beta_2P_{10}(t) = \alpha_2P_1(t) \tag{11}$$

$$P_{11}'(t) + \beta_3P_{11}(t) = \alpha_3P_1(t) \tag{12}$$

$$P'_{12}(t) + \beta_5 P_{12}(t) = \alpha_5 P_1(t) \tag{13}$$

$$P'_{13}(t) + \beta_6 P_{13}(t) = \alpha_6 P_1(t) \tag{14}$$

$$P'_{14}(t) + \beta_7 P_{14}(t) = \alpha_7 P_1(t) \tag{15}$$

$$P'_{15}(t) + \beta_8 P_{15}(t) = \alpha_8 P_1(t) \tag{16}$$

With initial conditions at time $t = 0$

$$\begin{aligned} P_i(t) &= 1 \; for \;\; i = 0 \\ &= 0 \; for \;\; i \neq 0 \end{aligned}$$

6. Solution of Differential Equations

6.1. Laplace Transform Method

When examining systems with intricate reliability features, the Laplace transform approach is very helpful since it offers a mathematical foundation for simplifying the issue. It enables you to evaluate system availability under various circumstances by accounting for elements like repair rates, failure rates, and other relevant metrics.

Solving above equations after taking Laplace transforms of equations (1)–(16), the following Laplace transforms of state probabilities are obtained:

$$P_1(s) = XP_0(s)$$

$$P_2(s) = (\frac{\alpha_1}{s+\beta_1})P_0(s)$$

$$P_3(s) = (\frac{\alpha_2}{s+\beta_2})P_0(s)$$

$$P_4(s) = (\frac{\alpha_3}{s+\beta_3})P_0(s)$$

$$P_5(s) = (\frac{\alpha_5}{s+\beta_5})P_0(s)$$

$$P_6(s) = (\frac{\alpha_6}{s+\beta_6})P_0(s)$$

$$P_7(s) = (\frac{\alpha_7}{s+\beta_7})P_0(s)$$

$$P_8(s) = (\frac{\alpha_8}{s+\beta_8})P_0(s)$$

$$P_9(s) = X(\frac{\alpha_1}{s+\beta_1})P_0(s)$$

$$P_{10}(s) = X(\frac{\alpha_2}{s+\beta_2})P_0(s)$$

$$P_{11}(s) = X(\frac{\alpha_3}{s+\beta_3})P_0(s)$$

$$P_{12}(s) = X(\frac{\alpha_5}{s+\beta_5})P_0(s)$$

$$P_{13}(s) = X(\frac{\alpha_6}{s+\beta_6})P_0(s)$$

$$P_{14}(s) = X(\frac{\alpha_7}{s+\beta_7})P_0(s)$$

$$P_{15}(s) = X(\frac{\alpha_8}{s+\beta_8})P_0(s)$$

where

$$X = \frac{\alpha_4}{s + C_1 - \sum_{i=1}^{3}\delta_i\beta_i - \sum_{j=5}^{8}\delta_j\beta_j} \tag{17}$$

Taking Laplace transform of equation (1), using in initial conditions and relations (17), we get:

$$P_0(s) = \left[s + C_0 - \left(X\beta_1 + \sum_{i=1}^{3}\beta_i\left(\frac{\alpha_i}{s+\beta_i}\right) - \sum_{i=5}^{8}\left(\frac{\alpha_i}{s+\beta_i}\right)\beta_i\right)\right]^{-1} \tag{18}$$

Laplace transform of Availability function $Av(t)$ is given by:

$$\begin{aligned} Av(s) &= P_0(s) + P_1(s) \\ &= [1 + X]P_0(s) \end{aligned}$$

where $P_0(s)$ is given by equation (18)
Inversion of $Av(s)$ gives the Availability function $Av(t)$.

6.2. Matrix Method

Redundancy and repair procedures are two examples of complex systems with many states and transitions that may be well-modeled utilizing the matrix technique with Markov analysis. It offers a methodical approach to computing reliability indicators such as availability. However, because calculating the balancing equation for bigger systems can get rather difficult, it could be necessary to use mathematical software tools or programming to carry out the computations.

Taking matrix, B, the matrix of coefficients of probabilistic states, the differential equations may be written as;

$(\theta I - B)\bar{P}(k,t) = \bar{0},\ where\ \theta = \frac{d}{dt}, \bar{0}$ is the null matrix.

$\bar{P}(k,t) = [P_0(t)\quad P_1(t).......P_{15}(t)]^T$ and $I_{16\times16}$ is the identity matrix.

Let M be the matrix such that $M^{-1}BM = D,$ where $D = (d_0, d_1, ..., d_{15})$ be the diagonal matrix of eigen values of the matrix B.

$M^{-1}(\theta I - B)\bar{P}(k,t) = M^{-1}\bar{0} = \bar{0}$ gives

$$M^{-1}(\theta I - B)MM^{-1}\bar{P}(k,t) = \bar{0},\ i.e., (\theta I - M^{-1}BM)M^{-1}\bar{P}(k,t) = \bar{0}$$

$(\theta I - D)E(k,t) = \bar{0}$, where $E(k,t) = M^{-1}\bar{P}(k,t)$

$$\frac{d}{dt}E(k,t) - DE(k,t) = \bar{0}$$

Equation is a matrix linear differential equation in variable $E(k,t)$. Solution of the equation is

$E(k,t)e^{-Dt} = K......(i)$, for some constant K.

Matrix B =

$$
\begin{bmatrix}
-C_0 & \beta_4 & \beta_1 & \beta_2 & \beta_3 & \beta_5 & \beta_6 & \beta_7 & \beta_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_4 & -C_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_1 & \beta_2 & \beta_3 & \beta_5 & \beta_6 & \beta_7 & \beta_8 \\
\alpha_1 & 0 & -\beta_1 & \mu_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_2 & 0 & 0 & -\beta_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_3 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_5 & 0 & 0 & 0 & 0 & -\beta_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_6 & 0 & 0 & 0 & 0 & 0 & -\beta_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_7 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 \\
0 & \alpha_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_5 & 0 & 0 & 0 \\
0 & \alpha_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_6 & 0 & 0 \\
0 & \alpha_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_7 & 0 \\
0 & \alpha_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_8
\end{bmatrix}
$$

Taking particular value $t = 0$ as the initial condition, we get

$$K = M^{-1}\bar{P}(k,0)e^{-Dt} ....(ii)$$

From *(i)* and *(ii)*

$$E(k,t) = M^{-1}\bar{P}(k,0)$$

As, $E(k,t) = M^{-1}\bar{P}(k,t)$, it follows

$$M^{-1}\bar{P}(k,t) = e^{Dt}M^{-1}\bar{P}(k,0)$$

$$\bar{P}(k,0) = M\left(1 + Dt + \frac{D^2t^2}{2!} + ...\right)M^{-1}\bar{P}(k,0) = \left(1 + Bt + \frac{B^2t^2}{2!} + ...\right)M^{-1}\bar{P}(k,0)$$

$$= 1 + N_1 t + \frac{N_2 t^2}{2!} + ... + \frac{N_n t^n}{n!} ,.......(iii)$$

where $N_n = B^n\bar{P}(k,0)$

The initial conditions make it clear that $\bar{P}(k,0)$ is the column matrix $(1 \quad 0 \quad 0.......0)^T$ of order 16×1. Note that $B\bar{P}(k,0)$ is just the 1st column of the matrix B. Let us denote this column matrix by

$$B_1 = (a_{11} \ a_{12} ......a_{116})^T .$$

$B^2\bar{P}(k,0) = B.B\bar{P}(k,0) = B.B,$ is again a column matrix, let us denote it by,

$$B_2 = (b_{11} \ b_{12} ......b_{116})^T .$$

Let $B^{r-1}\bar{P}(k,0) = B_{r-1} = (P_{11} \ P_{12} ......P_{116})^T$.

It follows from mathematical induction that

$$B^r\bar{P}(k,0) = B.B_{r-1} = (I_{11} \ I_{12} ......I_{116})^T \text{ for some } I'_{ij}s.$$

Note that the identity (iii) is a column matrix, Probability of different stages are;

$$P_0(t) = P(0,t) = 1 + a_{11}t + \frac{b_{11}t^2}{2!} + ...$$

$$P_1(t) = P(1,t) = 1 + a_{21}t + \frac{b_{21}t^2}{2!} + ...$$

$$P_i(t) = P(i,t) = 1 + a_{i1}t + \frac{b_{i1}t^2}{2!} + ...$$

There are two working states of the complex system. Since $P_0(t)$ and $P_1(t)$ are the only working states of the system,

$$\begin{aligned} Av(t) &= P_0(t) + P_1(t) = P(0,t) + P(1,t) \\ &= 1 + (a_{11} + a_{21})t + (b_{11} + b_{21})\frac{t^2}{2!} + ... \end{aligned}$$

7. Conclusion

In this paper, the application of Markov process in finding the Availability of the complex system has been discussed. A transition diagram shows the relationship between the system and its units. A complicated system's availability can be ascertained using a variety of data analysis methods and mathematical models. Mean Time Between Failures (MTBF) and Mean Time To Repair (MTTR) are some standard metrics for monitoring availability. You can compute these metrics and learn more about the availability of the system by gathering data on system breakdowns, downtime, and repair times. The differential equations for the system are converted into the Laplace domain using the Laplace method, allowing algebraic operations to be carried out to find the system's response. The time-domain solution is then obtained by applying the inverse Laplace transform. Laplace transforms are often not used for availability calculations or steady-state reliability analysis; instead, they are mostly utilized for dynamic system analysis. The matrix approach uses a collection of matrices and mathematical equations to depict the reliability structure of the system. The steady-state availability, or long-term probability of the system being operational, is found by solving these equations. The matrix approach was created expressly to analyze availability and reliability of systems. It works great for simulating intricate systems, such multi-component repairable systems.

In conclusion, there are differences between the applications of the Laplace transform method and the matrix method in the engineering field. For the analysis of time-dependent behavior in dynamic systems, the Laplace transform approach is utilized, whereas the matrix method is designed especially for steady-state reliability and availability analysis in static or quasi-static systems. Availability has advantages for many facets of operations and company, such as competitive advantage, cost-effectiveness, customer happiness, and dependability. For many organizations, achieving high availability is a strategic objective that can be attained through appropriate planning, maintenance, and reliability engineering techniques.

8. Discussion

Understanding availability in a complex system is essential to evaluating its performance and dependability. The frequency with which a system is up and ready to carry out its intended tasks is measured by its availability. Determining availability is crucial for a number of reasons in complex systems, including data centers, transportation networks, industrial plants, and healthcare facilities.

- *Reliability Analysis:* Calculations of availability aid in the reliability evaluation of complicated systems. Understanding the system's availability frequency can help you spot any possible deficiencies and prospective opportunities for development.

- *Risk Prevention:* Analysis of availability is useful in risk management. It can support the identification of possible hazards, the creation of mitigation plans, and the establishment of emergency response protocols to preserve high availability in vital systems.
- *Cost Reduction:* Availability analysis can identify areas that may require investments in order to increase reliability, but it can also point out redundant information that can be removed in order to save operating expenses.
- *Downtime Reduction and Performance Optimization:* By identifying the reasons behind breakdowns in systems, you can focus on areas where downtime may be minimized and operational efficiency can be increased. The efficiency of a complicated system can be maximized by using availability measurements to find areas that require improvement and bottlenecks.
- *Business Continuity:* Keeping sophisticated systems that are vital to corporate operations highly available is crucial for maintaining business continuity. Assessments of availability can help direct business continuity and catastrophe recovery plans. System availability is subject to stringent restrictions and requirements in many businesses, particularly in the energy, healthcare, and financial sectors. For compliance, an accurate measurement of availability is necessary.
- *Predictive Maintenance:* Predictive maintenance techniques can be put into practice using availability data. Through the process of tracking the availability of specific subsystems or components, you may reduce unplanned downtime and arrange maintenance during the most efficient period.
- *Decision Making:* Decision-makers can use availability data to determine whether to spend money on redundancy, backup systems, or system enhancements in order to reach desired availability levels.

In the final analysis, performance, risk management, cost optimization, and reliability all depend on a complex system's capacity to locate availability. It helps businesses to make wise choices and guarantee that their systems fulfil contractual and operational requirements.

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