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2-absorbing hyperideals and homomorphisms in join hyperlattices

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Abstract

In this paper, we investigate hyperlattices, which arise by replacing one (or both) binary operation(s) in a lattice with hyperoperation(s). Many authors have studied prime generalizations of ideals in rings and lattices. In this paper we focus on the prime generalizations of hyperideals in join hyperlattices. We introduce the notions of 2-absorbing, primary 2-absorbing primary, etc., in join hyperlattices and explore their interrelations. We establish that the intersection of two prime hyperideals is 2-absorbing, and the intersection of two Q-primary hyperideals is 2-absorbing primary. Finally, we study the of homomorphic images and pre-images of various types of hyperideals in join hyperlattices.

Key words and phrases: Hyperlattice, prime hyperideal, radical.

Mathematics Subject Classification (2010): Primary 06D35, 20N20

1. Introduction

The concept of a binary operation pertains to a system wherein two elements combine to produce another element. However, in many naturally occurring phenomena, the combination of two elements may yield more than one possible outcome. In such a scenario, the concept of hyperoperations proves more applicable than binary operations. A hyperoperation on a set $\mathbb{H}$ is a mapping $\circ : \mathbb{H}^2 \to \mathcal{P}^*(\mathbb{H})$, where $\mathcal{P}^*(\mathbb{H})$ denotes the collection of non-empty subsets of $\mathbb{H}$. The theory of hyperstructures was initially introduced by Marty [1–2] in 1934. Subsequently, various researchers have contributed to

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its development (see [3]). Preliminary definitions and propositions concerning hyperstructures can be seen in [4, 5] references, which further advanced and refined some notions. Pallavi et al. [6] studeied a different class of hyperstructure namely hypervector spaces over a hyperfield and extensively studied the properties of linear transformation. Also, the results discussed on lattice vector spaces [7] can be explored in the respective hyperstructures. Hyperlattice is a natural extension of a classical lattice. Out of the two binary operations $\vee$ and $\wedge$ of a lattice, at least one of the binary operations is taken as a hyperoperation. Konstantiniodou [8] et al. introduced the theory of hyperlattices. Later, the distributivity of P-hyperlattices was discussed in [9]. In [10], the prime ideal theorem for meet and join hyperlattices was proved. Rasouli et al. [11] considered hyperlattices, superlattices and their quotient structures with a regular relation and established a fundamental relation on a hyperlattice. In [12], the authors defined a topology on the collection of prime ideals and showed that it is a T_0-space. Asokkumar [13] obtained conditions under which the set of idempotents of a Krasner Hyperring form a hyperlattice, and also studied orthogonal idempotent elements. Indeed, the concept of pure ideals in hyperlattices and their algebraic, topological characterizations were obtained by Blaise et al. [14]. Lashkenari et al. [15] defined the completion on join hyperlattices and explored their properties. Lashkenari and Davvaz [16] explored the idea of semi prime ideals in ordered hyperlatttices. Davvaz and Lashkenari [17] established results on principle, regular and compact elements in hyperlattices. Kehayopulu [18] has explored the distributivity and modularity of different classes of hyperlattices. In [19–21], the authors have discussed the notions of different prime generalizations of ideals like 2-absorbing, primary, etc. in lattices. Pallavi et al. [22] have considered meet hyperlattices and established the properties of various prime generalizations of hyperideals.

In this paper, we introduce various generalizations of prime hyperideals, namely 2-absorbing, primary, 2-absorbing primary in join hyperlattices. furthermore, we study the properties of annihilators associated with these classes of hyperideals. We prove the results on homomorphic images and inverse images of 2-absorbing primary hyperideals. However, this situation does not hold in the case of a weak homomorphism. In particular, we show that the homomorphic image of a hyperideal under weak homomorphism need not be a hyperideal.

2. Preliminaries

Definition 2.1: [8] *Let $\mathbb{H}$ be a non-empty set, and $\mathcal{P}^*(\mathbb{H}) = \{A \subseteq H : A \neq \emptyset\}$, $\bigvee : \mathbb{H} \times \mathbb{H} \to \mathcal{P}^*(\mathbb{H})$ be a hyperoperation, and $\wedge : \mathbb{H} \times \mathbb{H} \to \mathbb{H}$ be a binary operation. Then $(\mathbb{H}, \bigvee, \wedge)$ is a join hyperlattice if the following conditions hold:*

1. $l_1 \in l_1 \bigvee l_1$ and $l_1 = l_1 \wedge l_1$;
2. $l_1 \bigvee (l_2 \bigvee l_3) = (l_1 \bigvee l_2) \bigvee l_3$ and $l_1 \wedge (l_2 \wedge l_3) = (l_1 \wedge l_2) \wedge l_3$;
3. $l_1 \bigvee l_2 = l_2 \bigvee l_1$ and $l_1 \wedge l_2 = l_2 \wedge l_1$;
4. $l_2 \in l_2 \wedge (l_1 \bigvee l_2) \cap l_2 \bigvee (l_1 \wedge l_2)$,

for all $l_1, l_2, l_3 \in \mathbb{H}$.

The relation '$\leq$' on $\mathbb{H}$ as follows:

$$l_1 \leq l_2 \text{ if and only if } l_1 \wedge l_2 = l_1.$$

Then $(\mathbb{H}, \leq)$ is a Poset [11].

Throughout, $(\mathbb{H}, \bigvee, \wedge)$ denotes a join hyperlattice.

Definition 2.2: [15] *A non-empty subset J of $\mathbb{H}$ is called a hyperideal if*

1. $l_1 \bigvee l_2 \subseteq J$; for all $l_1, l_2 \in J$,
2. $l_1 \in J, l_2 \in \mathbb{H}$, $l_2 \leq l_1$, then $l_2 \in J$, holds.

Definition 2.3: [15] *A proper hyperideal J of $\mathbb{H}$ is said to be prime if $l_1, l_2 \in \mathbb{H}$ and $l_1 \wedge l_2 \in J$ implies $l_1 \in J$ or $l_2 \in J$.*

Definition 2.4: [15] $\mathbb{H}$ *is said to be*

1. *distributive* if $l_1 \wedge (l_2 \bigvee l_3) = (l_1 \wedge l_2) \bigvee (l_1 \wedge l_3)$;
2. *s-distributive* if $l_1 \bigvee (l_2 \wedge l_3) = (l_1 \bigvee l_2) \wedge (l_1 \bigvee l_3)$,

for all $l_1, l_2, l_3 \in \mathbb{H}$, hold.

Theorem 2.5: [9] *Let $(L, \wedge, \vee)$ be a lattice and P a non-empty subset of L. We define a hyperoperation $\bigvee^P$ on L by*

$$l_1 \overset{P}{\bigvee} l_2 = l_1 \vee l_2 \vee P = \{l_1 \vee l_2 \vee p | p \in P\}.$$

Then $(L, \bigvee^P, \wedge)$ is a join hyperlattice if and only if for each $l_2 \in L$ there exists $p \in P$ such that $p \leq l_2$.

3. Classes of prime hyperideals in join hyperlattices

We denote the set of all hyperideals of $\mathbb{H}$ by $Id(\mathbb{H})$. We give some examples of join hyperlattices.

Example 3.1: *Let $\mathbb{N}$ be the set of all natural numbers and $\mathcal{P}(\mathbb{N})$ the set of all subsets of $\mathbb{N}$. Let $X = \{\phi, \{1,2\}\}$. Define the operations $\bigvee$ and $\wedge$ as follows:*

$$A \bigvee B = A \cup B \cup X = \{A \cup B \cup C \mid C \in X\} \text{and} A \wedge B = A \cap B$$

for any $A, B \in \mathcal{P}(\mathbb{N})$. Then $(\mathcal{P}(\mathbb{N}), \bigvee, \wedge)$ is a join hyperlattice.

Example 3.2: *Let $\mathbb{H} = D_{30} = \{1,2,3,5,6,10,15,30\}$, the set of all positive divisors of 30. The hyperoperation $\bigvee$ and the binary operation $\wedge$ on $\mathbb{H}$ are defined in Table 1. Then $(\mathbb{H}, \bigvee, \wedge)$ is a join hyperlattice.*

Definition 3.3: *Let $J \in Id(\mathbb{H})$. J is called 2-absorbing if whenever $l_1, l_2, l_3 \in \mathbb{H}$ with $l_1 \wedge l_2 \wedge l_3 \in J$, then either $l_1 \wedge l_2 \in J$ or $l_2 \wedge l_3 \in J$ or $l_1 \wedge l_3 \in J$.*

Remark 3.4: *Every prime hyperideal is 2-absorbing.*

Lemma 3.5: *Let $J_1, J_2 \in Id(\mathbb{H})$. Then $J_1 \cap J_2 \in Id(\mathbb{H})$.*

Proposition 3.6: *If Q and Q' are prime hyperideals of $\mathbb{H}$, then $Q \cap Q'$ is 2-absorbing.*

Proof. Let Q and Q' be prime hyperideals of $\mathbb{H}$ and $J = Q \cap Q'$. From Lemma 3.5, $J \in Id(\mathbb{H})$. Suppose that $l_1 \wedge l_2 \wedge l_3 \in J$. Then $l_1 \wedge l_2 \wedge l_3 \in Q$ and $l_1 \wedge l_2 \wedge l_3 \in Q'$. This means, $l_1 \wedge l_2 \in Q$ or $l_3 \in Q$ and $l_1 \wedge l_2 \in Q'$ or $l_3 \in Q'$.

If $l_3 \in Q$ and $l_3 \in Q'$, then $l_3 \in J$, and so $l_1 \wedge l_3 \in J$.

If $l_1 \wedge l_2 \in Q$ and $l_1 \wedge l_2 \in Q'$, then $l_1 \wedge l_2 \in J$.

Suppose that $l_1 \wedge l_2 \in Q$ and $l_3 \in Q'$. Since Q is prime hyperideal, we have, $l_1 \in Q$ or $l_2 \in Q$. If $l_1 \in Q$ then $l_1 \wedge l_3 \in Q$ and it follows that $l_1 \wedge l_3 \in Q'$. So $l_1 \wedge l_3 \in J$. If $l_2 \in Q$ then $l_2 \wedge l_3 \in Q$ and $l_2 \wedge l_3 \in Q'$. So $l_2 \wedge l_3 \in J$.

Definition 3.7: *For $J \in Id(\mathbb{H})$, we define the radical of J as the intersection of all prime hyperideals containing J and we denote it by $rad_\vee(J)$. If J is not contained in any prime hyperideal, then we take $rad_\vee(J) = \mathbb{H}$.*

Example 3.8: *Let $\mathbb{H} = \{0, l_1, l_2, l_3, l_4, 1\}$ and the hyperoperation $\bigvee$ and the binary operation $\wedge$ be defined as in Table 2:*

Then $(\mathbb{H}, \bigvee, \wedge)$ is a hyperlattice. Here, $I = \{0, l_1\}$ is a hyperideal of $\mathbb{H}$, and $rad_\vee(I) = \{0, l_1\}$ (see, Figure 1).

Table 1.

$\bigvee$	1	2	3	5	6	10	15	30
1	{1}	{2}	{3}	{5}	{6}	{10}	{15}	{30}
2	{2}	{1,2}	{6}	{10}	{3,6}	{5,10}	{30}	{15,30}
3	{3}	{6}	{1,3}	{15}	{2,6}	{30}	{5,15}	{10,30}
5	{5}	{10}	{15}	{1,5}	{30}	{2,10}	{3,15}	{6,30}
6	{6}	{3,6}	{2,6}	{30}	{1,2,3,6}	{15,30}	{10,30}	{5,10,15,30}
10	{10}	{5,10}	{2,10}	{30}	{15,30}	{1,2,5,10}	{6,30}	{3,6,15,30}
15	{15}	{30}	{5,15}	{3,15}	{10,30}	{6,30}	{1,3,5,15}	{2,6,10,30}
30	{30}	{15,30}	{10,30}	{6,30}	{5,10,15,30}	{3,6,15,30}	{2,6,10,30}	$\mathbb{H}$

$\wedge$	1	2	3	5	6	10	15	30
1	1	1	1	1	1	1	1	1
2	1	2	1	1	2	2	1	2
3	1	1	3	1	3	1	3	3
5	1	1	1	5	1	5	5	5
6	1	2	3	1	6	2	3	6
10	1	2	1	5	2	10	5	10
15	1	1	3	5	3	5	15	15
30	1	2	3	5	6	10	15	30

Table 2.

$\bigvee$	0	l_1	l_2	l_3	l_4	1
0	$\{0, l_1\}$	$\{l_1\}$	$\{l_2, l_3\}$	$\{l_3\}$	$\{l_4, 1\}$	{1}
l_1	$\{l_1\}$	$\{l_1\}$	$\{l_3\}$	$\{l_3\}$	{1}	{1}
l_2	$\{l_2, l_3\}$	$\{l_3\}$	$\{l_2, l_3\}$	$\{l_3\}$	$\{l_4, 1\}$	{1}
l_3	$\{l_3\}$	$\{l_3\}$	$\{l_3\}$	$\{l_3\}$	{1}	{1}
l_4	{1}	{1}	$\{l_4, 1\}$	{1}	$\{l_4, 1\}$	{1}
1	{1}	{1}	{1}	{1}	{1}	{1}

$\wedge$	0	l_1	l_2	l_3	l_4	1
0	0	0	0	0	0	0
l_1	0	l_1	0	0	0	l_1
l_2	0	0	l_2	l_2	l_2	l_2
l_3	0	l_1	l_2	l_3	l_2	l_3
l_4	0	0	l_2	l_2	l_4	l_4
1	0	l_1	l_2	l_3	l_4	1

Remark 3.9: *In a s-distributive hyperlattice, a hyperideal is equal to its radical. This need not be true in the case of non s-distributive hyperlattices. We show this in the following example.*

Example 3.10: *Let* $\mathbb{H} = \{0,a,b,c,d,e,f,g,h,i,1\}$. *The hyperoperation* $\bigvee$ *and the binary operation* $\wedge$ *on* $\mathbb{H}$ *are represented by the following Table 3:*

Then $(\mathbb{H},\bigvee,\wedge)$ is a non s-distributive join hyperlattice. Let $J = \{0,a,b,c,f\}$. Then we can see that $rad_{\vee}(J) = \{0,a,b,c,d,e,f,g,h,i\}$ *(see, Figure 2).*

Proposition 3.11: *Let* $J, J' \in Id(\mathbb{H})$. *We have the following*

1. If J is prime, then $rad_{\vee}(J) = J$.
2. $rad_{\vee}(rad_{\vee}(J)) = rad_{\vee}(J)$.
3. If $J \subseteq J'$, then $rad_{\vee}(J) \subseteq rad_{\vee}(J')$.

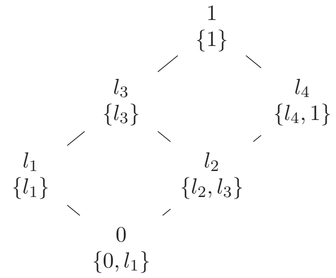

Figure 1.

Table 3.

$\bigvee$	0	a	b	c	d	e	f	g	h	i	1
0	$\{0, c\}$	$\{a, f\}$	$\{b, f\}$	$\{c\}$	$\{d, h\}$	$\{h, e\}$	$\{f\}$	$\{g\}$	$\{h\}$	$\{i\}$	$\{1\}$
a	$\{a, f\}$	$\{a, f\}$	$\{f\}$	$\{f\}$	$\{i\}$	$\{i\}$	$\{f\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{1\}$
b	$\{b, f\}$	$\{f\}$	$\{b, f\}$	$\{f\}$	$\{i\}$	$\{i\}$	$\{f\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{1\}$
c	$\{l_3\}$	$\{f\}$	$\{f\}$	$\{l_3\}$	$\{h\}$	$\{h\}$	$\{f\}$	$\{g\}$	$\{h\}$	$\{i\}$	$\{1\}$
d	$\{d, h\}$	$\{i\}$	$\{i\}$	$\{h\}$	$\{d, h\}$	$\{h\}$	$\{i\}$	$\{i\}$	$\{h\}$	$\{i\}$	$\{1\}$
e	$\{e, h\}$	$\{i\}$	$\{i\}$	$\{h\}$	$\{h\}$	$\{e, h\}$	$\{i\}$	$\{i\}$	$\{h\}$	$\{i\}$	$\{1\}$
f	$\{f\}$	$\{f\}$	$\{f\}$	$\{f\}$	$\{i\}$	$\{i\}$	$\{f\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{1\}$
g	$\{g\}$	$\{i\}$	$\{i\}$	$\{g\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{g\}$	$\{i\}$	$\{i\}$	$\{1\}$
h	$\{h\}$	$\{i\}$	$\{i\}$	$\{h\}$	$\{h\}$	$\{h\}$	$\{i\}$	$\{i\}$	$\{h\}$	$\{i\}$	$\{1\}$
i	$\{i\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{i\}$	$\{1\}$
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

$\wedge$	0	a	b	c	d	e	f	g	h	i	1
0	0	0	0	0	0	0	0	0	0	0	0
a	0	a	0	0	0	0	a	0	0	a	a
b	0	0	b	0	0	0	b	0	0	b	b
c	0	0	0	c	0	0	c	c	c	c	c
d	0	0	0	0	d	0	0	0	d	d	d
e	0	0	0	0	0	e	0	0	e	e	e
f	0	a	b	c	0	0	f	c	c	f	f
g	0	0	0	c	0	0	c	g	c	g	g
h	0	0	0	c	d	e	c	c	h	h	h
i	0	a	b	c	d	e	f	g	h	i	i
1	0	a	b	c	d	e	f	g	h	i	1

4. $rad_{\vee}(J \cap J') = rad_{\vee}(J) \cap rad_{\vee}(J')$.

Proof. The proofs of (1), (2) and (3) follow from the Definition 3.7.

(4) Clearly, $rad_{\vee}(J \cap J') \subseteq rad_{\vee}(J) \cap rad_{\vee}(J')$. Now, let $\alpha \in rad_{\vee}(J) \cap rad_{\vee}(J')$. If $\alpha \notin rad_{\vee}(J \cap J')$ then there exists $Q \in Id(\mathbb{H})$, Q is prime such that $J \cap J' \subseteq Q$ and $\alpha \notin Q$. Also, if $J \subseteq Q$, then $rad_{\vee}(J) \subseteq Q$, hence, $\alpha \in Q$, a contradiction. So $J \nsubseteq Q$ and $J' \nsubseteq Q$ and hence, $J \setminus Q \neq \varnothing$ and $J' \setminus Q \neq \varnothing$. Let $i \in J \setminus Q$ and $j \in J' \setminus Q$. Then $i \wedge j \in J$ and $i \wedge j \in J'$. Therefore, $i \wedge j \in J \cap J' \subseteq Q$, contradicts the fact that Q is prime.

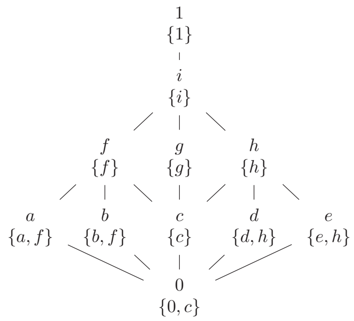

Figure 2.

Example 3.12: *Let $\mathbb{H} = \{0, l_1, l_2, l_3, l_4, 1\}$. Define the hyperoperation $\bigvee$ and binary operation $\wedge$ on $\mathbb{H}$ by Table 4:*

Then $(\mathbb{H}, \bigvee, \wedge)$ is a hyperlattice. Here, $I = \{0, l_2\}$ is a hyperideal of $\mathbb{H}$, and $rad_{\vee}(I) = \{0, l_2\}$ (see, Figure 3).

Definition 3.13: *$J \in Id(\mathbb{H})$ is called a primary hyperideal if whenever $l_1, l_2 \in \mathbb{H}$ and $l_1 \wedge l_2 \in J$, then $l_1 \in J$ or $l_2 \in rad_{\vee}(J)$.*

Example 3.14: *In Example 3.12, $I = \{0, l_1\}$, is a primary hyperideal.*

Evidently, every prime hyperideal of $\mathbb{H}$ is a primary hyperideal. The converse need not be true as shown in Example 3.15.

Example 3.15: *In Example 3.10, $J = \{0, a, b, c, f\}$ is a primary hyperideal but not a prime hyperideal as we see $d \wedge e \in J$ but $b \notin J$ and $e \notin J$.*

Theorem 3.16: *For any $J \in Id(\mathbb{H})$, $rad_{\vee}(J)$ is prime if and only if $rad_{\vee}(J)$ is primary.*

Proof. Suppose that $rad_{\vee}(J)$ is primary for some $J \in Id(\mathbb{H})$ and that $l_1 \wedge l_2 \in J$ for $l_1, l_2 \in \mathbb{H}$. Then either $l_1 \in rad_{\vee}(J)$ or $l_2 \in rad_{\vee}(rad_{\vee}(J)) = rad_{\vee}(J)$, shows that $rad_{\vee}(J)$ is prime. The other part follows from the definition.

Definition 3.17: *$J \in Id(\mathbb{H})$ is called a 2-absorbing primary hyperideal if whenever $l_1, l_2, l_3 \in \mathbb{H}$, $l_1 \wedge l_2 \wedge l_3 \in J$, then either $l_1 \wedge l_2 \in J$ or $l_2 \wedge l_3 \in rad_{\vee}(J)$ or $l_1 \wedge l_3 \in rad_{\vee}(J)$.*

Now we have the following Remark.

Remark 3.18:

1. If $J \in Id(\mathbb{H})$, is primary then J is 2-absorbing primary.
2. If $J \in Id(\mathbb{H})$ is 2-absorbing, then J is 2-absorbing primary.

Example 3.19: *Let $\mathbb{H} = \{0, l_1, l_2, l_3, \ldots, l_{18}, 1\}$. We define the hyperoperation $\bigvee$ and the binary operation on $\mathbb{H}$ as shown in the Figure 4. We give the lattice diagram as in Figure 4. In the diagram, the meet of two*

Table 2.

$\bigvee$	0	l_1	l_2	l_3	l_4	1
0	$\{0, l_2\}$	$\{l_1, l_3\}$	$\{l_2\}$	$\{l_3\}$	$\{l_4\}$	$\{1\}$
l_1	$\{l_1, l_3\}$	$\{l_1, l_3\}$	$\{l_3\}$	$\{l_3\}$	$\{1\}$	$\{1\}$
l_2	$\{l_2\}$	$\{l_3\}$	$\{l_2\}$	$\{l_3\}$	$\{l_4\}$	$\{1\}$
l_3	$\{l_3\}$	$\{l_3\}$	$\{l_3\}$	$\{l_3\}$	$\{1\}$	$\{1\}$
l_4	$\{l_4\}$	$\{1\}$	$\{l_4\}$	$\{1\}$	$\{l_4\}$	$\{1\}$
1	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

$\wedge$	0	l_1	l_2	l_3	l_4	1
0	0	0	0	0	0	0
l_1	0	l_1	0	0	0	l_1
l_2	0	0	l_2	l_2	l_2	l_2
l_3	0	l_1	l_2	l_3	l_2	l_3
l_4	0	0	l_2	l_2	l_4	l_4
1	0	l_1	l_2	l_3	l_4	1

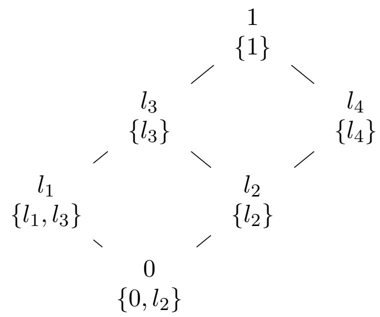

Figure 3.

elements is depicted as in a classical lattice. In contrast, the join of two elements is represented as a set written below the lattice elements. For instance, $0 \bigvee 0 = \{0, l_2\}, l_1 \bigvee l_2 = \{l_6\}, l_8 \bigvee l_8 = \{l_8, l_{13}\}, l_5 \bigvee l_{12} = \{l_{12}, l_{15}\}$.

Here, $J = \{0, l_2\} \in Id(\mathbb{H})$ and $rad(J) = \mathbb{H} \setminus \{l_{12}, l_{15}, 1\}$. Then J is 2-absorbing primary. But J is not a 2-absorbing hyperideal, since $l_9 \wedge l_{10} \wedge l_{12} = 0$, and neither $l_9 \wedge l_{10} \in J$ nor $l_9 \wedge l_{12} \in J$ nor $l_{10} \wedge l_{12} \in J$. In Example 3.20, we show a hyperideal which is J-absorbing primary but not primary.

Example 3.20: *Let* $\mathbb{H} = \{0, l_1, l_2, l_3, \cdots, l_{18}, 1\}$. *We define both the operations on* $\mathbb{H}$ *as shown in the Figure 3.5. Here,* $J = \{0, l_1\}$ *is a hyperideal and* $rad(J) = \{0, l_1, l_5, l_9\}$. *We can see that J is J-absorbing primary. But it is not a primary hyperideal, since* $l_2 \wedge l_4 \in J$ *but* $l_2 \notin J$ *and* $l_4 \notin J$.

Theorem 3.21: *If* $J \in Id(\mathbb{H})$ *such that* $rad_{\vee}(J)$ *is prime, then J is 2-absorbing primary.*

Proof. Suppose that $l_1 \wedge l_2 \wedge l_3 \in J$, where $l_1, l_2, l_3 \in \mathbb{H}$.

Case (1): Suppose that $l_1 \wedge l_2 \notin rad_{\vee}(J)$. Since $rad_{\vee}(J)$ is a prime, we have $l_3 \in rad_{\vee}(J)$. Then $l_1 \wedge l_3 \in rad_{\vee}(J)$ and $l_2 \wedge l_3 \in rad_{\vee}(J)$.

Case (2): Suppose that $l_1 \wedge l_2 \in rad_{\vee}(J)$. Since $rad_{\vee}(J)$ is prime, either $l_1 \in rad_{\vee}(J)$ or $l_2 \in rad_{\vee}(J)$. So $l_1 \wedge l_3 \in rad_{\vee}(J)$ or $l_2 \wedge l_3 \in rad_{\vee}(J)$. Hence, J is a 2-absorbing primary hyperideal of $\mathbb{H}$.

Corollary 3.22: *Let* $J \in \mathbb{H}$. *Then* $rad_{\vee}(J)$ *is a* **2**-*absorbing hyperideal of* $\mathbb{H}$ *if and only if* $rad_{\vee}(J)$ *is* 2-*absorbing primary.*

Definition 3.23: *A primary hyperideal J of* $\mathbb{H}$ *is called P-primary if P is the only prime hyperideal such that* $J \subseteq P$.

Remark 3.24: *If* $J \in Id(\mathbb{H})$ *is Q-primary, then* $rad_{\vee}(J) = Q$.

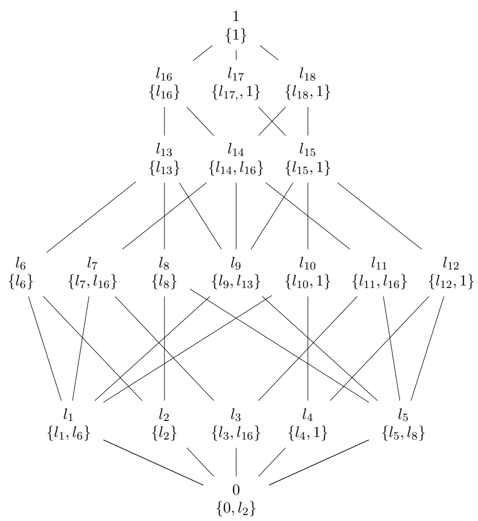

Figure 4.

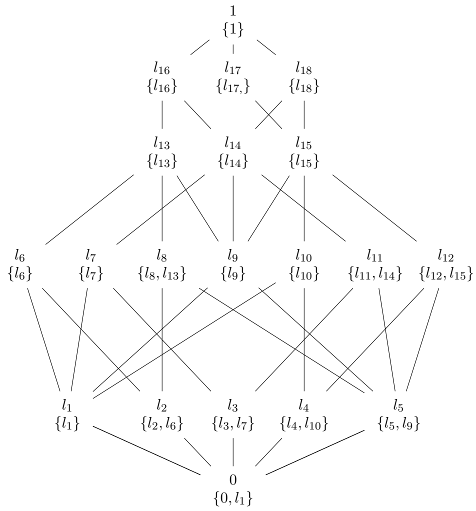

Figure 5.

Proposition 3.25: *Let Q and Q' be prime hyperideals in $\mathbb{H}$ and $J, J' \in Id(\mathbb{H})$ such that J is Q-primary and J' is Q'-primary. Then $J \cap J'$ is 2-absorbing primary.*

Proof. Let $I = J \cap J'$. Since J and J' are Q, Q'-primary respectively, it follows from Proposition 3.11(4) and Remark 3.24, we get $rad_{\vee}(I) = Q \cap Q'$. To see I is 2-absorbing primary, let $l_i (1 \le i \le 3) \in \mathbb{H}, l_1 \wedge l_2 \wedge l_3 \in I$ with $l_2 \wedge l_3 \notin rad_{\vee}(I)$ and $l_1 \wedge l_3 \notin rad_{\vee}(I)$. If any of l_1 or l_2 or l_3 is in $rad_{\vee}(I)$, then we get either $l_2 \wedge l_3 \in rad_{\vee}(I)$ or $l_1 \wedge l_3 \in rad_{\vee}(I)$, a contradiction. Therefore, $l_i (1 \le i \le 3) \notin rad_{\vee}(I)$. As $I \subseteq rad_{\vee}(I)$, we get $l_1 \wedge l_2 \wedge l_3 \in Q \cap Q'$. Then by Proposition 3.6, we get $l_1 \wedge l_2 \in Q \cap Q'$. This means $l_1 \wedge l_2 \in Q$ and $l_1 \wedge l_2 \in Q'$. Since Q and Q' are prime hyperideals, we get $l_1 \in Q, l_2 \in Q'$ and $l_1 \notin Q', l_2 \notin Q$.

Suppose that $l_1 \notin J$. As J is a primary hyperideal, and $l_1 \wedge l_2 \wedge l_3 \in J$, we get $l_2 \wedge l_3 \in Q$. Since $l_2 \in Q'$, so $l_2 \wedge l_3 \in Q'$, yields $l_2 \wedge l_3 \in Q \cap Q'$, a contradiction. Hence, $l_1 \in J$. Similarly, we can show $l_2 \in J'$. Thus, $l_1 \wedge l_2 \in I$.

4. Annihilator hyperideals

Definition 4.1: [23] *An element $l \in \mathbb{H}$ is called distributive, if*

$$l \wedge (l_1 \bigvee l_2) = (l \wedge l_1) \bigvee (l \wedge l_2),$$

for all $l_1, l_2 \in \mathbb{H}$ holds.

Definition 4.2: *Let $J, J' \in Id(\mathbb{H})$ and $x \in \mathbb{H}$. We define,*

$$[J : x] = \{l_1 \in \mathbb{H} : l_1 \wedge x \in J\}$$

$$[0 : x] = \{l_2 \in \mathbb{H} : x \wedge l_2 = 0\}$$

and

$$[J : J'] = \{l_1 \in \mathbb{H} : l_1 \wedge l_2 \in J \forall l_2 \in J'\}.$$

Proposition 4.3: *Let $\mathbb{H}$ be a distributive hyperlattice and $J, J' \in Id(\mathbb{H})$. Then $[J : J'] \in Id(\mathbb{H})$.*

Proof. Suppose that $l_1, l_2 \in [J_1 : J_2]$ and $x \in J_2$. Then $l_1 \wedge x \in J_1$ and $l_2 \wedge x \in J_1$. Now $(l_1 \wedge x) \bigvee (l_2 \wedge x) \in J_1$ and since $\mathbb{H}$ is distributive, we get $(l_1 \wedge x) \bigvee (l_2 \wedge x) = (l_1 \bigvee l_2) \wedge x \subseteq J_1$. Hence, $l_1 \bigvee l_2 \subseteq [J_1 : J_2]$.

Suppose that $x \in J_2, l_2 \in [J_1 : J_2], l_1 \in \mathbb{H}$ with $l_1 \le l_2$. Then $l_2 \wedge x \in J_1$, and so $l_1 \wedge x = (l_2 \wedge x) \wedge (l_1 \wedge x) \in J_1$.

The following Corollary is straightforward.

Corollary 4.4: *If $I \in Id(\mathbb{H})$ and $x \in \mathbb{H}$, a distributive element, then the set $[I : x] \in Id(\mathbb{H})$.*

For any $y \in I$, where $I \in Id(\mathbb{H})$, $y \wedge x \in I$, and hence, $y \in [I : x]$. More formally, we have the following.

Corollary 4.5: *Let $I \in Id(\mathbb{H})$ and $x \in \mathbb{H}$. Then $I \subseteq [I : x]$.*

Remark 4.6: *Let $J_1, J_2 \in Id(\mathbb{H})$. Then $J_1 \subseteq [J_1 : J_2]$.*

Corollary 4.7: *Let $\mathbb{H}$ be a distributive hyperlattice and $J_1, J_2 \in Id(\mathbb{H})$. If J_1 is a prime hyperideal and $J_2 \nsubseteq J_1$, then $[J_1 : J_2] = J_1$*

Proof. By Remark 4.6, $J_1 \subseteq [J_1 : J_2]$. Let $x \in [J_1 : J_2]$. Then $x \wedge y \in J_1$ for all $y \in J_2$. In particular, for $z \in J_2 \setminus J_1$, we get $x \wedge z \in J_1$. Since $z \notin J_1$ and J_1 is prime, we get $x \in J_1$. Therefore, $[J_1 : J_2] \subseteq J_1$.

The following corollary is straightforward.

Corollary 4.8: *Let I be a prime hyperideal of $\mathbb{H}$ and $x \in \mathbb{H} \setminus I$ be a distributive element. Then $I = [I : x]$.*

Proposition 4.9: *Let $x \in \mathbb{H}$ be a distributive element and J_1 a **2**-absorbing hyperideal of $\mathbb{H}$. Then $[J_1 : x]$ is 2-absorbing.*

Proof. By Corollary 4.4, $[J_1 : x]$ is a hyperideal of $\mathbb{H}$. Suppose that $l_1 \wedge l_2 \wedge l_3 \in [J_1 : x]$ where $l_i (1 \leq i \leq 3) \in \mathbb{H}$. Then $(l_1 \wedge l_2) \wedge l_3 \wedge x \in J_1$. Then either $(l_1 \wedge l_2) \wedge l_3 \in J_1$ or $(l_1 \wedge l_2) \wedge x \in J_1$ or $l_3 \wedge x \in J_1$.

Case (1): Suppose that $(l_1 \wedge l_2) \wedge l_3 \in J_1$. Since J_1 is 2-absorbing, we have $l_1 \wedge l_2 \in J_1$ or $l_2 \wedge l_3 \in J_1$ or $l_1 \wedge l_3 \in J_1$. As $J_1 \subseteq [I : x]$, we get $l_1 \wedge l_2 \in [J_1 : x]$ or $l_2 \wedge l_3 \in [J_1 : x]$ or $l_1 \wedge l_3 \in [J_1 : x]$.

Case (2): Suppose that $(l_1 \wedge l_2) \wedge x \in J_1$. Since J_1 is 2-absorbing, we get $l_1 \wedge l_2 \in J_1$ or $l_2 \wedge x \in J_1$ or $l_1 \wedge x \in J_1$. So we get $l_1 \wedge l_2 \in [J_1 : x]$ or $l_1 \in [J_1 : x]$ or $l_2 \in [J_1 : x]$.

Case (3): If $l_3 \wedge x \in J_1$, then $l_3 \in [J_1 : x]$.

Proposition 4.10: *Let J_1 be a primary hyperideal of $\mathbb{H}$ and $x \notin rad_\vee(J_1)$ be a distributive element in $\mathbb{H}$. Then $[J_1 : x]$ is primary.*

Proof. By Corollary 4.4, $[J_1 : x]$ is a hyperideal of $\mathbb{H}$. To prove $[J_1 : x]$ is primary, let $l_1 \wedge l_2 \in [J_1 : x]$ for some $l_1, l_2 \in \mathbb{H}$. Then $(l_1 \wedge l_2) \wedge x \in J_1$. As J_1 is primary, and $x \notin rad_\vee(J_1)$, we get $l_1 \wedge l_2 \in J_1$. Again, from the fact that J_1 is primary we get $l_1 \in J_1$ or $l_2 \in rad_\vee(J_1)$. Since $J_1 \subseteq [J_1 : x]$, we get $l_1 \in [J_1 : x]$ or $l_2 \in rad_\vee[J_1 : x]$.

Proposition 4.11: *Let $J_1, J_2, J_3 \in Id(\mathbb{H})$. If $J_1 \subseteq J_2 \cup J_3$ then $J_1 \subseteq J_2$ or $J_1 \subseteq J_3$.*

Proposition 4.12: *Let $\varnothing \neq J \in Id(\mathbb{H}.)$ Then the following statements are equivalent:*

1. J is 2-absorbing primary.
2. If $l_1, l_2 \in J$ with $l_1 \wedge l_2 \notin rad_\vee(J)$, then $[J : l_1 \wedge l_2] \subseteq [J : l_1]$ or $[J : l_1 \wedge l_2] \subseteq [rad_\vee(J) : l_2]$.
3. If $l_1 \in \mathbb{H}, J' \in Id(\mathbb{H})$ with $l_1 \wedge J' \nsubseteq rad_\vee(J)$, then $[J : l_1 \wedge J'] \subseteq [J : J']$ or $[J : l_1 \wedge J'] \subseteq [rad_\vee(J) : l_1]$.
4. If $J_1, J_2 \in Id(\mathbb{H}), J_1 \cap J_2 \nsubseteq J$, then either $[J : J_1 \cap J_2] \subseteq [rad_\vee(J) : J_1]$ or $[J : J_1 \cap J_2] \subseteq [rad_\vee(J) : J_2]$.
5. If $J_1, J_2, J_3 \in Id(\mathbb{H})$, $J_1 \cap J_2 \cap J_3 \subseteq J$, then either $J_1 \cap J_2 \subseteq J$ or $J_2 \cap J_3 \subseteq rad_\vee(J)$ or $J_1 \cap J_3 \subseteq rad_\vee(J)$.

Proof. (1) $\Rightarrow$ (2)

Suppose that $l_1, l_2 \in \mathbb{H}$ with $l_1 \wedge l_2 \notin rad_\vee(J)$. Let $l \in [J : l_1 \wedge l_2]$. Then $l \wedge l_1 \wedge l_2 \in J$. Since J is 2-absorbing primary, either $l \wedge l_1 \in J$ or $l \wedge l_2 \in rad_\vee(J)$. That is, $l \in [J : l_1]$ or $l \in [rad_\vee(J) : l_2]$. And so $[J : l_1 \wedge l_2] \subseteq [J : l_1] \cup [rad_\vee(J) : l_2]$. Then by Proposition 4.11, we get $[J : l_1 \wedge l_2] \subseteq [J : l_1]$ or $[J : l_1 \wedge l_2] \subseteq [rad_\vee(J) : l_2]$.

(2) $\Rightarrow$ (3)

Suppose that $l_1 \in \mathbb{H}, J_1 \in Id(\mathbb{H})$ with $l_1 \wedge J_1 \nsubseteq rad_\vee(J)$. Let $l \in [J : l_1 \wedge J_1]$. Then $l \wedge l_1 \wedge J_1 \subseteq J$. And so $J_1 \subseteq [J : l \wedge l_1]$. Now, if $l \wedge l_1 \in rad(J)$, then $l \in [rad_\vee(J) : l_1]$. If $l \wedge l_1 \notin rad(J)$, then by (2), we get $[J : l \wedge l_1] \subseteq [J : l]$ or $[J : l \wedge l_1] \subseteq [rad_\vee(J) : l_1]$, and hence, $J_1 \subseteq [J : l]$ or $J_1 \subseteq [rad_\vee(J) : l_1]$. If $J_1 \subseteq [rad_\vee(J) : l_1]$, then $l_1 \wedge J_1 \subseteq rad_\vee(J)$, a contradiction. So $[J : l \wedge l_1] \subseteq [J : l]$. Then $J_1 \wedge l \subseteq J$. That is, $l \in [J : J_1]$. Therefore, $l \in [J : J_1] \cup [rad_\vee(J) : l_1]$. So $[J : l_1 \wedge J_1] \subseteq [J : J_1] \cup [rad_\vee(J) : l_1]$. Then by Proposition 4.11, we get $[J : l_1 \wedge J_1] \subseteq [J : J_1]$ or $[J : l_1 \wedge J_1] \subseteq [rad_\vee(J) : l_1]$.

(3) $\Rightarrow$ (4)

Suppose that $J_1, J_2 \in Id(\mathbb{H})$ with $J_1 \cap J_2 \nsubseteq J$. Let $l \in [J : J_1 \cap J_2]$. Then $l \wedge J_1 \wedge J_2 \subseteq J$ and so $J_2 \subseteq [J : l \wedge J_1]$. If $l \wedge J_1 \subseteq rad_\vee(J_1)$, then $l \in [rad_\vee(J) : J_1]$. If $l \wedge J_1 \nsubseteq rad_\vee(J_1)$, then $[J : l \wedge J_1] \subseteq [J : J_1]$ or $[J : l \wedge J_1] \subseteq [rad_\vee(J) : l]$. If $[J : l \wedge J_1] \subseteq [J : J_1]$, then $J_2 \subseteq [J : J_1]$. Then we get $J_1 \cap J_2 \subseteq J$, a contradiction. Therefore, $[J : l \wedge J_1] \subseteq [rad_\vee(J) : l]$, which gives $J_2 \subseteq [rad_\vee(J) : l]$. So $[J : J_1 \cap J_2] \subseteq [rad_\vee(J) : J_1] \cup [rad_\vee(J) : J_2]$. That is, $l \in [rad_\vee(J) : J_2]$. Thus, by Proposition 4.11, $[J : J_1 \cap J_2] \subseteq [rad_\vee(J) : J_1]$ or $[J : J_1 \cap J_2] \subseteq [rad_\vee(J) : J_2]$.

(4) $\Rightarrow$ (5)

Suppose that $J_1, J_2, J_3 \in Id(\mathbb{H})$ with $J_1 \cap J_2 \nsubseteq J$. Then either $[J : J_1 \cap J_2] \subseteq [rad_\vee(J) : J_1]$ or $[J : J_1 \cap J_2] \subseteq [rad_\vee(J) : J_2]$. Since $J_1 \cap J_2 \cap J_3 \subseteq J$, we have $J_3 \subseteq [J : J_1 \cap J_2]$ and so $J_3 \subseteq [rad_\vee(J) : J_1]$ or $J_3 \subseteq [rad_\vee(J) : J_2]$. Therefore, $J_1 \cap J_3 \subseteq rad_\vee(J)$ or $J_1 \cap J_3 \subseteq rad_\vee(J)$.

(5) $\Rightarrow$ (1) Obvious.

5. Homomorphisms in join hyperlattices

It is noted that the authors [3] defined homomorphism between two algebraic hyperstructures in more than one way. Similarly, we define weak homomorphism in a hyperlattice. In Example 5.2, we show that the homomorphic image of a hyperideal under weak homomorphism need not be a hyperideal again.

Definition 5.1: *Let $\mathbb{H}$ and $\mathbb{H}'$ be two join hyperlattices. A mapping $\eta : \mathbb{H} \to \mathbb{H}'$ is a weak homomorphism if the following conditions satisfy:*

1. $\eta(u \bigvee v) \subseteq \eta(u) \bigvee \eta(v)$,
2. $\eta(u \wedge v) = \eta(u) \wedge \eta(v)$,

for all $u, v \in \mathbb{H}$. If Condition 1 holds with equality, then we say η is a homomorphism.

Example 5.2: *Let $\mathbb{H} = \{0, l_1, l_2, 1\}$. The hyperoperations $\bigvee_1, \bigvee_2$ and binary operation $\wedge$ on $\mathbb{H}$ are defined in Table 5:*

$\mathbb{H}_1 = (\mathbb{H}, \bigvee_1, \wedge)$ and $\mathbb{H}_2 = (\mathbb{H}, \bigvee_2, \wedge)$ are join hyperlattices. Let $f : \mathbb{H}_1 \to \mathbb{H}_2$ be defined as $f(0) = 0, f(l_1) = l_1, f(l_2) = l_2, f(1) = 1$. It can be seen that f is a bijective weak homomorphism. Note that $I = \{0\}$ is a hyperideal of $\mathbb{H}_1$. But $f(I)$ is not a hyperideal of $\mathbb{H}_2$, because $f(0) \bigvee_2 f(0) = 0 \bigvee_2 0 = \{0, l_2\} \nsubseteq \{0\} = f(I)$.

Proposition 5.3: [24] *Let $\mathbb{H}$ and $\mathbb{H}'$ be two join hyperlattices and let $\eta : \mathbb{H} \to \mathbb{H}'$ be a homomorphism.*

1. *If $J' \in Id(\mathbb{H}')$ and $\eta^{-1}(J') \neq \varnothing$, then $\eta^{-1}(J) \in Id(\mathbb{H})$.*
2. *If η is an isomorphism and $J \in Id(\mathbb{H})$, then $\eta(J) \in Id(\mathbb{H}')$.*

Proposition 5.4: [24] *Let $\mathbb{H}$ and $\mathbb{H}'$ be two join hyperlattices and let $\eta : \mathbb{H} \to \mathbb{H}'$ be a homomorphism.*

1. *If J' is a prime hyperideal of $\mathbb{H}'$ and $\eta^{-1}(J') \neq \varnothing$, then $\eta^{-1}(J)$ is a prime hyperideal of $\mathbb{H}$.*
2. *If $\eta : \mathbb{H} \to \mathbb{H}'$ is an isomorphism and J is a prime hyperideal of $\mathbb{H}$, then $\eta(J)$ is a prime hyperideal of $\mathbb{H}'$.*

Table 5.

$\bigvee_1$	0	l_1	l_2	1
0	{0}	$\{l_1\}$	$\{l_2\}$	{1}
l_1	$\{l_1\}$	$\{l_1\}$	{1}	{1}
l_2	$\{l_2\}$	{1}	$\{l_2\}$	{1}
1	{1}	{1}	{1}	{1}

$\bigvee_2$	0	l_1	l_2	1
0	$\{0, l_2\}$	$\{l_1, 1\}$	$\{l_2\}$	{1}
l_1	$\{l_2, 1\}$	$\{l_1, 1\}$	{1}	{1}
l_2	$\{l_2\}$	{1}	$\{l_2\}$	{1}
1	0	{1}	{1}	{1}

$\wedge$	0	l_1	l_2	1
0	0	0	0	0
l_1	0	l_1	0	l_1
l_2	0	0	l_2	l_2
1	0	l_1	l_2	1

Theorem 5.5: *Let* $\mathbb{H}$ *and* $\mathbb{H}'$ *be two join hyperlattices and let* $\eta : \mathbb{H} \to \mathbb{H}'$ *be an isomorphism. Then*

1. $\eta^{-1}(rad_\vee(J')) = rad_\vee(\eta^{-1}(J'))$ *where* J' *is a hyperideal of* $\mathbb{H}'$.
2. $\eta(rad_\vee(J)) = rad_\vee(\eta(J))$ *where* J *is a hyperideal of* $\mathbb{H}$.

Proof. 1. Let Q_i''s be the prime hyperideals containing J'. Then

$$\eta^{-1}(rad_\vee(J')) = \eta^{-1}\left(\bigcap Q_{i'}\right) = \bigcap \eta^{-1}(Q_{i'}) = rad_\vee(\eta^{-1}(J'))$$

$\left(\text{because each } \eta^{-1}\ (Q_{i'})\text{'s are prime hyperideals containing } \eta^{-1}(J')\right)$

2. Let Q_i's be the prime hyperideals containing J. Then

$$\eta(rad_\vee(J)) = f\left(\bigcap Q_i\right) = \bigcap \eta(Q_i) = rad_\vee(\eta(J))$$

$\left(\text{because each } \eta(Q_i)\text{'s are prime hyperideals containing } \eta(J)\right)$

Proposition 5.6: *Let* $\mathbb{H}$ *and* $\mathbb{H}'$ *be two join hyperlattices and let* $\eta : \mathbb{H} \to \mathbb{H}'$ *be an isomorphism.*

1. If J' is a 2-absorbing primary hyperideal of $\mathbb{H}'$, then $\eta^{-1}(J)$ is a 2-absorbing primary hyperideal of $\mathbb{H}$.
2. If J is a 2-absorbing primary hyperideal of $\mathbb{H}$, then $\eta(J)$ is a 2-absorbing primary hyperideal of $\mathbb{H}'$.

Proof. 1. Let J' be a 2-absorbing hyperideal of $\mathbb{H}'$. Let $l_i (1 \le i \le 3) \in \mathbb{H}$ such that $l_1 \wedge l_2 \wedge l_3 \in \eta^{-1}(J')$. Clearly, $\eta(l_1) \wedge \eta(l_2) \wedge \eta(w) \in J'$. Since J' is a 2- absorbing primary hyperideal, $\eta(l_1) \wedge \eta(l_2) \in J'$ or $\eta(l_2) \wedge \eta(l_3) \in rad_\vee(J')$ or $\eta(l_1) \wedge \eta(l_3) \in rad_\vee(J')$. This implies that, $\eta(l_1 \wedge l_2) \in J'$ or $\eta(l_2 \wedge l_3) \in rad_\vee(J')$ or $\eta(l_1 \wedge l_3) \in rad_\vee(J')$. That is, $l_1 \wedge l_2 \in \eta^{-1}(J')$ or $l_2 \wedge l_3 \in \eta^{-1}(rad_\vee(J'))$ or $l_1 \wedge l_3 \in \eta^{-1}(rad_\vee(J'))$. Now by Theorem 5.5, $l_1 \wedge l_2 \in \eta^{-1}(J')$ or $l_2 \wedge l_3 \in rad_\vee(\eta^{-1}(J'))$ or $l_1 \wedge l_3 \in rad_\vee \eta^{-1}(J')$.

2. Let J be a 2-absorbing hyperideal of $\mathbb{H}$. Let $l_{1'}, l_{2'}, l_{3'} \in \mathbb{H}'$ such that $l_{1'} \wedge l_{2'} \wedge w' \in \eta(J)$. Then there exist $l_i (1 \le i \le 3) \in \mathbb{H}$ such that $\eta(l_1) = l_{1'}, \eta(l_2) = l_{2'}$ and $\eta(l_3) = l_3'$. Then $\eta(l_1 \wedge l_2 \wedge l_3) \in \eta(J)$ and so, $(l_1 \wedge l_2 \wedge l_3) \in J$. Since J is 2- absorbing primary, we have $l_1 \wedge l_2 \in J$ or $l_2 \wedge l_3 \in rad_\vee(J)$ or $l_1 \wedge l_3 \in rad_\vee(J)$.

Then $\eta(l_1 \wedge l_2) \in \eta(J)$ or $\eta(l_2 \wedge w) \in \eta(rad_\vee(J))$ or $\eta(l_1 \wedge l_3) \in \eta(rad_\vee(J))$. Then by Theorem 5.5, $\eta(l_1) \wedge \eta(l_2) \in \eta(J)$ or $\eta(l_2) \wedge \eta(w) \in rad_\vee(\eta(J))$ or $\eta(l_1) \wedge \eta(w) \in rad_\vee(\eta(J))$. This shows that $l_{1'} \wedge l_{2'} \in \eta(J)$ or $l_{2'} \wedge l_3' \in rad_\vee(\eta(J))$ or $l_{1'} \wedge l_3' \in rad_\vee(\eta(J))$.

The downward arrows in Figure 6 represent various possible generalizations obtained for the prime hyperideals in a join hyperlattice.

6. Conclusion

In this paper, we have considered join hyperlattices as a generalization of classical lattices. We have defined the classes of prime hyperideals viz., 2-absorbing, primary in a join hyperlattice and studied several properties. As future scope, we try to study various radical properties arising from the generalized prime hyperideals in join hyperlattices. Furthermore, one can explore the hyperlattice aspects of essential elements in a lattice as discussed in [25]. In [26], authors studied the combinatorial aspects of superfluous elements in a lattice. One can discuss the notion of superfluous elements in a hyperlattice and obtain their possible connections to semiprime ideals in a hyperlattice.

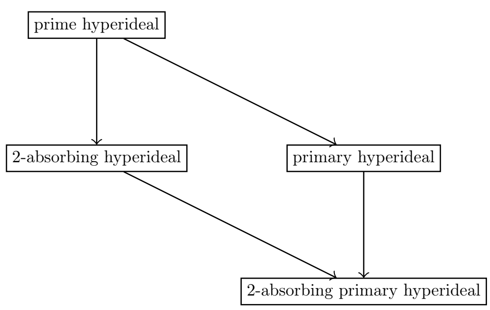

Figure 6.

7. Acknowledgments

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