Results in Nonlinear Analysis 6 (2023) No. 4, 189–200
https://doi.org/10.31838/rna/2023.06.02.016
Available online at www.nonlinear-analysis.com

Results in Nonlinear Analysis
ISSN 2636-7556

Peer Reviewed Scientific Journal

Routing optimization model for delivering multi meal products with time-dependent

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Abstract

Catering is a popular business, particularly in big cities of Indonesia, such as Jakarta and Medan. This business can provide an alternative for office workers, households, and even school students to enjoy home-cooked food according to taste with many choices or variants. The catering distribution service must consider the time-dependent between each delivery to minimize the delivery time. This paper uses a time-dependent logistic system to find the best set of routes taken by a fleet of vehicles carrying meals to satisfy a particular group of clients concerning service time constraints, which could be expressed as a routing problem regarding a fleet of vehicles. The issue's primary goal is to reduce the overall travel costs (time or distance associated with travel) and operational costs (number of vehicles utilized). Because of the widely varying sizes of the demands from distinct types of clients, we have to deal with a heterogeneous vehicle fleet in this regard. Heterogeneous Vehicle Routing Problem (HVRP) is the name for this type of issue. The issue is described using an integer programming model. The model's solution is presented in this paper using a viable neighborhood technique based on a reduced gradient approach. We solve a heterogenous vehicle routing problem to show that the model and the method proposed for the catering distribution service is appropriate.

Keywords: Vehicle routing problem, Heterogenic fleet, Time restricted, Feasible neighborhood search, Integer programming

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Received June 16, 2023; Accepted July 11, 2023; Online August 4, 2023

1. Introduction

To be away from home for more than a few hours necessitates finding somewhere to stay. The distance between their location and a food source prevents them from returning home for regular meals. Therefore, we need organizations that can provide us with food on demand. The catering business is responsible for these functions.

Catering is a service providing food based on contractual agreements with customers for a specified period of time. Catering businesses include:

1. Activities of food service contractors (for example, for transportation companies);
2. Activities of catering services based on agreements in sports facilities and similar facilities;
3. Activities of a canteen or cafeteria (for example, for factories, offices, hospitals, or schools) on the basis of concessions;
4. Catering service activities that serve households or individuals.

This paper focuses on activity No. 4.

For catering businesses, a logistical system is essential for strategic service planning since many clients prefer to have their meals delivered to their homes. These businesses compete with one another for consumers based on service quality, particularly speed of delivery, while also trying to keep their logistical costs as low as possible. Customers value fast shipping timeframes, and aggregators that meet such demands may gain an edge in the market.

For its business, the catering firm employs a wide fleet of vehicles. The carrying capacity of various vehicles varies. Heterogeneous VRP (HVRP) is a subset of VRP that considers a fleet with a wide variety of vehicle types (B. Golden et al., 1984). From a practical standpoint, it is helpful to generalize in this way since numerous different vehicle types may meet the needs of the vast majority of clients (Hoff et al., 2010; Koç et al., 2016). The High-Value Route Planning System (HVRP) aims to identify a fleet composition and accompanying routing plan that results in the lowest possible operational expenses.

Vehicle routing problems (VRP) center on the logistics system's capacity to fulfill a collection of customer requests through the deployment of a fleet of vehicles, aiming to do so in the shortest possible time. Because of the population and vehicle traffic increase, logistics companies face increasing difficulties keeping up with demand (Savelsbergh & Van Woensel, 2016). The rising fuel consumption of vehicles due to congestion is a major contributor to the higher costs logistics companies face. This well-known combinatorial optimization problem asks you to satisfy a set of customers through the most efficient use of a fleet of vehicles while adhering to various constraints.

When discussing the VRP, most academic works assume that vehicle travel times are constant regardless of when they occur in the future (Groër et al., 2009; Kuo et al., 2009; Laporte et al., 1992). Hence, the minimum distance of the whole vehicle's journey is the trip duration between two nodes. In contrast, in the time-dependent vehicle routing problem (TDVRP), the vehicle's speed or the time it takes to go between the two nodes is contingent on the moment the journey begins. The swiftest route between two points may be clogged with vehicles when a vehicle arrives at one of them. The quickest route for a vehicle to drive between the two nodes might take considerable time. During that span of time, road traffic may not exist on the longer route between the two nodes. This means the vehicle taking the longer route between the two nodes may arrive at its destination sooner (K. C. Tan et al., 2001).

Either soft time windows (wherein both early and late arrivals are penalized) or hard time windows (wherein neither are tolerated) may be applied to a VRP with Time Windows (VRPTW). Security patrol services, bank dispatches, postal shipments, factory waste pickups, supermarket delivery, school bus navigation, and urban newspaper delivery are all examples of industries that have difficulties fitting their schedules into tight time slots. Dial-a-ride difficulties are notable instances of soft time windows problems. The soft variation is the more realistic option when there is nowhere for vehicles to remain at the client's premises.

We provide a mixed-integer nonlinear programming model for the time-dependent vehicle routing problem with time windows (TDVRPTW) in order to lower both fixed and variable costs. Strict schedule constraints and vehicle capacity are taken into consideration. All recent TDVRPTW research makes the assumption that traffic would be constant over a specific time period. Here, we permit the journey duration to take over one interval since there is the potential that it will be prolonged owing to the far lengths between nodes. This implies that our model has to accommodate several repetitions of the same time period of traffic circumstances.

By factoring in the availability of other clients and the traffic circumstances, we can determine the most cost-effective time to deliver to each client.

The following are some of our most significant contributions:

- Making a model for TDVRPTW that incorporates smart journey periods. The overall situation considers traffic situations at various periods of the day and is described as time-dependent. The term "smart" is used to describe the journey because the time it takes to get from one node to the other varies and is determined by the pre-existing conditions and the predetermined timeframe of traffic. The focus is that figuring out how long it takes to get from one node to the next is not centered on a simple parameter but rather requires some additional complexity to come as close to reality as possible, and our method evaluates an accurate estimate.
- Using dynamic timings to solve a difficult form of the VRP. Vehicles of varying sizes are used by the corporation that is the subject of this study in order to transport items to clients. The capacity of every kind of vehicle varies. HVRP, or Heterogeneous VRP, is the name given to the form of VRP that anticipates a diverse fleet of vehicles proposed initially by (B. Golden et al., 1984). Practically speaking, this generality is critical since the majority of consumer demand is met by a variety of vehicle types (Hoff et al., 2010; Koç et al., 2016). It is the goal of the HVRP to determine the best possible fleet composition and the best possible route plan in order to reduce overall costs. This paper's issue is referred to as TDHVRP because it requires the company's management to consider consumers' service time.

2. Related Works

The Vehicle Routing Problem refers to the logistics system that coordinates the distribution of food based on the location of a fleet of vehicles (VRP). By reducing the overall trip cost (as a function of journey periods or lengths) and the operating cost (as a consequence of the total of vehicles employed), this widely recognized discrete optimization issue seeks to meet clients' request as soon as possible. To address the issue of truck routing, (Dantzig & Ramser, 1959) originally proposed VRP. Subsequently, the VRP model has been used to resolve various practical difficulties in logistics. The transportation of perishable agricultural goods is the focus of a new application presented by (Ji et al., 2021). To boost patron contentment overall, they created a nonlinear optimization model. (Bräysy & Gendreau, 2005; B. L. Golden et al., 2002; Kumar & Panneerselvam, 2012; Toth & Vigo, 2002b, 2002a; Vidal et al., 2012) provided a complete overview of the Vehicle Routing Problem (2014). A comprehensive history and update on VRP were discussed by (Gendreau & Tarantilis, 2010) in 2010.

The vehicle routing problem (VRP) is a class of optimization problems in which a fleet of vehicles based out of one or more depots must deliver goods to a group of consumers, each of whom has a differing need (Respen et al., 2019). Determining a selection of paths for capacitated trucks to minimize overall cost while satisfying consumer requests is the focus here (Ban et al., 2019). Transportation network design issues come in many kinds. Recent studies have mostly focused on permanently installed VRPs, which have been in place over the last decade (Mourgaya & Vanderbeck, 2006). Parameters, such as trip length, were considered to stay constant throughout the issues. Since static solutions were impractical, dynamic issues were created for application in the actual world. Parameters like consumer needs and/or customer activity are often regarded as dynamic in scientific articles researching dynamic issues (Keskin et al., 2019). Recent studies have seen an uptrend in research on the dynamic trip time routing challenge (F. Wang et al., 2021). In reality, the time it takes for a consumer to go from one location to another varies greatly (Liu et al., 2020). For instance, variable routing priority with variable trip durations is the norm in cities with heavy traffic. An example of a dynamic vehicle routing issue is the time-dependent vehicle routing problem (TDVRP) in congested areas. Considering that the journey period between two nodes relies on traffic situations and is provided as a function of the vehicle deploying time, this issue aims to reduce the overall journey period and the expenses that result from it. Time and money spent getting in between two given nodes might fluctuate based on several variables. The weather and, more crucially, traffic are two examples. Here, an assessment of journey durations across paths is the most important consideration in selecting the next client for service. When faced with such a situation, the driver may make an informed decision on which route to take by consulting traffic laws, including speed limits.

Until the early 2000s, time dependence in vehicle route optimization was only studied in a few cases. Using the time of day as a step function, (Malandraki & Daskin, 1992) developed a method for calculating journey times. The main problem with this strategy is that the first-in-first-out (FIFO) condition is not guaranteed. In other words, because vehicles departing one node may take longer than those departing another, depending on the commencement time, one node may be farther away from another node than another. To address this issue, (Hill & Benton, 1992) created the first journey period model to account for fluctuating speed of vehicles over a journey, hence introducing the FIFO property of the travel time function. (Ichoua et al., 2003) utilized a refined version of this model to show that dynamic routing is superior to static routing. The impact of time-dependent journey periods on reducing traffic was studied by (Kok et al., 2012), who concluded that effective strategies for avoiding road congestion reduce the number of cases in which clients are kept waiting. Employees must put in extra time at work. The time-dependent VRP with time windows (TDVRPTW) was given a recursive local search approach by (Hashimoto et al., 2008). Computational research for various issue scenarios may take up to a thousand nodes. Many researchers have looked at the TDVRPTW, including (Balseiro et al., 2011; Harwood et al., 2013). A hybrid ant colony/insertion heuristic method built by the former has been tested on up to 100 client's issue situations. To address the issue of the traveling salesperson, the latter developed rapid estimations of trip times that rely on the current day and time. According to their findings, there are considerable time savings to be had by using their calculations. By combining an ant colony search with a tabu search strategy, (Zhang et al., 2014) solved the TDVRP's synchronous pickup and delivery problem. A total of one hundred clients were taken into consideration throughout their project. The so-called pollution routing issue has garnered much attention in the previous decade and the environmental consequences of routing. To find the most efficient path for lowering emissions, (Kuo, 2010) devised a simulated annealing technique that considers the different periods it takes to traverse the various edges. Benchmarks with up to 100 clients are used to generate computational findings. Time-dependent VRPs were used to examine the tradeoff between trip times and Carbon dioxide emission. (Jabali et al., 2012) investigated this issue. (Franceschetti et al., 2013) also studied the time-dependent pollutant routing issue. The authors presented an integer linear programming formulation for situations with no traffic congestion. (Soysal et al., 2015) handled the time-dependent two-echelon VRP issue while also considering ecological consequences using a mixed integer linear programming (MILP) technique. All previous research has focused on the deterministic version of the TDVRP. Compared to other types of problem settings, the study dealing with stochastic ones is more limited. (Lecluyse et al., 2009) developed the tabu search metaheuristic used in the TDVRP with stochastic trip times. (Nahum & Hadas, 2009) developed a more robust savings strategy for the stochastic TDVRP. Taş et al., 2014, proposed a tabu search and adaptive big neighborhood search metaheuristic with fuzzy time constraints and stochastic travel durations for the TDVRP.

The time-dependent multi-depot heterogeneous vehicle routing problem arises in changing traffic conditions and joint distribution of vehicles from various depots. Simultaneously, the customer's service time, location, heterogeneous vehicles, fuel usage, and other factors must be considered. Tableware suppliers employed a range of vehicles to distribute the goods based on consumer demand. Vehicles are arranged complying with regional express quantities by a salesperson for the express delivery business. The route's traffic flow situation also influences vehicles' maximum speeds in actual time. During peak times, delivery vehicles move slowly; off-peak times, they move quickly. Time-dependent VRP (TDVRP) and multi-depot VRP (MDVRP) are the two major topics in contemporary VRP research, and they are both addressed in this study. (Beasley, 1981) proposes TDVRP as a start. Specifically, it looked at the effect of vehicle departure time on vehicle travel time; after TDVRP's first studies, the time-dependent function that (Ichoua et al., 2003) developed was widely used in the subsequent studies, piecewise functions are used to represent the vehicle's speed (Figliozzi, 2012; C. Wang et al., 2015).

Road network route planning, public transportation trip planning, and vehicle (especially vessel) routing are all examples of TDVRP routing issues that may be used for various applications. Consider it as an issue that emerges in many real-world scenarios: a vehicle routing and scheduling issue that must be solved simultaneously. A fleet of vehicles will need to make numerous stops in order to serve a group of clients, and it is crucial to plan which clients each vehicle will need to deliver and in what order in order to keep costs to a minimum in light of factors like available space and how long each stop will take to complete (Ellabib et al., 2002). It's necessary to find a way to minimize the cost of assigning vehicles to a trip and the resulting routing cost. Furthermore, researchers are encouraged to leverage the time-dependent and stochastic properties of journey time to solve the logistical challenges that arise in every-day life because of the availability of improved travel time estimate methods. Numerous transportation issues have demonstrated the need to consider time dependence while trying to solve the problem of network links with unpredictable journey times (e.g., (Duan et al., 2015; Gao & Chabini, 2006; Lecluyse et al., 2009; Miller-Hooks & Mahmassani, 2003; Prakash, 2018; Rabbani et al., 2018; Rajabi-Bahaabadi et al., 2015; Sarasola et al., 2016; X. Tan et al., 2005; Vodopivec & Miller-Hooks, 2017)).

The TDVRP has been the topic of intense ongoing research for both heuristic and precise optimization techniques. Among the first studies of VRPTW solution strategies are those by (Desrochers et al., 1990; B. L. Golden & Assad, 1986). Appropriate assessment approaches were the major emphasis of (Cordeau et al., 2001; Desrosiers et al., 1995). (Cook & Rich, 1999; Larsen, 1999) give additional information on these strategies. It's common knowledge that heuristics have become the most favored solution method because of the VRPTW's intricacy and vast application to real-world circumstances. A variant of the artificial bee colony technique (Shi et al., 2012) suggests solving the VRPTW. (Rincon-Garcia et al., 2017) have developed a metaheuristic for the VRPTW that combines large neighborhood search with variable neighborhood search. For the TDVRP problem, (Gmira et al., 2021) recommended using a tabu search strategy.

3. Description of the Problem

Using a graphical format, the TDVRP is explained as follows: Let's assume that $G=(V,E)$ is a connected cycle with $n+1$ attainable nodes in a specific time limit and a set E of arcs with nonnegative weights that accurately indicate travel lengths and associated travel times. Choose one of the nodes to serve as the storage facility. Every i^{th} node, excluding the depot, requires a service of size qi.

This research examines the problem confronting a service company in Medan, North Sumatra Province, Indonesia. The following diagram may be used to illustrate the basic structure of TDHVRP. Let $G=(V,A)$ be a fully connected directed acyclic graph with vertex set to $V=\{0,1,\ldots,n\}$ and routes set to $A=\{(i,j);i,j\in V,i\neq j\}$. The distance (or trip cost) c_{ij} is specified for every possible path $(i,j)\in A$. The depot, often known as vertex 0 $(i=0)$, is the hub from which the whole fleet operates. The clients' vertexes constitute the set defined by V_c. There is a daily demand $W_i\geq 0$ that is constant, a service time $s_i\geq 0$, and a service time window $[a_i,b_i]$ for each vertex $i\in V_c$. In particular, the demand $w=0$ and service time $t=0$.

This is a heterogeneous issue, as previously stated, there are m distinct vehicle types in K vehicle fleet, each with capacity Q_m. Vehicle type m may make use of as many as nm different vehicles. If K_m is a collection of vehicle types m, then precisely one vehicle is assigned to each client. At the depot $(i=0)$, provide an arrival and departure timeframe for vehicles with $[a_0,b_0]$. A vehicle's arrival time at customer i is a_i and its departure time is b_i. Set price for Each type of vehicle is f_m fm. Additionally, each vehicle k in the routes incurs a fixed purchase cost f_k. The central depot is the starting and ending point for each route, and it must meet the time frame requirements, i.e., A vehicle cannot begin serving client i until after a_i and before b_i have completed their respective journeys. a_i may be able to arrive before the vehicle does and wait for service.

Every client node $i\in V_c$ has a minimum service frequency F_i measured in days t per period, a daily demand W_i, and a service frequency σ^t. The node's daily demand is a factor in calculating the demand that has accrued between visits, denoted by the variable w_i. Since more goods pile up with less frequent service and, as a result, more time is needed to load/unload at each stop, the cost of halting at a node i, τ_i is a function of the schedule's periodicity. Each arc $(i,j)\in A$ has a predetermined travel expense (denoted by c_{ij}). We have a fleet of vehicles, K, with a certain capacity, C, and we need to schedule our operations over a period of time, T, where T is the number of workdays.

A service benefit (in monetary terms), δ^t, may be associated with service selection and used as an incentive to provide service more regularly. In this way, we can claim that the rate of change in demand w_i is directly proportional to the increase in service gain.

Following is our formalization of the choice variables:

- $x_{0j}^k = \begin{cases} 1 & \text{if vehicle type } k \in K \text{ to deliver from depot to customer } j \in V_c \\ 0 & \text{otherwise} \end{cases}$
- $x_{ij}^m = \begin{cases} 1 & \text{if vehicle type } m \in K_m \text{ to deliver for } (i,j) \in V_c, i \notin j \\ 0 & \text{otherwise} \end{cases}$
- $z_0^m = \begin{cases} 1 & \text{if vehicle type } m \in K \text{ is available and active at depot} \\ 0 & \text{otherwise} \end{cases}$
- l_i^m arrival time for vehicle type $m \in K_m$ at customer $i \in V_c$ (nonnegative continuous variable)
- u_i^m duration of service of vehicle type $m \in K_m$ at customer $i \in V_c$ (nonnegative continuous variable)

4. The Mathematical Model

To begin, we must identify the objective function. Choosing the best vehicle path to meet consumer demand and keep costs down is an important choice. The objective function specifying the lowest possible travel expenses is expressed (1).

In this fundamental structure, the caterer's management seeks to reduce overall costs by using the vehicles available for various types of deliveries. The overall price includes the expense of all vehicles utilized and the expense of securing a vehicle throughout the day's planning horizon.

$$\text{Minimize } \sum_{j\in V_c} c_{oj} \sum_{k\in K} x_{0j}^k + \sum_{(i,j)\in V_c} \sum_{m\in K_m} \sum_{t\in T} (\tau_{ij}^t \sigma_{ij}^t - w_{ij}^t \delta_{ij}^t) x_{ijm}^t + \sum_{m\in K_m} f_m z_o^m \tag{1}$$

Subject to

$$\sum_{k\in K} x_{0j}^k = 1, \qquad \forall j \in V_c \tag{2}$$

$$\sum_{k\in K} \sum_{j\in V} x_{ij}^k = 1, \qquad \forall i \in V_c \tag{3}$$

$$\sum_{i\in V} x_{ij}^k - \sum_{i\in V} x_{ji}^k = 1; \qquad \forall j \in V_c, \forall \mathrm{k} \in \mathrm{K} \tag{4}$$

$$x_{ij}^m \le z_0^m, \qquad (i,j) \in V_c, \forall m \in K_m \tag{5}$$

$$\sum_{j\in V_c} x_{1j}^k \le 1; \qquad \forall k \in K \tag{6}$$

$$\sum_{i\in V_c, i>1} x_{i1}^k \le 1; \qquad \forall k \in K \tag{7}$$

$$\sum_{i\in V_c} d_i \sum_{j\in V_c} x_{ij}^m \le Q_m\,; \qquad \forall m\in K_m \tag{8}$$

$$x_{ij}^m (l_i^m + u_i^m + s_i + t_{ij} - l_j^m) = 0\,; \qquad \forall m\in K_m, (i,j)\in A \tag{9}$$

$$l_i^m \le a_i \sum_{j\in V_c} x_{ij}^m\,; \qquad \forall m\in K_m, i\in V_c \tag{10}$$

$$a_i \sum_{j\in V_c} x_{ij}^m \le l_i^m + u_i^m \le b_i \sum_{j\in V_c} x_{ij}^m\,; \qquad \forall m\in K_m, i\in V_c \tag{11}$$

$$\sum_{j\in V_c} w_j x_{oj}^m \le n_m\,; \qquad \forall m\in K_m \tag{12}$$

$$x_{0j}^k, x_{ij}^m, z_0^m \in \{0,1\}; \qquad \forall i\in V, \forall j\in V_c, \forall k\in K, \forall m\in K_m \tag{13}$$

$$l_i^m, u_i^m \ge 0; \qquad \forall i\in V_c, \forall m\in K_m \tag{14}$$

The implementation of constraints (2) and (3) guarantees that only one vehicle may enter and leave each client node and return to the depot at any one time. Applying a flow conservation equation to Constraint (4) is what has needed to ensure that vehicle paths remain stable over time. According to constraint (5), just one vehicle of the appropriate sort that is now available for service should be used to provide each customer. Under constraints (6) and (7), the total number of routes that start and end at the central depot is limited to one for each vehicle type. The trucks' capacity to make deliveries must be respected in constraint (8). An optimal solution is found in constraint (9) when travel time between customers on the same route is comparable to the total travel time to and from each client. Every buyer has a certain time to purchase before being subject to constraints (10) and (11). Constraint (12) prevents the number of active vehicles from going over the stockpile capacity of the main garage. There is a constraint (13) for expressing the discrete variables, and we utilize Constraint (14) to specify the continuous variables.

5. The Proposed Method

The main steps that must be carried out in each iteration of the method are as follows. (by producing a viable descent direction, *p*)

1. Get reduced gradient gA = ZTg
2. Approximate the Hessian reduction, i.e. $G_A \doteqdot Z^T G Z$
3. Calculate solution for the system of equations ZTGZpA = –ZTg by breaking the system GApA = – gA.
4. Find search direction p = ZpA.
5. Perform a row search to find the approximation to a*, where

$$f\left(x+\alpha^* p\right) = \min_{\substack{\alpha \\ \{x+\alpha p \text{ feasibel}\}}} f\left(x+\alpha p\right)$$

For example, Z is not limited to only one shape since it is the sole restriction on Z (algebraically), and it has a complete column rank. The form of Z that represents the actual operation is as follows:

$$Z = \begin{bmatrix} -W \\ I \\ 0 \end{bmatrix} = \begin{bmatrix} -b^{-1}S \\ I \\ 0 \end{bmatrix} \begin{matrix} \}m \\ \}s \\ \}n-m-s \end{matrix} \tag{15}$$

The next part will, however, take this simple illustration as an example. However, it should be highlighted that only S and triangular (LU) factorizations of B can be utilised for computational purposes. The Z matrix calculation is never done.

As can be seen from the preceding discussion of steps A through D in equation (15), the fundamental benefit of the Z transformation is that it does not bring extra conditioning into the minimization issue. This method has been included in code when Z is expressly kept as a dense matrix. The LDV factorization of the $[BS]$ matrix allows for the extension to a linear constraint with a sparse distribution that is specified in advance.

$$[B\ S] = [L\ O]DV$$

Using the product form of L and V to store the triangle (L), diagonal (D), and orthogonal ($D^{1/2}V$). This factorization is always denser than the LU factorization of B, but only if S contains more than 1 or 2 columns. Hence, for expediency, We propose using Z in (15). However, it is clear (thanks to the unpleasant B-1) that B has to be protected to the fullest extent possible.

6. Procedure Summary

Following is a brief description of the optimization procedure. The following is assumed:

1. Eligible vector x satisfies $[B\ S\ N]x = b,\ l\ \ x\ \ u$.
2. The value of the corresponding function $f(x)$ and the gradient vector $g(x) = [gB\ \ gs\ \ gN]T$.
3. A number of superbase variables, $s(0\ \ s\ \ n\ \ m)$.
4. Factorization, LU, on the basis matrix B ($m \times m$).
5. The factorization, RTR, of the quasi-Newtonian approximation to the s × s matrix is ZTGZ (Note that G, Z and ZTGZ are never really calculated).
6. Get a vector π that satisfies $BT = gB$.
7. Reduced gradient vector $h = gs - ST$.
8. Small positive convergence tolerance TOLRG and TOLDJ.

7. The Algorithm

Step 1. Get row i^*a row, which has a basic non-feasible solution such that $\delta_{i^*} = \min\{f_i, 1 - f_i\}$

Step 2. Perform a pricing operation

$$v_{i^*}^T = e_{i^*}^T B^{-1}$$

(Column selection is often referred to as the pricing operation. During pricing, the d_j reduced costs of the nonbasic variables are computed)

Step 3. Move the point of nonbasic variable j from its bound
Calculate the maximum movement of nonbasic j, $\sigma_{ij} = v_{i^*}^T \alpha_j$
Eventually the column j^* is to be increased from LB or decreased from UB. If none, go to next i^*.

Step 4. Solve $B\alpha j^* = \alpha j^*$ for αj^*
Step 5. Do ratio test for the basic variables in order to stay feasible due to the releasing of nonbasic j* from its bounds.

Step 6. Do exchange basis

Step 7. If row $i^* = \{\varnothing\}$ go to Stage 2, otherwise

Repeat from step 2.

Stage 2.

Step 1. Adjust integer infeasible super basics by fractional steps to reach complete integer feasibility.

Step 2. Adjust integer feasible super basics. This phase aims to conduct a neighborhood search to verify local optimality.

8. Computational Illustration

The catering problem considered in this paper was taken from a Catering Company located in Medan city, the capital of North Sumatra Province, Indonesia. We obtained the results as shown in Table 1 to Table 3.

Table 1. Result of delivering food to cater

		Customer								
Vehicle	Customer	0	1	2	3	4	5	6	7	8
1	0		1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	1	0.00000		1.97403	0.00000	0.00000	0.01299	0.01299	0.00000	0.00000
	2	0.00000	0.05195		1.00000	0.98701	0.00000	0.00000	0.00000	0.00000
	3	1.00000	0.00000	0.00000		0.00000	0.00000	0.00000	0.00000	0.00000
	4	0.00000	0.92208	0.06494	0.00000		0.00000	0.00000	0.01299	0.00000
	5	0.00000	0.01299	0.00000	0.00000	0.00000		0.98701	0.00000	0.00000
	6	0.00000	0.00000	0.00000	0.00000	0.01299	0.00000		0.97403	0.01299
	7	0.00000	0.01299	0.00000	0.00000	0.00000	0.00000	0.00000		0.98701
	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.98701	0.00000	0.01299	
2	0		0.00000	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	1	0.00000		1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	2	1.00000	0.00000		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	3	0.00000	0.00000	0.00000		1.00000	0.00000	1.00000	0.00000	0.00000
	4	0.00000	1.00000	0.00000	0.00000		0.00000	0.00000	0.00000	0.00000
	5	0.00000	0.00000	0.00000	0.00000	0.00000		0.00000	1.00000	0.00000
	6	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000		0.00000	0.00000
	7	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000		1.00000
	8	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000	
3	0		0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	0.00000	0.00000
	1	0.00000		0.50000	0.00000	0.50000	0.00000	0.00000	0.00000	0.00000
	2	0.00000	0.50000		0.00000	0.00000	0.25000	0.25000	0.00000	0.00000
	3	0.00000	0.00000	0.00000		0.00000	0.00000	0.00000	0.25000	0.75000
	4	0.00000	0.50000	0.00000	0.00000		0.00000	0.00000	0.50000	0.00000
	5	0.00000	0.00000	0.00000	0.75000	0.50000		0.75000	0.00000	0.00000
	6	0.00000	0.00000	0.00000	0.25000	0.00000	0.50000		0.25000	0.00000
	7	0.75000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000		0.25000
	8	0.25000	0.00000	0.50000	0.00000	0.00000	0.25000	0.00000	0.00000	
4	0		1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	1	0.00000		1.00000	0.00000	1.00000	0.00000	0.00000	0.00000	0.00000
	2	0.00000	1.00000		0.00000	0.00000	0.00000	0.00000	0.00000	0.00000
	3	0.00000	0.00000	0.00000		0.00000	0.00000	0.00000	1.00000	0.00000
	4	0.00000	0.00000	0.00000	1.00000		0.00000	0.00000	0.00000	0.00000
	5	0.00000	0.00000	0.00000	0.00000	0.00000		0.00000	0.00000	1.00000
	6	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000		0.00000	0.00000
	7	1.00000	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000		0.00000
	8	0.00000	0.00000	0.00000	0.00000	0.00000	0.00000	1.00000	0.00000	

Table 2. Arrival time from each node

	Customer							
Vehicle	1	2	3	4	5	6	7	8
1	39.74026	69.48052	40.00000	20.64935	20.00000	10.25974	10.00000	19.87013
2	40.00000	40.00000	60.00000	20.00000	10.00000	30.00000	10.00000	20.00000
3	0.00000	0.00000	30.00000	0.00000	0.00000	59.87013	110.00000	100.12987
4	60.25974	10.51948	0.00000	99.35065	110.00000	19.87013	0.00000	0.00000

Table 3. Duration of service at each node

	Customer							
Periode	1	2	3	4	5	6	7	8
1	9.50649	39.80519	10.00000	10.02597	11.97403	12.00000	9.98701	11.97403
2	0.00000	12.00000	12.00000	12.00000	10.00000	10.00000	10.00000	10.00000

EXIT -- OPTIMAL SOLUTION FOUND.
NO. OF ITERATIONS 418 OBJECTIVE VALUE 4.2181818181818E+02
NORM OF X 1.100E+02 NORM OF PI 1.260E+02
1
PROBLEM NAME VRP-CATE OBJECTIVE VALUE 4.2181818182E+02
STATUS OPTIMAL SOLN ITERATION 418
NUMBER .COLUMN. AT ...ACTIVITY... .OBJ GRADIENT. ..LOWER LIMIT. ..UPPER LIMIT.
REDUCED GRADNT M+J

9. Conclusions

The most pressing issues in supply chain management and distribution are addressed in this study. This research aimed to construct a model for the heterogeneous vehicle routing problem (HVRP) with time frames that considered service choice. An encouragement to provide service to clients has been imposed due to this issue. In order to solve the model, the suggested technique makes use of the closest neighbor heuristic algorithm.

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