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Mathematical model of long jump

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Abstract

This paper presents a mathematical model of long jump. We determine the model of velocity, maximum distance, length of time in the air, maximum height, and optimum take-off angle. The modeling is done by assuming that the initial velocity is constant and the long jumper can determine the take-off angle when take-off and landing. The results of this modeling can explain physically how a jumper can do a very spectacular jump and get the maximum distance. We find that the optimum take-off angle is about 35.26°.

Keywords: mathematical model, long jump, velocity, distance, angle

1. Introduction

Mathematical modelling is the process of formulating real world situations in mathematical terms. Mathematical modelling takes observed real world behavior or phenomena and describes them using mathematical equations. Mathematical models can be found in everywhere, not only in science, but also in the social sciences and business. By constructing mathematical models, we can often explain real world behaviors and predict how sensitive real world situations are to certain changes [1]. The following is a summary of the standard steps for constructing a mathematical model: (i) identifying the problem, (ii) deriving the model, including the constants and variables involved, and making the assumptions about which variables to include in the model and the interrelationships between the variables, (iii) solving the equations and interpreting the model, (iv) verifying the model, and (v) refine the model [2], [3].

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Projectile motion has been exploited many times over the centuries in many different ways. If an object is fired, shot, thrown or kicked up in the air, it only come down under the influence of gravity. Therefore, the motion can be utilized to analyze the similar motion in a wide variety of sports, including many Olympic events such as high jump, triple jump, shooting, badminton, football, basketball, and long jump. Long jump involves projectile motion and mathematics can be used to investigate the optimum take-off angle and the maximum long jump. Long jumper try to project their body for maximum horizontal distance beyond a take-off line. While in the air the jumper is essentially a projectile in free flight and so the flight distance achieved is determined by the conditions at take-off [4].

One of the important thing to be considered in long jump is take-off angle. However, the optimum take-off angle that produces the greatest jump distance is not well understood. Red and Zogaib (1997) [5] calculated the optimum release angle by combining the measured relation between release speed and release angle for the jumper. The study calculated the optimum release angle in around $37°$. Tan and Zumerchik (2000) [6] determined the optimum take-off angle by assuming that the jumper loses a constant friction of translational kinetic energy at take-off. The predicted optimum take-off angle for a top male jumper is about $33°$. Further, Linthorne (2001) [7] calculated the release angles $31° - 35°$. Linthorne et al. (2005) [4] pointed out that the take-off speed that the athlete is able to produce decreases with increasing take-off angle. For the long jumpers in the study, this lowered the optimum take-off angle by $18° - 23°$. Tsuboi (2010) [8] derived the optimum take-off angle based on the maximization problem of the flight distance.

Although there have been numerous studies of long jump, but there has been minimal mathematical research on it. The aims of this study are to develop mathematical model of long jump and to determine and analyze the model of velocity, maximum distance, length of time in the air, maximum height, and optimum take-off angle.

2. Jump Velocity in the Horizontal Plane

There are four phases in long jump, namely run-up, take-off, aerial, and landing. In mechanics, the optimum angle of a projectile is $45°$ in order to get the maximum distance in a horizontal plane. However, the assumption does not work in the long jump. In this case, the long jumper use the kinetic energy of run-up in order to take off with large launching velocity. In fact, the jumper does not jump and land on the ground vertically. Most long jumpers use the angles of $20° - 30°$ at their take-off. Therefore, the center of gravity is set at the same height at the center of the mass (see Figure 1).

The diagram of speed and take-off angle is illustrated in Figure 2, where v_0 is the initial velocity before jumping, v_1 is the velocity when jumping, and α is the take-off angle. Let $E_0 = \frac{1}{2}mv_0^2$ is the kinetic energy before making a jump. The energy does not fully work. There is an energy reduction

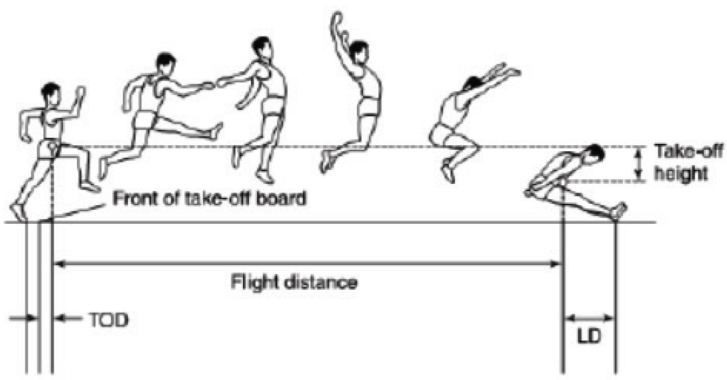

Figure 1: The center of mass of an athlete when take-off and landing.

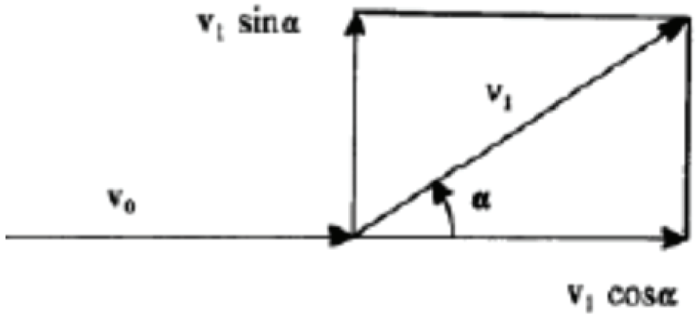

Figure 2: Diagram of speed and take-off angle.

with coefficient γ and the other part is converted into vertical motion $\Delta E = \frac{1}{2} m v_1^2 \sin^2 \alpha$. Based on the conservation law of energy, we have

$$E_1 = E_0 - \gamma E_0 - \Delta E .$$

Hence, $E_1 = \frac{1}{2} m v_0^2 - \gamma \frac{1}{2} m v_0^2 - \frac{1}{2} m v_1^2 \sin^2 \alpha$. Thus, we obtain

$$v_1^2 = v_0^2 - \gamma v_0^2 - v_1^2 \sin^2 \alpha .$$

By solving this equation, the following model of jump velocity is obtained.

$$v_1 = \frac{(1-\gamma)^{0.5}}{(1+\sin^2 \alpha)^{0.5}} v_0 .$$

3. Maximum Distance

The diagram of jump position is illustrated in Figure 3. From the figure it can be observed that the long jumper landed with his feet jutting forward to get the maximum distance. At this position, the center of gravity of the jumper is at a distance h below the position before making a jump and at a distance b between the center of gravity and the foot. Hence, $h = a - b \sin \beta$, where β is the landing angle.

Thus, the long jump distance model is the sum of the previous parabolic equation, the distance after passing the horizontal line, and the length of the foot. We have

$$v' = v_1 \sin \alpha - \frac{1}{2} g t' .$$

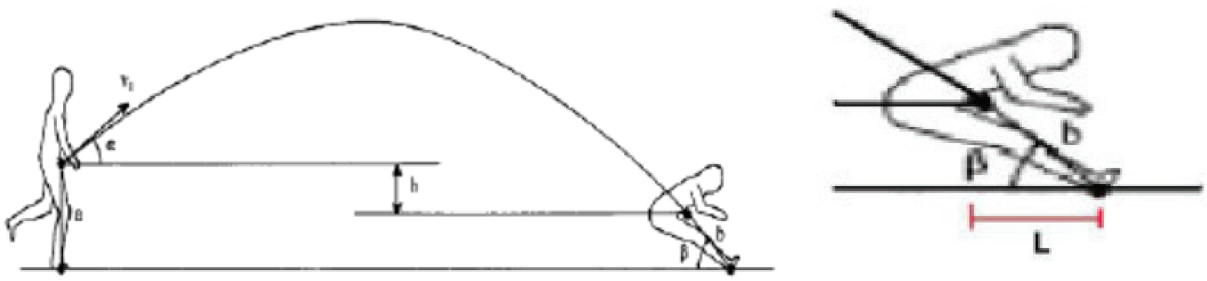

Figure 3: Diagram of jump position.

Since the velocities are in the opposite direction, the values are negative:

$$-v' = -v_1 \sin\alpha - \frac{1}{2} gt'$$
$$-y = -v_1 t' \sin\alpha - \frac{1}{2} gt'^2.$$

With $y = h$, we have $-h = -v_1 t' \sin\alpha - \frac{1}{2} gt'^2$, or

$$v_1 t'^{\sin\alpha} + \frac{1}{2} gt'^2 - h = 0.$$

The positive solution of the equation is

$$t' = \frac{-v_1 \sin\alpha + \sqrt{(v_1 \sin\alpha)^2 + 2gh}}{g}.$$

By substituting the solution into the distance equation, we obtain

$$x = v_1 \cos\alpha \frac{-v_1 \sin\alpha + \sqrt{(v_1 \sin\alpha)^2 + 2gh}}{g}$$

Thus, the model can be formulated as follows.

$$R = \frac{v_1 \cos\alpha}{g} (v_1 \sin\alpha + (v_1^2 \sin^2\alpha + 2gh)^{1/2}) + L,$$

where $L = b\cos\beta$ is an additional distance when the long jumper lands. From the formulation of v_1, the model can be rewritten as

$$R = \left(A \frac{\sin 2\alpha}{1 + \sin^2\alpha} \right) \left(1 + \left(1 + B \frac{\sin 2\alpha}{1 + \sin^2\alpha} \right)^{1/2} \right) + L,$$

where $B = h/A$. From the model, the landing angle β can be made as small as possible while supporting the jumper's body.

4. Length of Time in the Air and the Maximum Height

The length of time in the air until touch the ground is affected by the position of the foot when landing. When the long jumper reaches the ground, the vertical position is zero, i.e., $y = 0$. Hence, we have $0 = v_1 t \sin\alpha - \frac{1}{2} gt^2$, or

$$t_R = \frac{2v_1 \sin\alpha}{g}.$$

There is another very short time after the jumper passes the horizontal line from the first center of mass, that is t'. Therefore, the following total time is obtained

$$t = \frac{v_1 \sin\alpha + \sqrt{(v_1 \sin\alpha)^2 + 2gh}}{g},$$

Since the position of an object at y axis is

$$y = v_1 t \sin\alpha - \frac{1}{2} g t^2,$$

by substituting the total time t into the equation, we have the following maximum height.

$$h = \frac{(v_1 \sin\alpha)^2}{2g} = \frac{(1-\gamma)(v_0 \sin\alpha)^2}{2g(1+\sin^2\alpha)}.$$

Dari model tersebut, prestasi lompat jauh bergantung pada kecepatan awal dan sudut lompatan. Sudut lompatan optimal sekitar 35.6° dari titik pusat gravitasi pelompat.

5. Optimum Take-off angle

Let $R = \dfrac{v_1^2 \sin 2\alpha}{g}$ and $A = \dfrac{(1-\gamma)v_0^2}{g}$. Then, we obtain

$$R = \frac{A \sin 2\alpha}{1+\sin^2\alpha}.$$

The first and second derivatives of R subject to α are given by

$$\frac{dR}{d\alpha} = \frac{3\cos 2\alpha - 1}{(1+\sin^2\alpha)^2} A \text{ and } \frac{d^2R}{d\alpha^2} = \frac{4(3\sin 4\alpha + 14\sin 2\alpha)}{(\cos 2\alpha - 3)^3} A.$$

By taking $\dfrac{dR}{d\alpha} = 0$, the following optimum take-off angle is obtained.

$$a_m = \frac{1}{2}\cos^{-1}\frac{1}{3} = 35.26°.$$

It can be observed that the angle is independent on γ and v_0.

6. Conclusions

Based on the conservation of energy law, the kinetic energy of the long jump is distributed to other forms such as heat energy and sound energy. The energy is also spent for vertical movement, since the running velocity will not be the same as the jumping velocity. The energy combination is formulated by multiplication between a constant γ and the kinetic energy. The other energy is used for vertical movement. Moreover, the velocity after take-off is affected by the initial velocity. The greater the initial velocity, the greater the velocity in the air. The model points out that the optimum jump is more affected by the initial velocity than the initial position. In the maximum distance model, the maximum jump is influenced by the take-off angle, and the optimum angle is 35.26°.

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