Results in Nonlinear Analysis 6 (2023) No. 3, 137–143
https://doi.org/10.31838/rna/2023.06.03.011
Available online at www.nonlinear-analysis.com

Results in Nonlinear Analysis
ISSN 2636-7556
Peer Reviewed Scientific Journal

On the uniformity space in ZB-algebras

Ahmed Talip Hussein, Karrar Saad Hamzah

Department of Mathematics, Faculty of Science, University of AL-Qadisiyah

Abstract

In this paper, we used sb-ideal of an zb-algebra B to make a uniform structure $(B, \mathfrak{R})$ and then the part $\mathfrak{R}$ induce an uniform topology T in B. We demonstrated that the pair (B, ϕ) is a topological zb-algebra and discussed related properties. Finally, we presented the translation of space (B,T_I) *and stud*ied some of its properties that we consider essential according to our knowledge.

Keywords: b-algebra, sb-ideal, Uniform topology on B, zb-algebra, Topological zb-algebra.

1. Introduction

Yoon and Kim [1] and Meng and Jun [2] offer two types of algebra which are BCI-algebras(and BCK-algebras). In [3] Sims presented a type of abstract algebra BCH-algebras. He has shown a specie of BCI-algebras is a favorable subtype BCH-algebras. In [4] Neggers and Hee presented an incoming algebra that they namely b-algebra and studied the algebraic properties of it. J. Neggers, Y. BAE, and H. Sik. [5] provided a larger notion and description of d-algebras than BCK-algebras, and analyzed the link between the two. They studied alternative topologies in the same manner that they studied synapses. Many mathematicians are still researching topological set theories in addition to topological algebraic structures. In section one, we provided a set of initial definitions that we believe will be useful to us in proving and formulating some theories and results in this section or the next, and among these concepts and definitions is the definition of b-Algebra and sb-ideal from which we know The set $[i]_I = \{j \in B \setminus i \sim j\}$ is the congruence class containing the element i. In section two, we have provided a definition (Uniformity in zb-algebras) which helps us to obtain a topology. And that this topology made the binary process continuous, and thus we got the topology of zb algebra, which

Email addresses: Ahmed.talip@qu.edu.iq (Ahmed Talip Hussein); karrar.s.hamzah@qu.edu.iq (Karrar Saad Hamzah)

Received July 19, 2023; Accepted August 7, 2023; Online October 18, 2023

we studied some of its properties and some of the relationships and examples of this space. Then we examined the properties of space (B, T_I) and that we found: If I and J are sb-ideals in a zb-algebra B and $I \subseteq J$. Then J is closed and open set of (B, T_I).

Definition 1.1: Let $B \neq \varphi$, $*$ be a binary operation, then the triple $(B;*,0)$ is said to be a b-algebra If the following conditions are met:

1. $a * a = 0$
2. $a * 0 = a$
3. $(a * b) * c = a * (c * (0 * b))$.

$\forall a, b, c \in B$ and element 0 is called zero element in B.

Definition 1.2: [6] The set $\varphi \neq S \subseteq B$ is called sub b-algebra of B if it is closed under the b-operation and S is ideal of B if:

(B1) $o \in S$
(B2) $s \in S$ and $r * s \in S$ reveal $s \in S$. for all $r, s \in B$.

Definition 1.3: [5] A non-empty subset A of a b-algebra $(B;*,0)$ is called a b-ideal if it holds (B1) and

(B3) $(a * b) \in A$ and $(d * a) \in A$, Imply $(b * d) \in A$, for arbitrary $a,b, d \in B$,

Definition 1.4: A b-ideal A of B is said to be a strongly b-ideal (sb-ideal) in B if the following condition are met:

(B4) $a * b \in A$ and $b * a \in A$ imply $(a * u) * (b * u) \in A$ and $(u * a) * (u * b) \in A$, $\forall a,b, u \in B$

Example 1.5: The set $B = \{0, \varepsilon, \delta, \gamma,\}$ with process * as specified below:

Table 1: Shows that the set B with the binary operation * is b - Algebra

$*$	0	ε	δ	γ
0	0	ε	δ	γ
ε	ε	0	γ	δ
δ	δ	γ	0	ε
γ	γ	δ	ε	0

From table (1) the set B with * form b-algebra and $A = \{0, \delta\}$ is sb- ideal

Definition 1.6: A b-algebra B is called zero b-algebra (zb-algebra) if satisfy the condition $0 * a = 0$ for all $a \in B$.

Definition 1.7: A relation $\sim$ on a zb-algebra B is called congruence if

1. A relation $\sim$ is an equivalence.
2. $\gamma \sim \sigma, \varepsilon \sim \delta$ then $\gamma * \varepsilon \sim \sigma * \delta$, for all $\gamma, \sigma, \varepsilon, \delta \in B$.

Remark 1.8: If I is sb-ideal from a b-algebra $(B,*, 0)$. $\forall \gamma, \sigma \in B$ and $\sim$ be relation on B such that it define by $\gamma \sim \sigma$ if and only if $\gamma * \sigma \in I$ and $\sigma * \gamma \in I$. Then $\sim$ is congruence relation on B.

Proof. Since $\gamma * \gamma = 0 \in I$, i.e., $\gamma \sim \gamma$, $\forall \gamma \in B$. If $\gamma \sim \sigma$ and $\sigma \sim \varepsilon$, then $\gamma * \sigma$, $\sigma * \gamma \in I$ and $\sigma * \varepsilon$, $\varepsilon * \sigma \in I$. By (B3) $\gamma * \varepsilon$, $\varepsilon * \gamma \in I$ and hence $\gamma \sim \varepsilon$. So $\sim$ is transitive. It is easy to prove that $\sim$ symmetry relationship and by (B4) we obtained that $\sim$ is a congruence relation on B.

We defined The set $[i]_I = \{j \in B \setminus i \sim j\}$ is the congruence class containing the element i and $i \sim j \Leftrightarrow [i]_I = [j]_I$. The set $B / I = \{[i]_I / i \in B\}$ is all equivalence classes of B.

Lemma 1.9: If S be a sb-ideal of a zb-algebra $(B; *, 0)$. Then $S = [0]_S$.

Proof. If $\sigma \in S$, then $\sigma * 0$, $0 * \sigma \in S$ by (B2) and hence $\sigma \in [0]_S$, i.e., $S \subseteq [0]_S$.
Since

$$\begin{aligned}[0]S &= \{\sigma \in B \mid \sigma \sim 0\} \\ &= \{\sigma \in B \mid \sigma * 0,\ 0 * \sigma \in S\} \\ &= \{\sigma \in B \mid i * 0 \in S\}\ (0 \in S) \\ &\subseteq S,\end{aligned} \tag{B1}$$

it follows that $S = [0]_S$.

Proposition 1.10: If S is sb-ideal from zb-algebra $(B; *, 0)$ and define $[\sigma]_S * [x]_S = [\sigma * x]_S, \forall \sigma, x \in B$, then $(B/S; *, 0)$ is a quotient zb-algebra.

Proof. Since $\sigma * x \sim \sigma' * x'$ for any $\sigma \sim \sigma'; x \sim x'$.(By ~ is a congruence on B) This means that $[\sigma]_S * [x]_S = [\sigma * x]_S$ is well-defined. The rest is trivial.

A mapping $f : (B_1; *, 0_1) \rightarrow (B_2; \text{o}, 0_2)$ is called a b-homomorphism(hom.) when $f(a*b) = f(a) \text{ o } f(b)$, $\forall a, b \in B1$ and a trivial hom. 0_1 when $0_1(a) = 0_2$, $\forall a \in B_1$. Then we denoted the set of all b-homomorphisms of B_1 into B_2 by b-Hom(B_1, B_2) (see [7]).

Proposition 1.11. Let S be a sb-ideal of the zb-algebra B. Then the mapping $\zeta: B \rightarrow B/S$ defined by $\zeta(\sigma) = [\sigma]_S$ is a b-morphism of B onto the quotient zb-algebra B/S and the kernel of ζ is precisely the set S.

Proof: Since $[\sigma * x]_S = [\sigma]_S * [x]_S$, ζ is a b-morphism. By Lemma (1.11) we know that:

$$\begin{aligned}Ker(\zeta) &= \{\sigma \in B / \zeta(\sigma) = [0]_S\} \\ &= \{\sigma \in B / [\sigma]_S = [0]_S\} \\ &= \{\sigma \in B / \sigma \sim 0\} \\ &= [0]_S \\ &= S.\end{aligned}$$

Definition 1.12: [9] A b-algebra $(B; *, 0)$ is said to be b-transitive if $\sigma * \ni = 0$ and $\ni * x = 0$ imply $\sigma * x = 0$.

Lemma 1.13: [9] Let B be a b-algebra. Then for any $\sigma, x \in B$. Then if $\sigma * x = 0$ it follows that $\sigma = x$.

Theorem 1.14: Let $\zeta: B_1 \rightarrow B_2$ be b-morphism of a b-algebra B_1 onto a b-transitive b-algebra B_2, it follows that $B_1 / Ker\ \zeta \cong B_2$.

Proof. Let $\alpha : B_1 / Ker\ \zeta \rightarrow B_2$ such that $\alpha([\sigma]_{Ker\,\zeta}) = \zeta(\sigma)$. If $[\sigma]_{Ker\,\zeta} = [x]_{Ker\,\zeta}$ then $\sigma * x$, $x * \sigma \in Ker\ \zeta$, and so $\zeta(\sigma) * \zeta(x) = 0 = \zeta(x) * \zeta(\sigma)$. Lemma (1.15) we have $\zeta(\sigma) = \zeta(x)$, i.e., $\alpha([\sigma]_{Ker\,\zeta}) = \alpha([x]_{Ker\,\zeta})$. Thus α is well-defined. So $\forall x \in B_2$, $\exists\ \sigma \in B_1$, $\ni x = \zeta(\sigma)$ since ζ is inclusive. It follows that $\alpha([\sigma]_{Ker\,\zeta}) = \zeta(\sigma) = x$, so α is inclusive. If $[\sigma]Ker\ f \neq [x]Ker\ \zeta$ then either $\sigma * x \notin Ker\zeta$ or $x * \sigma \notin Ker\ \zeta$. Assume that $\sigma * x \notin Ker\ \zeta$. Thus $\zeta(\sigma) * \zeta(x) = \zeta(\sigma * x) \neq 0$ and hence $\zeta(\sigma) \neq \zeta(x)$. Then α is one-one. Since $\alpha([\sigma]_{Ker\,\zeta} * [x]_{Ker\,\zeta}) = \alpha([\sigma * x]_{Ker\,\zeta}) = \zeta(\sigma * x) = \zeta(\sigma) * \zeta(x) = \alpha([\sigma]_{Ker\,\zeta}) * \alpha([x]_{Ker\,\zeta})$. α is b-morphism. So $B_1 / Ker\zeta \cong B_2$. So we omit the proof.

2. Uniformity in zb-algebras

Henceforth, B is a zb-algebra also $\breve{N}, K \subseteq B \times B$. Define:

$$\breve{N} \text{ o } K = \{(b_1, b_2) \in B \times B \,/\, \textit{for some } b_3 \in B, (b_1, b_3) \in \breve{N} \textit{ and } (b_3, b_2) \in K\}.$$
$$\breve{N}\text{-}1 = \{(b_1, b_2) \in B \times B \,/\, (b_2, b_1) \in \breve{N}\}.$$
$$\Delta = \{(b, b) \,/\, b \in B\}.$$

Definition 2.1: [4] By a uniformity on B, we refer to a non-empty collection $\mathcal{X}$ of subsets of B× B that meets the following criteria.:

($N1$) $\Delta \subseteq Й$, $\forall Й \in X$,
($N2$) if $Й \in X, \Rightarrow Й$ -1 $\in X$,
($N3$) if $Й \in X, \Rightarrow \exists\ V \in X, \ni K$ o $K \subseteq Й$,
($N4$) if $Й, K \in X \Rightarrow Й \cap K \in X$,
(N5) if $Й \in X, Й \subseteq K \subseteq B \times B \Rightarrow K \in X$.

A couple (B, X) is referred to as a uniform structure.

Theorem 2.2: If I is a sb-ideal from zb-algebra B. If we define

$$Й_I = \{(\sigma, \text{x}) \in B \times B \ / \ \sigma * \text{x} \in I \ and \ \text{x} * \sigma \in I\}$$

and let

$$\Psi = \{Й_I \ / \ I \text{ is a sb-ideal of } B\}.$$

Then Ψ fulfillment of stipulations ($N1$)~ ($N4$).

Proof: ($N1$): *If* $(\sigma, \sigma) \in \Delta, \Rightarrow (\sigma, \sigma) \in Й_I$ since $\sigma * \sigma = 0 \in I$. Hence $\Delta \in Й_I$ for any $Й_I \in \Psi$.

($N2$): For any $Й_I$ in Ψ,

$$\begin{aligned}(\sigma, \text{x}) \in Й_I &\leftrightarrow \sigma * \text{x} \in I \ and \ \text{x} * \sigma \in I,\\ &\leftrightarrow \text{x} \sim \sigma,\\ &\leftrightarrow (\text{x}, \sigma) \in Й_I,\\ &\leftrightarrow (\sigma, \text{x}) \in (Й–1)_I\end{aligned}$$

Hence $(Й–1)_I = Й_I \in \Psi$.

($N3$): Since the relation ~ is transitivity. Then $Й_I$ o $Й_I \subseteq Й_I$. $\forall Й_I \in \Psi$,

($N4$): $\forall Й_I, Й_J \in \Psi$, then:

$$\begin{aligned}(\sigma, \text{x}) \in Й_I \cap Й_J &\leftrightarrow (\sigma, \text{x}) \in Й_I \ and \ (\sigma, \text{x}) \in Й_J.\\ &\leftrightarrow \sigma * \text{x}; \sigma * \text{x} \in I \cap J,\\ &\leftrightarrow \sigma \sim \text{x},\\ &\leftrightarrow (\sigma, \text{x}) \in Й_{I\cap J}.\end{aligned}$$

Thus $Й_I \cap Й_J = Й_{I\cap J} \in \Psi$. This establishes the theorem.

Theorem 2.3: The set $X = \{Й \subseteq B \times B \ / \ \exists\ Й_I \in \Psi$ such that $Й_I \subseteq Й\}$ fulfills the conditions for a uniformity on B.

Proof: The collection X fulfills the provisions ($N1$) ~ ($N4$) (By theorem 3.2). Also Let $K \in X$ and $K \subseteq Ж \subseteq B \times B$. Then $\exists\ K\ I \subseteq K \subseteq Ж$. Thus $Ж \in X$ *and* X fulfills ($N5$).

Let $\sigma \in B$ and $Й \in X$ Define:

$$Й[\sigma] = \{\text{x} \in B \ / \ (\sigma, \text{x}) \in Й\}.$$

Theorem 2.4: The collection $Й_\sigma = \{ Й[\sigma] \ / \ Й \in X, \sigma \in B\}$ is neighborhood base at σ, making B a topological space.

Proof: Since $(\sigma, \sigma) \in Й$, then $\sigma \in Й[\sigma]$ for each $\sigma \in B$. Since $Й_1[\sigma] \cap Й_2[\sigma] = (Й_1 \cap Й_2)[\sigma]$ the intersection of neighborhood is also a neighborhood. Finally, if $Й[\sigma] \in Й_\sigma$ then by $N3$ there exists $K \in X$ such that K o $K \subseteq Й$. Hence for any $\text{x} \in K[\sigma]$, $K[\text{x}] \subseteq Й[\sigma]$.

Theorem 2.5: The set $\phi = \{O \subseteq B / \ \forall\ \sigma \in O, \exists\ Й \in X, Й[\sigma] \subseteq O\}$ is a topology on B. Where B is a zb-algebra.

Proof: From the definition of ф, Easy proof that ϕ is closed under arbitrary unions. Also if O, $\breve{И} \in \phi$ and $\sigma \in O \cap \breve{И}$. Then $\exists$ K, $\Upsilon \in X$ such that $K[\sigma] \subseteq O$ and $\Upsilon[\sigma] \subseteq \breve{И}$. Let $Q = K \cap \Upsilon$. Then $Q \in X$. Also $Q[\sigma] \subseteq K[\sigma] \cap \Upsilon[\sigma]$ and so $\Upsilon[\sigma] \subseteq O \cap \breve{И}$. Therefore $O \cap \breve{И} \in \phi$. So ϕ is a topology on B.

Definition 2.6: If(B,X) is a uniform structure. Then the topology ϕ *is* termed the uniform topology on B deduced from X.

Example 2.7: If $B = \{0, \sigma, \mathfrak{r}, \vartheta\}$ is zb-algebra with the table below:

Table 4: Shows that the binary operation * with the set B is zb - Algebra

*	0	σ	$\mathfrak{r}$	ϑ
0	0	0	0	0
σ	σ	0	σ	σ
$\mathfrak{r}$	$\mathfrak{r}$	$\mathfrak{r}$	0	$\mathfrak{r}$
ϑ	ϑ	ϑ	ϑ	0

Then {0} and B are the only sb-ideals of B. We can see that $U_{\{0\}} = \Delta$, and $U_B = B \times B$: Therefore $\Psi = \{U_{\{0\}}, U_B\}$ and $X = \{U \subseteq B \times B \ / \ U_I \subseteq U$ for some $U_I \in \Psi\}$, If we take $U = U_I$ and $I = \{0\}$. Then $U_I = \Delta$. If we take $U = U_I$, then $U[\sigma] = \{\sigma\}$, $\forall\ \sigma \in B$ and we obtain $\phi = 2B$, the discrete topology. Moreover, if we take B as a sb-ideal of B, so $U[\sigma] = B$, *for all u belong to* B and obtain $\phi = \{\phi, B\}$, the indiscrete topology.

Let B be a zb-algebra and F, H be subsets of B. We define a set $F * H$ as follows:

$$F * H = \{f * h / f \in F, h \in H\}.$$

Definition 2.8: Let B be a zb-algebra and ϕ be a topology on B. Then pair (B, ϕ) is termed topological zb -algebra if the function " ∗ " is continuous with regard to ϕ this definition is rewarded the following property:

(C): If $\breve{И}$ is a set in the topology ϕ and $\sigma, \mathfrak{r} \in B \ni \sigma * \mathfrak{r} \in \breve{И}$, then $\exists\ \breve{И}_1, \breve{И}_2 \in \phi \ni \sigma \in \breve{И}_1, \mathfrak{r} \in \breve{И}_2$ and $\breve{И}_1 * \breve{И}_2 \subseteq \breve{И}$.

Theorem 2.9: The pair (B, ϕ) is a topological zb-algebra.

Proof: Let $\sigma * \mathfrak{r} \in O$, with $\sigma, \mathfrak{r} \in B$ and $O \in \phi$. Then $\exists\ \breve{И} \in X$, $\breve{И}[\sigma * \mathfrak{r}] \subseteq O$, I is a sb-ideal in B where $\breve{И}_I \subseteq \breve{И}$. We will show that:

$$\breve{И}_I[\sigma] * \breve{И}_I[\mathfrak{r}] \subseteq \breve{И}[\sigma * \mathfrak{r}].$$

Indeed, for any $\vartheta \in \breve{И}_I[\sigma]$ and $\varsigma \in U_I[\mathfrak{r}]$ we have that $\sigma \sim I\ \vartheta$ and $\mathfrak{r} \sim I\ \varsigma$. Since $\sim I$ is a congruence relation, Thus $\sigma * \mathfrak{r} \sim I\ \vartheta * \varsigma$. $\Rightarrow (\sigma * \mathfrak{r}, \vartheta * \varsigma) \in \breve{И}_I \subseteq \breve{И}$. So $\vartheta * \varsigma \in \breve{И}_I[\sigma * \mathfrak{r}] \subseteq \breve{И}[\sigma * \mathfrak{r}]$. $\Rightarrow \vartheta * \varsigma \in O$.

Theorem 2.10: [8] If $S \subseteq P(B \times B)$ such that $\forall\ \breve{И} \in S$ meet the following conditions:

(a) $\Delta \subseteq \breve{И}$,
(b) $\breve{И} - 1 \in S$,
(c) $\exists\ K \in S, \ni K \text{ o } K \subseteq \breve{И}$.

Then there exists a unique uniformity $Ђ$, for which S is a sub base.

Theorem 2.12: If $Ђ = \{\breve{И}_I \ / \ I$ is sb-ideal in a zb-algebra $B\}$, then $Ђ$ is a sub base for a uniformity of B.

Proof: It is lucid that the set $Ђ$ fulfills the axioms of the theory.

In Example 2.8, we can see that $Ђ = \{\breve{И}_{\{0\}} = \Delta, \breve{И}_B = B \times B\}$.

Theorem 2.13: Let B be zb-algebra and $\Lambda = \{I \subseteq B \ / \ I$ is a sb-ideals in $B\}$, which is closed under intersection. Then any I in Λ is open and closed in B.

Proof: Let $I \in \Lambda$ and $x \in Ic$. So $x \in \breve{Y}_I[x]$ from that we obtain that $Ic \subseteq \cup \{\breve{Y}_I[x] \ / \ x \in Ic\}$. We plea that $\breve{Y}_I[x] \subseteq Ic$ for all $x \in Ic$. Let $\ni \in \breve{Y}_I[x]$, then $x \sim I \ni$. Hence $x * \ni \in I$. If $\ni \in I$, then $x \in I$, since I is a sb-ideal of B, which is a contradiction. So $\ni \in Ic$ and we obtain

$$\cup \{\breve{Y}_I[x] \ / \ x \in Ic\} \subseteq Ic.$$

Hence $Ic = \cup \{\breve{Y}_I[x] \ / \ x \in Ic\}$. Since $\breve{Y}_I[x]$ is open for $x \in B$, I is a closed subset in B. We show that $I = \cup \{\breve{Y}_I[x] \ / \ x \in I\}$. If $x \in I$ then $x \in \breve{Y}_I[x]$ and hence $I \subseteq \cup \{\breve{Y}_I[x] \ / \ x \in I\}$. Given $x \in I$, if $\ni \in \breve{Y}_I[x]$, then $x \sim I \ni$ and so $\ni * x \in I$. Since $x \in I$ and I is a sb-ideal of B, we have $\ni \in I$. And upon it we get $\cup \{\breve{Y}_I[x] \ / \ x \in I\} \subseteq I$, i.e., I is an open in B.

Theorem 2.14: If $I \in \Lambda$. Then $\breve{Y}_I[\sigma]$ is open and closed in a zb-algebra B.

Proof: We confirm that $(\breve{Y}_I[\sigma])c$ is open. If $x \in (\breve{Y}_I[\sigma])c$, then $\sigma * x \in Ic$ or $x * \sigma \in Ic$. We postulate that $x * \sigma \in Ic$. By applying Theorems 2.10 and 2.11, we obtain $(\breve{Y}_I[x] * \breve{Y}_I[\sigma]) \subseteq \breve{Y}_I[x * \sigma] \subseteq Ic$. We prove that $\breve{Y}_I[\sigma] \subseteq (\breve{Y}_I[a])c$. *If* $\varsigma \in UI[b]$, then $\varsigma * \sigma \in (\breve{Y}_I[\varsigma] * \breve{Y}_I[\sigma])$. *Hence* $\varsigma * \sigma \in Ic$ then we obtain $\varsigma \in (\breve{Y}_I[\sigma])c$, proving that $(\breve{Y}_I[\sigma])\, c$ is open. Hence $\breve{Y}_I[\sigma]$ is closed and since $\breve{Y}_I[\sigma]$ is open. Thus $\breve{Y}_I[\sigma]$ *is a* closed and open in B.

Theorem 2.15: Let $T\Lambda$ be the uniform topology by Λ. Then $T\Lambda = T\{J\}$, where $J = \cap\{I \ / \ I \in \Lambda\}$.

Proof: Let $\Im$ and Ω be as Theorems 2.4, Theorems 2.5, respectively. Put $\Lambda_0 = \{J\}$. Define $\Omega_0 = \{\breve{Y}_J\}$ and $\Im_0 = \{\breve{Y} \ / \ \breve{Y}_J \subseteq \breve{Y}\}$. Let $O \in T_\Lambda$. Given an $\sigma \in O$, $\exists\, \breve{Y} \in \Im$, $\ni \breve{Y}[\sigma] \subseteq O$. By $J \subseteq I$, then $\breve{Y}_J \subseteq \breve{Y}_I$, to any sb-ideal I of B. Since $\breve{Y} \in \Im$, $\exists\, I \in \Lambda$, $\ni \breve{Y}_I \subseteq \breve{Y}$. So $\breve{Y}_J[\sigma] \subseteq \breve{Y}_I[\sigma] \subseteq O$. *Thus* $O \in T_J$ (By $\breve{Y}_J \in \Omega_0$). So $T\Lambda \subseteq T_J$. On the contrary, if $\Upsilon \in T_J$, then $\forall \sigma \in \Upsilon$, $\exists\, \breve{Y} \in \Im_0, \ni \breve{Y}[\sigma] \subseteq \Upsilon$. So $\breve{Y}_J[\sigma] \subseteq \Upsilon$, hence Λ is locked beneath intersection, $J \in \Lambda$. Thus $\breve{Y}_J \in \Im$ and so $\Upsilon \in T\Lambda$. then $T_J \subseteq T_\Lambda$.

Theorem 2.17: Let I and J be sb-ideals in a zb-algebra B and $I \subseteq J$. Then J is closed and open set of (B, TI).

Proof: Put $\Lambda = \{I, J\}$. Thus $T_\Lambda = T_I$ and J is closed and open in (B, T_I).(By Theorem 2.16).

Theorem 2.18: Let I, J be sb-ideals in a zb-algebra B. Then $T_I \subseteq T_J$ if $J \subseteq I$.

Proof: Let $J \subseteq I$. Put $\Lambda_1 = \{I\}$, $\Psi_1 = \{\breve{Y}_I\}$, $\Re_1 = \{\breve{Y} / \breve{Y}_I \subseteq \breve{Y}\}$ and $\Lambda_2 = \{J\}$, $\Psi 2 = \{\breve{Y}_J\}$, $\Re_2 = \{\breve{Y} \ / \ \breve{Y}_J \subseteq \breve{Y}\}$. Let $O \in T_I$. Then $\forall \sigma \in O$, $\exists\, \breve{Y} \in \Re_1$, $\ni \breve{Y}[\sigma] \subseteq O$. Since $J \subseteq I$, So $\breve{Y}_J \subseteq \breve{Y}_I$. $\breve{Y}_I[\sigma] \subseteq O$ implies $\breve{Y}_J[\sigma] \subseteq O$. This proves that $\breve{Y}_J \in \Re_2$ and so $O \in T_J$. Thus $T_I \subseteq T_J$.

Recall that a uniform space $(B, \Re)$ is said to be totally bounded if for each $\breve{Y} \in \Re$, there exist $a1, \ldots, an \in B$, $\ni B = \bigcup_{i=1}^{n} \breve{Y}[a_i]$, and (B, K) is said to be compact if any open cover of B has its finite sub cover.

Theorem 2.19: Let (B, TI) be the topological space where I is a sb-ideal in B. Then the next conditions are on a par with:

1. (B, T_I) is compact,
2. (B, T_I) is totally bounded,
3. $\exists\, P = \{a_1, \ldots, a_n\} \subseteq B$, $\ni \forall\, a \in B\ \forall ai \in P$ with $a * a_i \in I$ and $ai * a \in I$, $\forall i = 1, \ldots, n$.

Proof: $1 \Rightarrow 2$: It is clear by [9].

$2 \Rightarrow 3$: Let $\breve{Y}_I \in X$. $\Rightarrow \exists\, a_1, \ldots, a_n \in I \ni B = \bigcup_{i=1}^{n} \breve{Y}[a_i]$. (By the space (B, T_I) is totally bounded). If $a \in B$, then $\exists\, ai$, $\ni a \in \breve{Y}_I[a_i]$, subsequently $a * a_i$, $a_i * a \in I$.

3 ⇒ 1: $\forall\ a \in B$, by presumption, $\forall a_i \in P$ with $a * a_i$, $a_i * a \in I$. Hence $a \in \check{Ӵ}_I\ [a_i]$. Thus $B = \bigcup_{i=1}^{n} \check{Ӵ}\ [a_i]$. Now let $B = \bigcup_{\alpha\in\Omega} O_\alpha$ for each O_a is open in B. Then $\forall\ a_i \in B\ \exists\ a_i \in \Omega$, ∍ $a_i \in O_{\alpha_i}$, since O_{α_i} is an open. Hence $\check{Ӵ}_I\ [a_i] \subseteq O_{\alpha_i}$. Hence $B = \bigcup_{i=1}^{n} \check{Ӵ}\ [a_i] \subseteq \bigcup_{i=1}^{n} O_{\alpha_i}$, i.e., $B = \bigcup_{i=1}^{n} O_{\alpha_i}$, which means that (B, T_I) is compact.

Theorem 2.20: If I is a sb-ideal of a zb-algebra B. Then $\check{Ӵ}_I\ [\sigma]$ is a compact set in a topological space (B, T_I), for any $\sigma \in B$.

Proof: Let $\check{Ӵ}_I\ [\sigma] \subseteq \bigcup_{\alpha\in\Omega} O_\alpha$ where $\forall O\alpha$ is an open in B. Since $\sigma \in \check{Ӵ}_I\ [\sigma]$, then $\exists\ \alpha \in \Omega$, ∍ $\alpha \in O_a$. Hence $\check{Ӵ}_I\ [\sigma] \subseteq O_a$, proving that $\check{Ӵ}_I\ [\sigma]$ is compact.

3. Conclusions

Since Yoon D.S. Kim HS [1] uniform structures in BCL algebra has become a very interesting research topic recently. Basically, they made the binary process of the Zb-Algebra continuous structure with topological work obtained for a unified structure and got many useful and interesting results. Motivated by this idea, we propose, to be taught investigated characterizations of topological zb-algebra and also discussed and studied the separation axioms, some basic properties, connectedness and quotients of topological zb-algebra and generalized topological zb-algebra.

References

[1] Neggers, J., Jun, Y. B., Hee, S. K., "*On d-ideals in d-algebras*", Math. Slovaca, 49 (3), (1999), 243–251.
[2] Young. B. J., Hee. S. K., "*Uniform structures in positive implication algebras*", Int. Math. J., 2, (2002), 215–2.
[3] Yoon, D. S., Kim, H. S., "*Uniform structures in BCL – algebras*", Commun. Korean, Math.Soc., 17 (3), (2002), 403407, https://doi.org/10.4134/CKMS.2002.17.3.403
[4] Ahmed, T. H., Emad, A. S, "*Some Properties of BP-Space*", Mathematics and Statistics, 10 (1), (2022), 195–200, https://doi.org/10.13189/ms.2022.100118.
[5] Habeeb, K., Haider, J., Ahmed, T., "*Limit Sets and Cartan D – space*", Journal of AL-Qadisiyah for computer science and mathematics, 9 (2), (2017), 134–140, https://doi.org/10.29304 /jqcm.2017.9.2.305.
[6] Ahmed, T. H., Habeeb, K. A., Haider, J., A, "*Uniformity in d-algebras*", Earthline Journal of Mathematical Sciences, 2 (1), (2019), 207–221, https://doi.org/10.34198/ejms.2119.207221
[7] Ahmed, T. H, "*The orbits and minimal sets in d – algebra*", Journal of Physics: Conference Series, 1664, (2020), 1742–6596, https://doi.org/10.1088/1742-6596/1664/1/012048
[8] Sims B.T., "*Fundamentals of Topology*", Macmillan Publishing Co., Inc., New York, (1976).
[9] Yamini C., Kailasavalli S., "*Fuzzy B – Ideals On B – algebra*", International, Journal of Mathematical Archive, 5 (2), (2014), 227–233, www.ijma.info
[10] Neggers, J., Hee, S. K., "*On B-algebras*", Mate. Vesnik, vol.54, pp.21-29, 2002.
[11] Meng, J. Y. Jun, B., "*BCK-algebras*", Kyung Moon Sa, Seoul, Korea, pp. 293–377, 1994. https://doi.org/10.1007/978-94-017-2422-7_6.