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# Proposed theorems on a lorentzian Kähler space-time manifold admitting Bochner curvature tensor

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## Abstract

The objective of this paper is to investigate the Lorentzian Kähler space-time manifold that is Bochner flat. We have demonstrated that a Lorentzian Kähler space-time manifold with Bochner flatness is also an Einstein manifold. Furthermore, we have established that the energy-momentum tensor is covariantly constant when the manifold satisfies the Einstein field equation with a cosmological constant. Additionally, we have determined that the energy-momentum tensor of a perfect fluid Lorentzian Kähler space-time manifold exhibits hybrid characteristics. In the final section, we analyse the behaviour of a dust fluid Lorentzian Kähler space-time manifold where the Bochner curvature vanishes.

*Key words and phrases:* Differential equations, partial differential equations, nonlinear equations, cosmology, theory of relativity, Lorentzian Kähler space-time manifolds, Bochner curvature tensor, Einstein space, perfect fluid space-time, Einstein field equation, energy momentum tensor.

## 1. Introduction

A space-time is defined as a connected four-dimensional semi-Riemannian manifold denoted by ($M^4$, $g$), where $g$ represents the Lorentzian metric with a signature of (–, +, +, +). This metric characterizes the geometry of space-time. The study of space-time manifold provides the opportunity to extend the

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theory with the help of differential equations, partial differential equations, nonlinear equations, cosmology, and the theory of relativity. B. O' Neill in 1983 [1] started the investigation of astrophysics, cosmology, and general relativity. Furthermore, a number of authors explored space-time in a variety of ways. In 2015, De and Velimirovic [2] investigated Spacetimes with a semisymmetric energy-momentum tensor. In 2009, Ahsan and Siddiqui [3] conducted research exploring the relationship between the concircular curvature tensor and fluid spacetimes. Güler and Demirbag in 2016 [4] developed generalised quasi-Einstein spacetimes. In 2014, Arslan et al. [5] conducted research that specifically examined generalized Robertson-Walker spacetimes. Recently, in 2023 B. B. Chaturvedi et al. [6] studied Novel theorems for a Bochner Flat Lorentzian Kähler Space-time Manifold with $\eta$-Ricci-Yamabe Solitons. $M$-projectively flat spacetimes were discussed by Zengin in 2012 [7]. Pseudo-$Z$ symmetric spacetime was researched by Mantica and Suh in 2014 [8]. Spacetime admitting $W_2$- curvature tensor, pseudo-projective curvature tensor, quasi-conformal curvature tensor, were all explored by Mallick et al. (see ref. [9]–[11]). Young Jin Suh in 2021 [12] investigated Spacetimes admitting pseudo-quasi-conformal curvature tensor. For the further studies on space time and solutions, we refer to ([21–26]).

A semi-Riemannian manifold $(M^n, g)$ with an even dimension, where the metric $g$ is Lorentzian is known as a Lorentzian Kähler manifold if it satisfies the conditions:

$$J^2(\omega_1) = -\omega_1,\ g(J\omega_1, J\omega_2) = g(\omega_1, \omega_2),\ and\ (\nabla_{\omega_1} J)\omega_2 = 0, \tag{1.1}$$

where $\omega_1$ and $\omega_2$ are vector fields, $J$ is a tensor field of type (1,1) with the property $J(\omega_1) = \omega_1$, and $\nabla$ is a Levi-Civita connection. The following relations hold in a Lorentzian Kähler manifold:

$$g(J\omega_1, \omega_2) = -g(\omega_1, J\omega_2), \tag{1.2}$$

$$S(J\omega_1, \omega_2) = -S(\omega_1, J\omega_2), \tag{1.3}$$

$$S(J\omega_1, J\omega_2) = S(\omega_1, \omega_2). \tag{1.4}$$

In this paper, we adopt the assumption that a four-dimensional Lorentzian Kähler manifold can be referred to as a Lorentzian Kähler space-time manifold. This assumption serves as the basis for our study throughout the paper.

S. Bochner [13] established the notion of the Bochner curvature tensor in 1949. The Bochner curvature tensor, denoted as $B$, is defined by the following equation:

$$\begin{aligned}
B(\omega_1,\omega_2,\omega_3,\omega_4) = R(\omega_1,\omega_2,\omega_3,\omega_4) - \frac{1}{2(n+2)}\Big\{ & S(\omega_1,\omega_4)g(\omega_2,\omega_3) - S(\omega_1,\omega_3)g(\omega_2,\omega_4) \\
& + g(\omega_1,\omega_4)S(\omega_2,\omega_3) - g(\omega_1,\omega_3)S(\omega_2,\omega_4) + S(J\omega_1,\omega_4)g(J\omega_2,\omega_3) \\
& - S(J\omega_1,\omega_3)g(J\omega_2,\omega_4) + S(J\omega_2,\omega_3)g(J\omega_1,\omega_4) - g(J\omega_1,\omega_3)S(J\omega_2,\omega_4) \\
& - 2S(J\omega_1,\omega_2)g(J\omega_3,\omega_4) - 2g(J\omega_1,\omega_2)S(J\omega_3,\omega_4)\Big\} \\
& + \frac{r}{(2n+2)(2n+4)}\Big\{g(\omega_2,\omega_3)g(\omega_1,\omega_4) - g(\omega_1,\omega_3)g(\omega_2,\omega_4) \\
& + g(J\omega_2,\omega_3)g(J\omega_1,\omega_4) - g(J\omega_1,\omega_3)g(J\omega_2,\omega_4) - 2g(J\omega_1,\omega_2)g(J\omega_3,\omega_4)\Big\},
\end{aligned} \tag{1.5}$$

where, $B(\omega_1,\omega_2,\omega_3,\omega_4) = g(B(\omega_1,\omega_2)\omega_3,\omega_4)$, $R(\omega_1,\omega_2,\omega_3,\omega_4) = g(R(\omega_1,\omega_2)\omega_3,\omega_4)$, $S$ represents the Ricci tensor, and $r$ denotes the scalar curvature of the manifold.

Several authors have studied the Bochner curvature tensor in different manifolds through different approaches. B. B. Chaturvedi and B. K. Gupta [14–17] explored Bochner Ricci semi-symmetric Hermitian manifold, C-Bochner curvature tensor on almost C(λ) manifolds, Bochner Ricci pseudo-symmetric Hermitian manifolds and Bochner curvature tensor on Kaehler-Norden manifolds. O. Kassabov [18] investigated Bochner flat almost Kähler manifolds. Abood (2010) [19] studied

Almost Hermitian manifolds with flat Bochner tensor. T. S. Chauhan et al. [19] researched Einstein-Kaehlerian space with recurrent Bochner curvature tensor.

This paper is organized as follows: The paper begins with an introduction in the first section, followed by an overview of a Lorentzian Kähler space-time manifold that has a Bochner curvature tensor equal to zero. We then proceed to explore various geometric characteristics of this manifold. Moving on to the third section, we explore cosmological models that exhibit a vanishing Bochner curvature tensor. Finally, in the last section, we examine a Lorentzian Kähler space-time manifold with a dust fluid, emphasizing its properties when the Bochner curvature tensor is zero.

## 2. Lorentzian Kähler Space-time manifold with Vanishing Bochner Curvature Tensor

If we take into account the space-time framework of general relativity, we can express equation (1.5) as follows:

$$\begin{aligned} B(\omega_1,\omega_2,\omega_3,\omega_4) = R(\omega_1,\omega_2,\omega_3,\omega_4) - \frac{1}{12}\Big\{& S(\omega_1,\omega_4)g(\omega_2,\omega_3) - S(\omega_1,\omega_3)g(\omega_2,\omega_4) \\ &+ g(\omega_1,\omega_4)S(\omega_2,\omega_3) - g(\omega_1,\omega_3)S(\omega_2,\omega_4) + S(J\omega_1,\omega_4)g(J\omega_2,\omega_3) \\ &- S(J\omega_1,\omega_3)g(J\omega_2,\omega_4) + S(J\omega_2,\omega_3)g(J\omega_1,\omega_4) - g(J\omega_1,\omega_3)S(J\omega_2,\omega_4) \\ &- 2S(J\omega_1,\omega_2)g(J\omega_3,\omega_4) - 2g(J\omega_1,\omega_2)S(J\omega_3,\omega_4)\Big\} \\ &+ \frac{r}{(10)(12)}\Big\{g(\omega_2,\omega_3)g(\omega_1,\omega_4) - g(\omega_1,\omega_3)g(\omega_2,\omega_4) \\ &+ g(J\omega_2,\omega_3)g(J\omega_1,\omega_4) - g(J\omega_1,\omega_3)g(J\omega_2,\omega_4) - 2g(J\omega_1,\omega_2)g(J\omega_3,\omega_4)\Big\}, \end{aligned} \tag{2.1}$$

If $B(\omega_1,\omega_2,\omega_3,\omega_4) = 0$, then equation (2.1) leads to

$$\begin{aligned} R(\omega_1,\omega_2,\omega_3,\omega_4) - \frac{1}{12}\Big\{& S(\omega_1,\omega_4)g(\omega_2,\omega_3) - S(\omega_1,\omega_3)g(\omega_2,\omega_4) \\ &+ g(\omega_1,\omega_4)S(\omega_2,\omega_3) - g(\omega_1,\omega_3)S(\omega_2,\omega_4) + S(J\omega_1,\omega_4)g(J\omega_2,\omega_3) \\ &- S(J\omega_1,\omega_3)g(J\omega_2,\omega_4) + S(J\omega_2,\omega_3)g(J\omega_1,\omega_4) - g(J\omega_1,\omega_3)S(J\omega_2,\omega_4) \\ &- 2S(J\omega_1,\omega_2)g(J\omega_3,\omega_4) - 2g(J\omega_1,\omega_2)S(J\omega_3,\omega_4)\Big\} \\ &+ \frac{r}{(10)(12)}\Big\{g(\omega_2,\omega_3)g(\omega_1,\omega_4) - g(\omega_1,\omega_3)g(\omega_2,\omega_4) \\ &+ g(J\omega_2,\omega_3)g(J\omega_1,\omega_4) - g(J\omega_1,\omega_3)g(J\omega_2,\omega_4) - 2g(J\omega_1,\omega_2)g(J\omega_3,\omega_4)\Big\} = 0. \end{aligned} \tag{2.2}$$

Taking a frame field over $\omega_1$ and $\omega_4$ in equation (2.2) and using equations (1.1), (1.2), (1.3) and (1.4), we obtain the following expression

$$S(\omega_2,\omega_3) = \frac{r}{10}g(\omega_2,\omega_3), \tag{2.3}$$

where $S$ represents the Ricci tensor and $r$ corresponds to the scalar curvature of the manifold. Consequently, we assert the subsequent theorem:

**Theorem 2.1:** *A Bochner flat Lorentzian Kähler space-time manifold is an Einstein space-time.*
The Einstein's field equation with cosmological constant [1], in terms of Ricci tensor $S$, scalar curvature $r$ and energy momentum tensor $T$ of type (0, 2) is given by

$$S(\omega_2,\omega_3) - \frac{r}{2}g(\omega_2,\omega_3) + \alpha g(\omega_2,\omega_3) = KT(\omega_2,\omega_3), \tag{2.4}$$

where $\alpha$ represents the cosmological constant, and $K$ corresponds to the gravitational constant.

By utilizing equations (2.3) and (2.4), We draw the following expression:

$$T(\omega_2,\omega_3)=\frac{1}{K}\left[\alpha-\frac{2}{5}r\right]g(\omega_2,\omega_3). \tag{2.5}$$

Taking covariant derivative of equation (2.5), we get

$$(\nabla\omega_5 T)(\omega_2,\omega_3)=\frac{-2}{5K}dr(\omega_5)g(\omega_2,\omega_3). \tag{2.6}$$

Since, from equation (2.3) a Bochner flat Lorentzian Kähler space-time manifold is Einstein, therefore we can write the differential equations

$$dr(\omega_5)=0, \tag{2.7}$$

for all $\omega_{5.}$
Putting equation (2.7) into equation (2.6), we derive the following expression:

$$(\nabla_{\omega 5}T)(\omega_2,\ \omega_3)=0. \tag{2.8}$$

Consequently, we assert the subsequent theorem:

**Theorem 2.2:** *The energy momentum tensor is covariant constant in a Bochner flat Lorentzian Kähler space-time manifold satisfying Einstein's field equation with cosmological constant.*

## 3. Cosmological Models with Vanishing Bochner Curvature Tensor

In this section, we investigate a perfect fluid in a Lorentzian Kähler space-time manifold that has a vanishing Bochner curvature tensor and satisfies Einstein's field equation without the inclusion of a cosmological constant.

The energy-momentum tensor $T$ describing a perfect fluid can be expressed in the following manner, as stated in the reference: [1]

$$T(\omega_2,\omega_3)=(\sigma+p)A(\omega_2)A(\omega_3)+pg(\omega_2,\omega_3), \tag{3.1}$$

where σ denotes the energy density and $p$ denotes the isotropic pressure. $A$ is a non-zero 1–*form* such that $g(\omega_2,\rho)=A(\omega_2)$, for all $\omega_2$ and $\rho$ is the velocity vector field of the flow, *i.e.*, $g(\rho,\rho)=-1$.

The equation that describes Einstein's field equation without the inclusion of a cosmological constant is stated as follows. [27]

$$S(\omega_2,\omega_3)-\frac{r}{2}g(\omega_2,\omega_3)=KT(\omega_2,\omega_3), \tag{3.2}$$

where, $r$ represents the scalar curvature of the manifold and $K\neq 0$.
Using equations (2.3), (3.1) and (3.2), we have

$$-\left(\frac{2}{5}r+Kp\right)g(\omega_2,\omega_3)=K(\sigma+p)A(\omega_2)A(\omega_3). \tag{3.3}$$

By contracting over $\omega_2$ and $\omega_3$ in equation (3.3), we obtain the following

$$r=\frac{5}{8}K(\sigma-3p). \tag{3.4}$$

Therefore, substituting equation (3.4) in equation (2.3), we can express the Ricci tensor $S$ of a Bochner flat Lorentzian Kähler space-time manifold as follows:

$$S(\omega_2, \omega_3) = \frac{1}{16} K(\sigma - 3p) g(\omega_2, \omega_3). \tag{3.5}$$

We know that the Ricci operator, denoted by $Q$, is defined in the following way

$$g(Q\omega_2,\omega_3) = S(\omega_2,\omega_3) \text{ and } S(Q\omega_2,\omega_3) = S^2(\omega_2,\omega_3). \tag{3.6}$$

Then, we get

$$A(Q\omega_2) = g(Q\omega_2,\rho) = S(\omega_2,\rho)$$

Thus from equation (3.5) and (3.6), we obtain

$$S(Q\omega_2,\omega_3) = S^2(\omega_2,\omega_3) = \left[\frac{K(\sigma-3p)}{16}\right]^2 g(\omega_2,\omega_3). \tag{3.7}$$

Taking contraction over $\omega_2$ and $\omega_3$ in equation (3.7), we obtain

$$\| Q \|^2 = \left[\frac{K(\sigma-3p)}{8}\right]^2. \tag{3.8}$$

Therefore, we can assert the following conclusion:

**Theorem 3.1:** *In a Bochner flat perfect fluid Lorentzian Kähler space-time manifold satisfying Einstein's field equation without a cosmological constant, the norm of Ricci operator is equal to* $\left[\frac{K(\sigma-3p)}{8}\right]$.

Now, if $\sigma = 3p$ (*i.e.*, the condition of a perfect fluid to be radiation fluid) in equation (3.8), then we can draw the following inference:

**Theorem 3.2:** *If a Bochner flat perfect fluid Lorentzian Kähler space-time manifold obeys Einstein's field equation without cosmological constant, then the length of the Ricci operator is zero if the perfect fluid is radiation fluid (i.e $\sigma = 3p$).*

Now, putting $\omega_2 = J\omega_2$ and $\omega_3 = J\omega_3$ in equation (3.2), the Einstein's field equation without cosmological constant becomes

$$S(J\omega_2, J\omega_3) - \frac{r}{2} g(J\omega_2, J\omega_3) = KT(J\omega_2, J\omega_3). \tag{3.9}$$

Subtracting equation (3.9) from equation (3.2) and using equations (1.1) and (1.4), we get

$$T(\omega_2, \omega_3) = T(J\omega_2, J\omega_3). \tag{3.10}$$

Consequently, we assert the subsequent theorem:

**Theorem 3.3:** *In a perfect fluid Lorentzian Kähler space-time manifold obeying Einstein's field equation without cosmological constant, the energy momentum tensor is hybrid.*

Putting $\omega_2 = \omega_3 = \rho$ in equation (3.3), we get

$$r = \frac{5}{2} K\sigma. \tag{3.11}$$

From equation (3.4) and (3.11), we get

$$\sigma + p = 0, \tag{3.12}$$

Thus from equation (3.12) we get $\sigma = -p$, which is termed as inflation or dark matter [20]. Consequently, we assert the subsequent theorem:

**Theorem 3.4:** *In a Bochner flat Lorentzian Kähler space-time manifold obeying Einstein's field equation without cosmological constant, the space-time represent dark matter or inflation.*

By employing equation (2.3) along with equation (2.4), we obtain the following expression

$$\left(\alpha - \frac{2}{5}r\right) g(\omega_2, \omega_3) = KT(\omega_2, \omega_3), \tag{3.13}$$

using equation (3.1) in equation (3.13), we get

$$\left(\alpha - \frac{2}{5}r - Kp\right) g(\omega_2, \omega_3) = K(\sigma + p)A(\omega_2)A(\omega_3). \tag{3.14}$$

Now, if we take Lorentzian Kähler space-time manifold with radiation fluid *i.e.*, $\sigma = 3p$, we get

$$\left(\alpha - \frac{2}{5}r - \frac{K\sigma}{3}\right) g(\omega_2, \omega_3) = \frac{4}{3}\sigma K A(\omega_2)A(\omega_3). \tag{3.15}$$

Contracting $\omega_2$ and $\omega_3$ in equation (3.15), we obtain

$$r = \frac{5}{2}\alpha. \tag{3.16}$$

Multiplying both side of equation (3.14) by $A(\omega_5)$ and then performing a contraction over $\omega_2$ and $\omega_5$, we obtain the following expression

$$r = \frac{5}{2}(\alpha + K\sigma). \tag{3.17}$$

From equation (3.16) and (3.17), we get

$$\sigma = 0 \textit{ i.e.}, p = 0, \tag{3.18}$$

This contradicts the assumption we made earlier.
Therefore, we can assert the following conclusion:

**Theorem 3.5:** *If the energy density of the perfect fluid in a Bochner flat Lorentzian Kähler space-time manifold, which satisfies Einstein's field equation with a cosmological constant, is non-zero, then the space-time is isotropic and homogeneous.*

## 4. Dust Fluid Lorentzian Kähler Space-time Manifold with Vanishing Bochner Curvature Tensor

The expression for the energy momentum tensor in the context of a dust model is stated as follows:

$$T(\omega_2, \omega_3) = \sigma A(\omega_2)A(\omega_3), \tag{4.1}$$

where, σ represents the energy density and $\rho$ represents the time-like unit flow vector field of the fluid, such that $A(\omega_2) = g(\omega_2, \rho)$ for all $\omega_2$, and $g(\rho, \rho) = -1$.

Thus using equation (3.2) and (4.1), the Einstein's field equation without cosmological constant can be expressed as

$$S(\omega_2, \omega_3) - \frac{r}{2} g(\omega_2, \omega_3) = K\sigma A(\omega_2)A(\omega_3), \tag{4.2}$$

contracting $\omega_2$ and $\omega_3$ in equation (4.2), we get

$$r = K\sigma. \tag{4.3}$$

Therefore, using equation (4.3) in equation (2.3), the Ricci tensor of a Bochner flat Lorentzian Kähler space-time manifold becomes

$$S(\omega_2, \omega_3) = \frac{K\sigma}{10} g(\omega_2, \omega_3). \tag{4.4}$$

Thus, in virtue of equation (3.7) and (4.4), we get

$$S(Q\omega_2, \omega_3) = \left[\frac{K\sigma}{10}\right]^2 g(\omega_2, \omega_3). \tag{4.5}$$

Contracting $\omega_2$ and $\omega_3$ in equation (4.5), we get

$$\| Q \|^2 = \left[\frac{K\sigma}{5}\right]^2. \tag{4.6}$$

Therefore, we can assert the following conclusion:

**Theorem 8:** *If a Bochner flat Lorentzian Kähler space-time manifold with dust cosmological model obeys Einstein's field equation without cosmological constant, then the norm of Ricci operator is equal to* $\left[\frac{K\sigma}{5}\right]$.

Now, using equation (2.3) and (4.2), we get

$$\frac{2}{5} rg(\omega_2, \omega_3) = -K\sigma A(\omega_2)A(\omega_3). \tag{4.7}$$

Taking contraction over $\omega_2$ and $\omega_3$ in equation (4.7), we get

$$r = \frac{5}{8} K\sigma. \tag{4.8}$$

By multiplying $A(\omega_5)$in equation (4.7) and then contracting over $\omega_2$ and $\omega_5$, we obtain

$$r = \frac{5}{2} K\sigma. \tag{4.9}$$

Now from equation (4.8) and (4.9), we get

$$\sigma = 0. \tag{4.10}$$

Therefore, from equation (4.1) and (4.10), we get

$$T(\omega_2, \omega_3) = 0. \tag{4.11}$$

Therefore, we can assert the following conclusion:

**Theorem 4.2:** *A Bochner flat dust fluid Lorentzian Kähler space-time manifold obeying Einstein's field equation without cosmological constant is vacuum.*

## 5. Conclusion

The results presented in this research paper have opened up a new direction in the study of Lorentzian Kähler space-time manifolds. The paper highlights the significance of a flat Bochner curvature tensor in such spaces. The results demonstrate how a Lorentzian Kähler space-time manifold with a flat Bochner curvature tensor can be transformed into an Einstein space-time. In the context of a cosmological model, when the Bochner curvature tensor vanishes in a Lorentzian Kähler space-time manifold containing a perfect fluid and dust fluid (without a cosmological constant), the Ricci operator can be expressed in terms of the gravitational constant, energy density, and pressure. Furthermore, the results (theorem 3.4) shed light on how the properties of a fluid influence the characteristics of this flat manifold.

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