



Occasionally weakly compatible mappings and common fixed points in Menger space

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Abstract

The purpose of this paper is to establish a common fixed point result in Menger space in two pairs and three pairs of self mappings by using occasionally weakly compatible mappings. Our first theorem generalizes the theorem of Sharma and Shahu [1] and B. Fisher et al. [2] and both theorems deduce some similar results in the literature.

Key words and phrases: Menger Space, compatible mappings, weakly compatible mappings, occasionally weakly compatible.

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1. Introduction

Probabilistic metric space is one of the important generalizations of metric space and is a dynamic area of research space in mathematics. The notion of probabilistic metric space is introduced by Karl's Menger [3] in 1942. The existence of a fixed point in Menger probabilistic metric space was first defined and obtained by Sehgal and Barucha Reid [4] in 1972. After that, so many fixed point theorems have been established by using single and multi-valued mappings in Menger probabilistic metric space by many authors some of them are ([5]–[20]).

The condition of commutativity weakened by Sessa [21] to weakly commuting in 1982. Jungck in 1986, generalized the weak commutativity to compatible [22] and then weakly compatible maps [23]

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in metric space. Singh and Jain [24] give the notion of weakly compatible mapping in Menger space as its extension. In 1991, S.N. Mishra [25] introduced the notion of compatible maps in Menger space. And Occasionally weakly compatible mapping in metric space was introduced by G. Jungck and B.E. Rhodes [16] and by AI. Thapagi and Naseer Sahzad [26] in 2008 and extended it by B.D. Pant et al. [27] in Menger space.

2. Preliminaries

Definition 2.1: [28] A left continuous and non-decreasing function $F : \mathbb{R} \rightarrow \mathbb{R}^+$ is said to be distribution function if it's $\inf_{x \in \mathbb{R}} F(x) = 0$ and $\sup_{x \in \mathbb{R}} F(x) = 1$.

Definition 2.2: [28] An ordered pair (K, F) is said to be **Probabilistic Metric Space** (briefly, PM-space) where K be an abstract set of elements and $F : K \times K \rightarrow L$ is distribution function defined by $(u, v) \rightarrow F(u, v)$ where $L = \{F(u, v) : u, v \in K\}$, if the distribution function $F(u, v)$, also denoted by $F(u, v)$, where $(u, v) \rightarrow K \times K$, satisfy following conditions:

- (PM 1) $F_{u,v}(0) = 0$; for every $u, v \in K$,
- (PM 2) $F_{u,v}(x) = 1$, for every $x > 0$ if and only if $u = v$,
- (PM 3) $F_{u,v}(x) = F_{v,u}(x)$, for every $u, v \in K$, and
- (PM 4) For every $u, v, w \in K$ and for every

$$x, y > 0, F_{u,v}(x) = 1, F_{v,w}(y) = 1 \Rightarrow F_{u,w}(x + y) = 1.$$

Here, $F(u, v)(x)$ represents the value of $F(u, v)$ at $x \in \mathbb{R}$.

Definition 2.3: [29] A Triangular norm is a binary operation on unit interval $[0, 1]$ which may be defined as a function $T : [0, 1] \times [0, 1] \rightarrow [0, 1]$ is called **Triangular norm** (shortly T-norm) if for all $a, b, c, d \in [0, 1]$ satisfies the following conditions:

- i. $T(0, 0) = 0$ and $t(a, 1) = a$ for every $a \in [0, 1]$, (boundary condition);
- ii. $t(a, b) = T(b, a)$ for every $a, b \in [0, 1]$, (commutativity);
- iii. if $a \leq c$ and $b \leq d$, then $T(a, b) \leq T(c, d)$, (monotonicity) and
- iv. $T(a, T(b, c)) = T(T(a, b), c)$ ($a, b, c \in [0, 1]$). (associativity).

Definition 2.4: [12] **Menger space** or Menger probabilistic metric space, is a triplet (K, F, T) , where (K, F) is a probabilistic metric space and t is a triangular norm which satisfies the condition:

$$(PM 5) F_{u,w}(x + y) \geq t(F_{u,v}(x), F_{v,w}(y)), \text{ for every } u, v, w \in K \text{ and } x, y > 0.$$

Definition 2.5: [14] A mapping $f : K \rightarrow K$ in Menger space (K, F, t) is said to be **continuous** at a point $u \in K$ if for every $\varepsilon > 0$ and $\lambda > 0$, there exists $\varepsilon_1 > 0$ and $\lambda_1 > 0$ such that if $F_{u,v}(\varepsilon_1) > 1 - \lambda_1$, then $F_{fu, fv}(\varepsilon) > 1 - \lambda$

Definition 2.6: [14] Let (K, F, T) be a **Menger space** and t be a continuous T-norm. Then,

- (a) A sequence $\{x_n\}$ in K is said to **converge** to a point x in K if and only if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = (N, \varepsilon) > 0$ such that $F_{x_n, x}(\varepsilon) > 1 - \lambda$ for all $n \geq N$. We write $\lim_{n \rightarrow \infty} x_n = x$.
- (b) A sequence $\{x_n\}$ in K is said to be a **Cauchy sequence** if for every $\varepsilon > 0$ and $\lambda > 0$, there exists an integer $N = (N, \varepsilon) > 0$ such that $F_{x_n, x_m}(\varepsilon) > 1 - \lambda$ for all $n, m \geq N$.
- (c) A Menger space (K, F, T) is said to be **complete** if every Cauchy sequence in K converges to a point in K .

Definition 2.7: [25] Two mappings $S, T : K \rightarrow K$ are said to be **compatible mappings** in Menger space (K, F, t) iff

$$\lim_{n \rightarrow \infty} F_{STx_n, TSx_n}(x) = 1 \quad \text{for all } x > 0$$

whenever $\{x_n\}$ is a sequence in K such that $\lim_{n \rightarrow \infty} Sx_n = \lim_{n \rightarrow \infty} Tx_n = z$ for some z in K .

Definition 2.8: [30] Let $S, T : K \rightarrow K$ be two self-mappings in Menger Space (K, F, t) . Then, $x \in K$ is said to be a coincidence point of S and T iff $Sx = Tx = w$ for some $x \in K$. And $w \in K$ is called a point of coincidence of S and T .

Example 1: Let $K = [0, 5]$ equipped with the usual metric $d(x, y) = |x - y|$ and define self-maps $S, T : [0, 5] \rightarrow [0, 5]$ by

$$S(x) = \begin{cases} 5 - x, & \text{if } 0 \leq x < 3, \\ 5, & \text{if } 3 \leq x \leq 5, \end{cases}$$

and

$$T(x) = \begin{cases} x, & \text{if } 0 \leq x < 3, \\ 5, & \text{if } 3 \leq x \leq 5, \end{cases}$$

Then, we see that for any $x \in [3, 5]$, x is a coincidence point and $STx = TSx$, showing S and T are weakly compatible.

Definition 2.9: [24] Two self-mappings $S, T : K \rightarrow K$ are said to be **weakly compatible mapping** or **coincidentally commuting** in Menger space (K, F, t) if they commute at their coincidence points, that is if $Sx = Tx$ then $STx = TSx$ for some $x \in K$.

In 2008, M.A. Al-Thapagi and N. Shahzad [26] introduced the concept of occasionally weakly compatible mappings in metric space and extended it in Menger space by B.D. Pant et al [27] as:

Definition 2.10: Two self-mappings $S, T : K \rightarrow K$ are said to be **occasionally weakly compatible mapping** (shortly **owc**) in Menger space (K, F, t) if there is a point x in K which is a coincidence point of S and T at which S and T commute.

Remark 2.1: Weakly compatible is occasionally weakly compatible but the converse is not true.

Example 2.2: Let \mathbb{R} be a usual metric space and define two self-mappings S and T by $S(x) = 3x$ and $T(x) = x^2$ for all $x \in \mathbb{R}$. We see here that $Sx = Tx$ for $0, 3$ And $ST0 = TS0$ but $ST3 \neq TS3$. So, S and T are not weakly compatible but occasionally weakly compatible.

3. Main Results

The followings lemmas help us to prove main theorem:

Lemma 3.1: [27] Let S and T be occasionally weakly compatible self-mappings of Menger space (K, F, t) . If S and T have a unique point of coincidence, $w = Sx = Tx$ then w is a unique common fixed point of S and T .

Lemma 3.2: [24] Let (K, F, t) be a Menger space. If there exists $k \in (0, 1)$ such that for all $x, y \in K$, $F_{x,y}(kv) \geq F_{x,y}(v)$, $\forall v > 0$ then $x = y$.

Now we prove our main theorem in four and six self-mappings by using occasionally weakly compatible mappings in Menger space.

Theorem 3.1: Let (K, F, t) be complete Menger space and $Q, S, R, T : K \rightarrow K$ be four mappings. Let (Q, R) and (S, T) be occasionally weakly compatible mappings. And if there exists a constant $k \in (0, 1)$ such that

$$[F_{Qx, Sy}(kv)]^2 \geq c_1 \min\{[F_{Rx, Qx}(v)]^2, [F_{Ty, Sy}(v)]^2, [F_{Rx, Ty}(v)]^2\} + c_2 \min\{F_{Rx, Qx}(v)F_{Rx, Sy}(v)F_{Qx, Ty}(v)F_{Sy, Ty}(v)\} + c_3 F_{Sy, Rx}(v)F_{Ty, Qx}(v) \quad (3.1.1)$$

for all $x, y \in K$, and $v > 0, c_1, c_2, c_3 > 0$ and $c_1 + c_2 + c_3 > 1$. Then there exists a unique point $w \in K$ such that $Qw = Rw = w$ and a unique point $k \in K$ such that $Sk = Tk = k$. Moreover, $k = w$ so that there is a unique common fixed point of Q, S, R and T .

Proof. Consider (Q, R) and (S, T) to be occasionally weakly compatible mappings. Then there exist $x, y \in K$ such that $Qx = Rx$ and $Sy = Ty$. We claim $Qx = Sy$. If $Qx \neq Sy$ then from 3.1.1

$$\begin{aligned} [F_{Qx, Sy}(kv)]^2 &\geq c_1 \min\{[F_{Qx, Qx}(v)]^2, [F_{Sy, Sy}(v)]^2, [F_{Qx, Sy}(v)]^2\} \\ &\quad + c_2 \min\{F_{Qx, Qx}(v)F_{Qx, Sy}(v)F_{Qx, Sy}(v)F_{Sy, Sy}(v)\} + c_3 F_{Sy, Qx}(v)F_{Sy, Qx}(v) \\ \text{or, } [F_{Qx, Sy}(kv)]^2 &\geq c_1 \min\{1, 1, [F_{Qx, Sy}(v)]^2\} + c_2 \min\{1, F_{Qx, Sy}(v)F_{Qx, Sy}(v), 1\} + c_3 F_{Sy, Qx}(v)F_{Sy, Qx}(v) \\ \text{or, } [F_{Qx, Sy}(kv)]^2 &\geq c_1 [F_{Qx, Sy}(v)]^2 + c_2 [F_{Qx, Sy}(v)]^2 + c_3 [F_{Qx, Sy}(v)]^2 \\ \text{or, } [F_{Qx, Sy}(kv)]^2 &\geq (c_1 + c_2 + c_3) [F_{Qx, Sy}(v)]^2 \end{aligned}$$

Which is a contradiction because $c_1 + c_2 + c_3 > 1$.

So, by lemma (3.2), we have

$Qx = Sy$, that is $Qx = Rx = Sy = Ty$. Let z be another point such that $Qz = Rz$, then by 3.1.1 $Qz = Rz = Sy = Ty$. So, $Qx = Qz$ and $w = Qx = Rx$ is the unique point of coincidence of Q and R . By lemma (3.1), w is the only common fixed point of Q and R . That is $w = Qw = Rw$. Similarly, there is a unique point $z \in K$ such that $z = Sz = Tz$. Assume that $w \neq z$. Then by 3.1.1, we have

$$\begin{aligned} [F_{w, z}(kv)]^2 &= [F_{Qw, Sz}(kv)]^2 \geq c_1 \min\{[F_{Rw, Qw}(v)]^2, [F_{Tz, Sz}(v)]^2, \\ &\quad [F_{Rw, Tz}(v)]^2\} + c_2 \min\{F_{Rw, Qw}(v)F_{Rw, Sz}(v)F_{Qw, Tz}(v)F_{Sz, Tz}(v)\} + c_3 F_{Sz, Rw}(v)F_{Tz, Qw}(v) \\ \text{or, } [F_{w, z}(kv)]^2 &= [F_{Qw, Sz}(kv)]^2 \geq c_1 \min\{[F_{w, w}(v)]^2, [F_{z, z}(v)]^2, \\ &\quad [F_{w, z}(v)]^2\} + c_2 \min\{F_{w, w}(v)F_{w, z}(v)F_{w, z}(v)F_{z, z}(v)\} + c_3 F_{z, w}(v)F_{z, w}(v) \\ [F_{w, z}(kv)]^2 &= [F_{Qw, Sz}(kv)]^2 \geq c_1 \min\{1, 1, [F_{w, z}(v)]^2\} + c_2 \min\{1, [F_{w, z}(v)]^2, 1\} + c_3 [F_{w, z}(v)]^2 \\ &\geq c_1 \min\{[F_{w, z}(v)]^2\} + c_2 \min\{[F_{w, z}(v)]^2\} + c_3 [F_{w, z}(v)]^2 \end{aligned}$$

$\geq (c_1 + c_2 + c_3) [F_{w, z}(v)]^2$ Which is a contradiction because $c_1 + c_2 + c_3 > 1$.

Therefore, by lemma (3.1) $z = w$ and z is a common fixed point of Q, S, R and T .

Uniqueness: Let $z_1 (z_1 \neq z)$ be another common fixed point of Q, S, R and T .

Taking $x = z$ and $y = z_1$ then from 3.1.1, we have

$$\begin{aligned} [F_{Qz, Sz_1}(kv)]^2 &\geq c_1 \min\{[F_{Rz, Qz}(v)]^2, [F_{Tz_1, Sz_1}(v)]^2, \\ &\quad [F_{Rz, Tz_1}(v)]^2\} + c_2 \min\{F_{Rz, Qz}(v)F_{Rz, Sz_1}(v)F_{Qz, Tz_1}(v)F_{Sz_1, Tz_1}(v)\} + c_3 F_{Sz_1, Rz}(v)F_{Tz_1, Qz}(v) \end{aligned}$$

On simplification, $[F_{z, z_1}(kv)]^2 \geq (c_1 + c_2 + c_3) [F_{z, z_1}(v)]^2$

Which is a contradiction because $c_1 + c_2 + c_3 > 1$. Thus, by lemma (3.1), $z = z_1$ and hence uniqueness of common fixed point.

The following example illustrate Theorem 3.1

Example 3: Let $K = [0, 20]$ with metric d defined by $d(x, y) = |x - y|$ and F is defined by

$$F_{x,y}(t) = \begin{cases} \frac{t}{t + |x - y|} & \text{for } t > 0 \\ 0 & \text{for } t = 0 \end{cases}$$

for all $x, y \in K$. Then, (K, F, t) be a complete Menger space with $t(a, b) = \min(a, b)$ for all $a, b \in [0, 1]$. Let $Q, R, S, T : K \rightarrow K$ be defined by

$$Q(x) = \begin{cases} 0 & \text{for } x = 0 \\ 4 & \text{for } 0 < x \leq 20 \end{cases}$$

$$S(x) = \begin{cases} 0 & \text{for } x = 0 \\ 7 & \text{for } 0 < x \leq 20 \end{cases}$$

$$R(x) = \begin{cases} 0 & \text{for } x = 0 \\ 11 - x & \text{for } 0 < x \leq 11 \\ x - 7 & \text{for } 11 < x \leq 20 \end{cases}$$

and

$$T(x) = \begin{cases} 0 & \text{for } x = 0 \\ 11 - x & \text{for } 0 < x \leq 11 \\ x - 4 & \text{for } 11 < x \leq 20 \end{cases}$$

Then Q, S, R and T satisfy all the conditions of the above Theorem 3.1 with $k \in (0, 1)$ and have a unique common fixed point $x = 0$ in K . That is, $Q(0) = 0 = R(0)$, $QR(0) = 0 = RQ(0)$ and $S(0) = 0 = T(0)$, $ST(0) = 0 = TS(0)$. Also, Q and S as well as R and T are owc. Hence, 0 is unique common fixed point of Q, S, R and T . These maps are discontinuous at 0.

If we put $Q = S$ and $R = T$ in theorem 3.1 then, we obtain

Corollary 3.1: Let (K, F, t) be Menger Space and $Q, R : K \rightarrow K$ be mappings. Let (Q, R) be occasionally weakly compatible mappings. And if there exist a constant $k \in (0, 1)$ such that

$$[F_{Qx, Qy}(kv)]^2 \geq c_1 \min\{[F_{Rx, Qx}(v)]^2, [F_{Ry, Qy}(v)]^2, [F_{Rx, Ry}(v)]^2\} + c_2 \min\{F_{Rx, Qx}(v)F_{Rx, Qy}(v)F_{Qx, Ry}(v)F_{Qy, Ry}(v)\} + c_3 F_{Qy, Rx}(v)F_{Ry, Qx}(v)$$

for all $x, y \in K$, and $v > 0$, $c_1, c_2, c_3 > 0$ and $c_1 + c_2 + c_3 > 1$. Then, there is a unique common fixed point of Q and R .

Here establishing the next theorem in six self-mappings may generalize and extends similar theorem in literature.

Theorem 3.2: Let (K, F, t) be a complete Menger space with $t(x, y) = \min(x, y)$ for all $x, y \in [0, 1]$ and $P, Q, R, S, T, U : K \rightarrow K$ be mappings such that

- i. The pairs (PQ, T) and (RS, U) are occasionally weak compatible,
- ii. There exists a constant $k \in (0, 1)$ such that

$$F_{PQx, RSy}(kv) \geq \min\{F_{Tx, Uy}(v), \frac{1}{2}[F_{PQx, Tx}(t) + F_{RSy, Uy}(v)], \frac{1}{2}[F_{PQx, Uy}(v) + F_{RSy, Tx}(v)]\}$$

for all $x, y \in K$ and $v > 0$.

Then, PQ, RS, T, U have a unique common fixed point in K . Furthermore, if the pairs (P, Q) and (R, S) are commuting pairs of mappings then P, Q, R, S, T, U have a unique common fixed point.

Proof. Here, (PQ, T) and (RS, U) are occasionally weak compatible. So, for all $x, y \in K$, we have $PQx = Tx$ and $RSy = Uy$. We claim $RSy = Uy$.

From condition (ii), we have

$$\begin{aligned} F_{PQx,RSy}(kv) &\geq \min\{(F_{PQx,RSy}(v), \frac{1}{2}[F_{PQx,PQx}(v) + F_{RSy,RSy}(v)], \frac{1}{2}[F_{PQx,RSy}(v) + F_{RSy,PQx}(v)]\}) \\ &\geq \min\{(F_{PQx,RSy}(v), 1, \frac{1}{2}[F_{PQx,RSy}(v) + F_{RSy,PQx}(v)]\}) \\ &\geq \min\{(F_{PQx,RSy}(v), 1, 1)\} \\ &\geq F_{PQx,RSy}(v) \end{aligned}$$

From lemma 3.2, we get, $PQx = RSy$. So,

$$PQx = Tx = RSy = Uy \quad (3.1)$$

Moreover, if there is another coincidence point z such that $PQz = Tz$ then from condition (ii), we get

$$PQz = Tz = RSy = Uy \quad (3.2)$$

Also, from relation (3.1) and (3.2) it follows that $PQx = PQz$ $z = x$. // Hence, $w = PQx = Tx$ for $w \in K$ is the unique point of coincidence of PQ and T .

By lemma (3.1), w is unique common fixed point of PQ and T .

Hence, $PQw = Tw = w$. Similarly, there is unique common fixed point $u \in K$ such that $u = RSu = Uu$.

Uniqueness: Suppose that $u \neq w$ then by condition (ii)

$$\begin{aligned} F_{w,u}(kv) = F_{PQw,RSu}(kv) &\geq \min\{(F_{Tw,Uu}(v), \frac{1}{2}[F_{PQw,Tw}(v) + F_{RSu,Uu}(v)], \frac{1}{2}[F_{PQw,Uu}(v) + F_{RSu,Tw}(v)]\}) \\ &\geq \min\{(F_{w,u}(v), \frac{1}{2}[F_{w,w}(v) + F_{u,u}(v)], \frac{1}{2}[F_{w,u}(v) + F_{u,w}(v)]\}) \\ &\geq \min\{(F_{w,u}(v), 1, \frac{1}{2}[F_{w,u}(v) + F_{u,w}(v)]\}) \\ &\geq \min\{(F_{w,u}(v), 1, F_{w,u}(v))\} \\ &\geq F_{w,u}(v) \end{aligned}$$

By lemma (3.2), $w = u$.

Hence, w is unique common fixed point of PQ, RS, T, U . Finally, we have to show that w is only the common fixed point of P, Q, R, S, T and U . If the pairs (P, Q) and (R, S) are commuting pairs.

We may write, $Pw = P(PQw) = P(QPw) = PQ(Pw)$ $Pw = w$. Also, $Qw = Q(PQw) = QP(Qw) = PQ(Qw)$ $Qw = w$. Similarly, we have $Rw = w$ and $Sw = w$.

Hence, P, Q, R, S, T and U have a unique common fixed point.

Example 4: The following example satisfies the condition of Theorem 3.2 with $k \in (0, 1)$ and $u = 1$ is a common fixed point of P, Q, R, S, T and U where consider $X = [0, 1]$ with usual metric space (X, d) defined by $d(x, y) = (x - y)^2$. And distribution function F is defined by

$$F_{x,y}(t) = \begin{cases} \frac{t}{t + |x - y|} & \text{for } t > 0 \\ 0 & \text{for } t = 0 \end{cases}$$

for all $x, y \in X$. Then, (X, F, t) be a complete Menger space with $t(a, b) = \min(a, b)$ for all $a, b \in [0, 1]$. Defining self mappings P, Q, R, S, T and U in X as:

$$Pu = \frac{u+1}{2}, Qu = \frac{2+3u}{5}, Ru = \frac{2u+1}{3}, Su = \frac{u+3}{4}, Tu = \frac{3u+1}{4} \text{ and } Uu = \frac{2u+3}{5}.$$

As the consequences of Theorem 3.2, we have following theorem in metric space:

Theorem 3.3: Let (K, d) be a complete metric space and P, R, T and U be self-mappings in K such that

- i. (P, T) and (R, U) are occasionally weakly compatible.
- ii. $d(Px, Ry) \leq \phi(\max\{(d(Tx, Ty), \frac{1}{2}[d(Px, Tx) + d(Ry, Uy)], \frac{1}{2}[d(Px, Uy) + d(Ry, Tx)]\})$
for all $x, y \in K$.

Then, P, R, T and U have a unique common fixed point.

4. Conclusions

Our theorem 3.1 generalizes the theorems of Sharma and Shahu [1] and B. Fisher et.al [2]. And theorem 3.2 extends and generalize theorem U. Rajopadhayaya et al [31] and G. Jungck and B.E. Rhodes [16]. Our result also generalizes and improves other similar results in literature.

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