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ON THE DISCRETE LAPLACE TRANSFORM

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Abstract

The objective of this paper is to introduce the discrete Laplace transform. Basic theorems related to this transformation are mentioned and the discrete Laplace transform of basic functions are given.

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1. Introduction and Preliminaries

The Laplace transform, one of the strongest integral transforms, has been regarded as a powerful tool for solving differential equation, both ordinary and partial [5, 8, 4]. The Laplace transform was, firstly used in the work of Euler who utilized the inverse Laplace transform for solving a second order linear differential equation in 1763. The modern Laplace transform was firstly used by Bateman in 1910 and Bemotein in 1920. In 1920's Doctch applied the Laplace transform to differential, integral and integro-differential equation giving this transform a more modern approach [4]. Analogous to the Laplace transform which is applied to continuous linear systems of differential equations, the Z-transform is applied to solve linear systems of difference equations. This transform was known to Laplace, but developed by Huriwicz in 1947 [6, 7]. In [2], Bohner et al. used the definition of the Laplace transform, on an arbitrary time scale, in order to specify the h-Laplace and consequently the discrete Laplace transform.

To the last extent of our knowledge, no body has attempted to go further and develop the theory of discrete Laplace transform in order to make it applicable to linear systems of difference equations. In this article we present some basic theorems of the discrete Laplace transform. In addition, we compute the discrete Laplace transforms of some basic functions.

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2. The Discrete Laplace Transform

We can define the discrete Laplace transform as the following:

Definition 2.1. *[2, 3, 1] The "L_d-transform" of a sequence $\{x_k\}_{k=0}^{\infty}$ is a function $X(p)$ of a complex variable defined by*

$$X(p) = L_d\{x_k\} = \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^{k+1}} \tag{2.1}$$

For all values of p for which the series converges.

Remark 2.2. *We say that the L_d-transform "exists" provided there is a number $N > 0$ such that :*

$$\sum_{k=0}^{\infty} \frac{x_k}{(p+1)^{k+1}} \quad \textit{converges for} \quad |p+1| > N. \tag{2.2}$$

If $R = \lim sup|x_k|^{\frac{1}{k}}$, then one of the following cases holds:

1. *If $0 < R < \infty$, the series (2.1) converges for $|p+1| > R$ and diverges otherwise;*
2. *If $R = 0,$, the series (2.1) converges for all values of p except possibly for $p = -1$;*
3. *If $R = 0$, the series (2.1) diverges everywhere.*

Definition 2.3. *The sequence $\{x_k\}_{k=0}^{\infty}$ is said to be "exponentially bounded" if there is an $M > 0$ and a $c > 1$ such that*

$$|x_k| \leq Mc^k \textit{ for } k \geq 0.$$

Theorem 2.4. *If the sequence $\{x_k\}$ is exponentially bounded, then the L_d-transform of $\{x_k\}$ exists.*

Proof. Assume that the sequence $\{x_k\}$ is exponentially bounded. Then there is an $M > 0$ and a $c > 1$ such that

$$|x_k| \leq Mc^k \text{ for } k \geq 0.$$

We have :

$$\sum_{k=0}^{\infty} |\frac{x_k}{(p+1)^{k+1}}| \leq \sum_{k=0}^{\infty} \frac{|x_k|}{|p+1|^{k+1}} \leq \frac{M}{p+1} \sum_{k=0}^{\infty} |\frac{c}{(p+1)}|^k$$

and the last series is a geometric series that converges when $|p+1| > c$.
It follows that the L_d-transform of the sequence $\{x_k\}$ exists. □

Lemma 2.5. *If $r > R$,the series (2.1) is uniformly convergent for values of p where $|p+1| \geq r$.*

Proof. Because $r > R$, there exists $\epsilon > 0$ such that
$r > R + \epsilon$ and since $= \lim sup|x_k|^{\frac{1}{k}}$ for the same ϵ there exists a natural number such that

$$|x_k| \leq (R+\epsilon)^k, \quad \text{for all} \quad k \geq n,$$

Now,

$$\left| \sum_{k=n}^{\infty} \frac{x_k}{(p+1)^{k+1}} \right| \leq \sum_{k=n}^{\infty} |x_k| |\frac{1}{p+1}|^{k+1} \leq \sum_{k=n}^{\infty} (\frac{R+\epsilon}{r})^{k+1}.$$

Therefore,

$$\left|\sum_{k=n}^{\infty}\frac{x_k}{(p+1)^{k+1}}\right| \leq \frac{r}{r-R-\epsilon}(\frac{R+\epsilon}{r})^{n+1} \to 0 \ as \ n \to \infty.$$

□

Below, we mention the basic theorems related to the Discrete Laplace Transform

Theorem 2.6. *(Linearity Property)*
If a and b are constant, then

$$L_d\{ax_k + by_k\} = aL_d\{x_k\} + bL_d\{y_k\}. \tag{2.3}$$

Proof. The proof is straightforward. □

Theorem 2.7. *(Discrete Laplace Transform of a shifted sequence)*
Let $\{x_k\}_{k=0}^{\infty}$ *be a sequence such that its Discrete Laplace Transform exists for* $|p+1| > R$ *and* n *a positive integer, then for* $|p+1| > R$ *, the following holds:*

$$L_d\{x_{k+n}\} = (p+1)^n L_d\{x_k\} - \sum_{m=0}^{n-1} x_m(p+1)^{n-m-1} \tag{2.4}$$

Proof. First observe that :

$$L_d\{x_{k+n}\} = \sum_{k=0}^{\infty}\frac{x_{k+n}}{(p+1)^{k+1}} = \frac{1}{p+1}\sum_{k=0}^{\infty}(p+1)^{-k}x_{k+n} = \frac{1}{p+1}\sum_{k=n}^{\infty}(p+1)^{-k+n}x_k$$

$$= \frac{(p+1)^n}{p+1}\left[\sum_{k=0}^{\infty}(p+1)^{-k}x_k - \sum_{m=0}^{n-1}(p+1)^{-m}x_m\right] = (p+1)^n\left[\sum_{k=0}^{\infty}\frac{x_k}{(p+1)^{k+1}} - \sum_{m=0}^{n-1}\frac{x_m}{(p+1)^{m+1}}\right]$$

Therefore, we have

$$L_d\{x_{k+n}\} = (p+1)^n L_d\{x_k\} - \sum_{m=0}^{n-1} x_m(p+1)^{n-m-1}.$$

□

Theorem 2.8. *(Initial Value and Final Value Theorem)*

1. *If* $X(p) = L_d\{x_k\}$ *exists for* $|p+1| > R$ *, then*

$$\lim_{p\to\infty}(p+1)X(p) = x_0, \lim_{p\to\infty} X(p) = 0 \tag{2.5}$$

2. *If* $X(p)$ *exists for* $|p+1| > 1$ *and* $pX(p)$ *is analytic at* $p = 1$ *, then*

$$\lim_{n\to\infty} x_n = \lim_{p\to 0} pL_d\{x_k\} \tag{2.6}$$

Proof. Since

$$L_d\{x_k\} = \sum_{k=0}^{\infty}\frac{x_k}{(p+1)^{k+1}},$$

we have

$$(p+1)L_d\{x_k\} = \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^k} = x_0 + \frac{x_1}{p+1} + \frac{x_2}{(p+1)^2} + \ldots$$

Thus,

$$\lim_{p\to\infty} (p+1)L_d\{x_k\} = x_0$$
$$\lim_{p\to\infty} (p+1)X(p) = x_0.$$

Consequently,

$$\lim_{p\to\infty} X(p) = \lim_{p\to\infty} \frac{x_0}{p+1} = 0.$$

Thus, the assertion in (2.5) is proved. Since

$$L_d\{x_{k+1} - x_k\} = \sum_{k=0}^{\infty} \frac{x_{k+1}}{(p+1)^{k+1}} - \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^{k+1}} = \lim_{n\to\infty} \sum_{k=0}^{n} \left(\frac{x_{k+1}}{(p+1)^{k+1}} - \frac{x_k}{(p+1)^{k+1}} \right)$$
$$= \lim_{n\to\infty} \left[-\frac{x_0}{p+1} + \left(\frac{1}{p+1} - \frac{1}{(p+1)^2} \right) x_1 + \right.$$
$$\left. \left(\frac{1}{(p+1)^2} - \frac{1}{(p+1)^3} \right) x_2 + \ldots + \left(\frac{1}{(p+1)^n} - \frac{1}{(p+1)^{n+1}} \right) x_n + \frac{x_{n+1}}{(p+1)^{n+1}} \right],$$

we have,

$$\lim_{p\to 0} L_d\{x_{k+1} - x_k\} = \lim_{p\to 0} \lim_{n\to\infty} \left[-\frac{x_0}{p+1} + (\frac{1}{p+1} - \frac{1}{(p+1)^2})x_1 + \right.$$
$$\left. \left(\frac{1}{(p+1)^2} - \frac{1}{(p+1)^3} \right) x_2 + \ldots + \left(\frac{1}{(p+1)^n} - \frac{1}{(p+1)^{n+1}} \right) x_n + \frac{x_{n+1}}{(p+1)^{n+1}} \right].$$

Thus,

$$\lim_{p\to 0} L_d\{x_{k+1} - x_k\} = \lim_{n\to\infty} \lim_{p\to 0} [-x_0 + x_{n+1}]$$

$$\lim_{p\to 0} L_d\{x_{k+1} - x_k\} = \lim_{n\to\infty} [-x_0 + x_{n+1}]$$

$$\lim_{p\to 0} L_d\{x_{k+1} - x_k\} = (\lim_{n\to\infty} x_{n+1}) - x_0.$$

Hence,

$$\lim_{p\to 0} [L_d\{x_{k+1}\} - L_d\{x_k\}] = (\lim_{n\to\infty} x_{n+1}) - x_0.$$

On the other side, using Theorem (2.7, we have

$$\lim_{p\to 0} [(p+1)L_d\{x_k\} - x_0 - L_d\{x_k\}] = (\lim_{n\to\infty} x_{n+1}) - x_0$$

$$\lim_{p\to 0} pL_d\{x_k\} - x_0 = (\lim_{n\to\infty} x_{n+1}) - x_0.$$

Hence, we have

$$\lim_{n\to\infty} x_{n+1} = \lim_{p\to 0} pL_d\{x_k\}$$

□

Definition 2.9. *[7] Let* $\{x_k\}_{k=0}^{\infty}$ *and* $\{y_k\}_{k=0}^{\infty}$ *be two sequences. Then the convolution of* x_k *and* y_k *is defined by*

$$x_k * y_k = \sum_{m=0}^{k} x_{k-m} y_m. \tag{2.7}$$

The following theorem presents the Discrete Laplace Transform of the convolution of two sequences.

Theorem 2.10. *(Convolution Theorem) If* $X(p)$ *exists for* $|p+1| > a$ *and* $Y(p)$ *exists for* $|p+1| > b$*, then:*

$$L_d\{x_k * y_k\} = (p+1) L_d\{x_k\} L_d\{y_k\}, \tag{2.8}$$

for $|p+1| > max\{a, b\}$.

Proof.

$$L_d\{x_k\}.L_d\{y_k\} = \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^{k+1}} \sum_{k=0}^{\infty} \frac{y_k}{(p+1)^{k+1}} = \frac{1}{(p+1)^2} \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^k} \sum_{k=0}^{\infty} \frac{y_k}{(p+1)^k}$$

$$= \frac{1}{(p+1)^2} \sum_{k=0}^{\infty} \sum_{m=0}^{k} \frac{x_{k-m} y_m}{(p+1)^k} = \frac{1}{p+1} \sum_{k=0}^{\infty} (\sum_{m=0}^{k} x_{k-m} y_m) \frac{1}{(p+1)^{k+1}} = \frac{1}{p+1} \sum_{k=0}^{\infty} \frac{x_k y_k}{(p+1)^{k+1}}$$

$$= \frac{1}{p+1} L_d\{x_k * y_k\}.$$

Therefore, we have

$$L_d\{x_k * y_k\} = (p+1) L_d\{x_k\}.L_d\{y_k\}.$$

Or

$$L_d\{x_k * y_k\} = (p+1) X(p).Y(p).$$

□

Example 2.11. *In this example, we find the Discrete Laplace Transform of* $\{x_k = 1\}$.

$$X(p) = L_d\{1\} = \sum_{k=0}^{\infty} \frac{1}{(p+1)^{k+1}}$$

$$= \frac{1}{(p+1)} \sum_{k=0}^{\infty} \frac{1}{(p+1)^k} = \frac{1}{(p+1)} \frac{1}{1 - \frac{1}{p+1}} = \frac{1}{p}, \quad \textit{for } |p+1| > 1.$$

3. Discrete Laplace Transform of Elementary Functions

In this section, we discuss the Discrete Laplace Transforms of some elementary discrete functions. In the following theorem, the Discrete Laplace Transforms of the discrete exponential function is given.

Theorem 3.1. *Let* a *be a real number. Then*

$$L_d\{a^k\} = \frac{1}{p+1-a}, \quad \textit{for } |p+1| > |a| \tag{3.1}$$

Proof.

$$L_d\{a^k\} = \sum_{k=0}^{\infty} \frac{a^k}{(p+1)^{k+1}} = \sum_{k=0}^{\infty} \frac{1}{(p+1)} \frac{a^k}{(p+1)^k} = \frac{1}{p+1} \sum_{k=0}^{\infty} (\frac{a}{(p+1)})^k$$

$$= \frac{1}{p+1} \frac{1}{1-\frac{a}{p+1}} = \frac{1}{p+1} \frac{(p+1)}{(p+1-a)} = \frac{1}{p+1-a}, \quad |p+1| > |a|.$$

□

Using Theorem 3.1 one can find the Discrete Laplace Transforms of some trigonometric functions.

Corollary 3.2. 1. *For a real number a and $|p+1| > 1$,*

$$L_d\{\cos(ak)\} = \frac{p+1-\cos a}{p^2 - 2p\cos a + 2} \tag{3.2}$$

$$L_d\{\sin(ak)\} = \frac{\sin a}{p^2 - 2p\cos a + 2} \tag{3.3}$$

2. *For a real number a and $|p+1| > \max\{e^a, e^{-a}\}$,*

$$L_d\{\cosh(ak)\} = \frac{p+1-\cosh(a)}{p^2 + 2(p+1)(1-\cosh(a))} \tag{3.4}$$

$$L_d\{\sinh(ak)\} = \frac{\sinh(a)}{p^2 + 2(p+1)(1-\cosh(a))} \tag{3.5}$$

Proof. The proof is verbatim of the proof of Theorem 3.1. □

Theorem 3.3. *If $L_d\{x_k\} = X(p)$exists for $|p+1| > r$,then for any constant $a \neq 0$, we have*

$$L_d\{a^k x_k\} = \frac{1}{a} X(\frac{p+1-a}{a}). \tag{3.6}$$

Proof.

$$X(p) = L_d\{a^k x_k\} = \sum_{k=0}^{\infty} \frac{a^k x_k}{(p+1)^{k+1}} = \sum_{k=0}^{\infty} \frac{1}{a} \frac{a^{k+1} x_k}{(p+1)^{k+1}} = \frac{1}{a} \sum_{k=0}^{\infty} \frac{x_k}{(\frac{p+1}{a})^{k+1}}$$

$$= \frac{1}{a} \sum_{k=0}^{\infty} \frac{x_k}{(\frac{p}{a} + \frac{1}{a})^{k+1}} = \frac{1}{a} X\left(\frac{p+1}{a} - 1\right).$$

Therefore, we have

$$L_d\{a^k x_k\} = \frac{1}{a} X\left(\frac{p+1-a}{a}\right).$$

□

Theorem 3.4. *If $X(p) = L_d\{x_k\}$ for $|p+1| > r$ then,*

$$L_d\{(k+n)^{\underline{n}} x_k\} = (-1)^n (p+1)^n \frac{d^n}{dp^n} X(p). \tag{3.7}$$

Proof.

$$X(p) = \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^{k+1}} = \sum_{k=0}^{\infty} (p+1)^{-k-1} x_k$$

$$X'(p) = -\sum_{k=0}^{\infty} (k+1)(p+1)^{-k-2} x_k$$

$$X''(p) = \sum_{k=0}^{\infty} (k+1)(k+2)(p+1)^{-k-3} x_k$$

$$X'''(p) = -\sum_{k=0}^{\infty} (k+1)(k+2)(k+3)(p+1)^{-k-4} x_k.$$

Taking the derivative continuously with respect to p, we reach

$$X^{(n)}(p) = (-1)^n \sum_{k=0}^{\infty} (k+1)(k+2)\dots(k+n)(p+1)^{-k-n-1} x_k.$$

Therefore, we get

$$(-1)^n X^{(n)}(p)(p+1)^n = \sum_{k=0}^{\infty} \frac{(k+1)(k+2)\dots(k+n)}{(p+1)^{k+1}} x_k.$$

Hence, we find

$$L_d\{(k+n)^{\underline{n}} x_k\} = (-1)^n (p+1)^n \frac{d^n}{dp^n} X(p).$$

□

Theorem 3.5. *For $n = 1, 2, 3, \dots,$*

$$L_d\{k^{\underline{n}}\} = \frac{n!}{p^{n+1}} \quad for\, |p| > 0. \tag{3.8}$$

Proof. We will use mathematical induction. For n=1,

$$L_d\{k\} = \sum_{k=0}^{\infty} \frac{k}{(p+1)^{k+1}} = \sum_{k=0}^{\infty} \frac{1}{(p+1)} . \frac{k}{(p+1)^k} = \frac{1}{(p+1)} \sum_{k=0}^{\infty} \frac{k}{(p+1)^k} = \frac{1}{(p+1)} \sum_{k=1}^{\infty} \frac{k}{(p+1)^k}$$

$$= \frac{1}{(p+1)} \sum_{k=0}^{\infty} \frac{k+1}{(p+1)^{k+1}} = \frac{1}{(p+1)} [\sum_{k=0}^{\infty} \frac{k}{(p+1)^{k+1}} + \sum_{k=0}^{\infty} \frac{1}{(p+1)^{k+1}}]$$

$$L_d\{k\} = \frac{1}{(p+1)} [L_d\{k\} + L_d\{1\}]$$

$$(p+1)L_d\{k\} - L_d\{k\} = L_d\{1\}$$

$$L_d\{k\}(p+1-1) = L_d\{1\}$$

$$pL_d\{k\} = \frac{1}{p}.$$

Thus,

$$L_d\{k\} = \frac{1}{p^2}.$$

Now, suppose it is true for $n \leq m$.

$$L_d\{k^{\underline{m+1}}\} = \sum_{k=0}^{\infty} \frac{k^{\underline{m+1}}}{(p+1)^{k+1}} = \sum_{k=0}^{\infty} k^{\underline{m+1}} \left(\frac{1}{p+1}\right)^{k+1}$$

$$= \left[-k^{\underline{m+1}}(\frac{1}{p(p+1)^k}|_0^\infty - \sum_{k=0}^{\infty}(m+1)k^{\underline{m}}(\frac{-1}{p})\frac{1}{(p+1)^{k+1}} \right] = \frac{1}{p}\sum_{k=0}^{\infty}(m+1)k^{\underline{m}}\frac{1}{(p+1)^{k+1}}$$

$$= \frac{m+1}{p}\sum_{k=0}^{\infty}k^{\underline{m}}\frac{1}{(p+1)^{k+1}} = \frac{m+1}{p}L_d\{k^{\underline{m}}\} = \frac{m+1}{p}\frac{m!}{p^{m+1}}.$$

Therefore, we have

$$L_d\{k^{\underline{m+1}}\} = \frac{(m+1)!}{p^{m+2}}.$$

□

Theorem 3.6. *For a positive integer n,*

$$L_d\left\{\binom{k}{n}\right\} = \frac{1}{p^{n+1}}. \tag{3.9}$$

Proof.

$$L_d\left\{\binom{k}{n}\right\} = L_d\left\{\frac{k^{\underline{n}}}{n!}\right\} = \frac{1}{n!}L_d\{k^{\underline{n}}\} = \frac{1}{n!}\frac{n!}{p^{n+1}} = \frac{1}{p^{n+1}}.$$

Therefore, we have

$$L_d\left\{\binom{k}{n}\right\} = \frac{1}{p^{n+1}}.$$

□

Definition 3.7. *[7]*

$$\delta_k(n) = \begin{cases} 1 \ if \ k = n \\ 0 \ if \ k \neq n \end{cases} \tag{3.10}$$

In the following theorem the Discrete Laplace Transform of the unit impulse sequence is given.

Theorem 3.8.

$$L_d\{\delta_k(n)\} = (\frac{1}{p+1})^{n+1}. \tag{3.11}$$

Proof.

$$L_d\{\delta_k(n)\} = \sum_{k=0}^{\infty}\frac{\delta_k(n)}{(p+1)^{k+1}} = \frac{1}{(p+1)^{n+1}}.$$

□

Below after we define the unit step sequence we percent its Laplace Transform.

Definition 3.9. *[7]*

$$u_k(n) = \begin{cases} 0 \ if \ 0 \leq k \leq n-1 \\ 1 \ if \ k \geq n \end{cases} \tag{3.12}$$

Theorem 3.10. *For any positive integer n and $|p+1| > 1$,*

$$L_d\{u_k(n)\} = \frac{1}{p(p+1)^n}. \tag{3.13}$$

Proof.

$$L_d\{u_k(n)\} = \sum_{k=0}^{\infty} \frac{u_k(n)}{(p+1)^{k+1}} = \sum_{k=n}^{\infty} \frac{1}{(p+1)^{k+1}} = \sum_{k=0}^{\infty} \frac{1}{(p+1)^{n+k+1}}$$

$$= \frac{1}{(p+1)^n} \sum_{k=0}^{\infty} \frac{1}{(p+1)^{k+1}} = \frac{1}{(p+1)^n} L_d\{1\} = \frac{1}{(p+1)^n} \frac{1}{p}.$$

□

Theorem 3.10 can be generalized as follows.

Theorem 3.11. *For any positive integer n,*

$$L_d\{x_{k-n}u_k(n)\} = (p+1)^{-n} L_d\{x_k\}. \tag{3.14}$$

Proof.

$$L_d\{x_{k-n}u_k(n)\} = \sum_{k=0}^{\infty} \frac{x_{k-n}u_k(n)}{(p+1)^{k+1}} = \sum_{k=n}^{\infty} \frac{x_{k-n}}{(p+1)^{k+1}} = \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^{k+n+1}}$$

$$= \frac{1}{(p+1)^n} \sum_{k=0}^{\infty} \frac{x_k}{(p+1)^{k+1}}.$$

Therefore, we have

$$L_d\{x_{k-n}u_k(n)\} = \frac{1}{(p+1)^n} L_d\{x_k\}.$$

□

Table 1. List of Discrete Laplace Transforms

Sequence	**d- Laplace Transforms**
1	$\frac{1}{p}$
a^k	$\frac{1}{p+1-a}$
k	$\frac{1}{p^2}$
k^2	$\frac{1}{p^2} + \frac{2}{p^3}$
$k^{\underline{n}}$	$\frac{n!}{p^{n+1}}$
$\sin(ak)$	$\frac{\sin a}{p^2-2p\cos a+2}$
$\cos(ak)$	$\frac{p+1-\cos a}{p^2-2p\cos a+2}$
$\sinh(ak)$	$\frac{\sinh a}{p^2+2(p+1)(1-\cosh a)}$
$\cosh(ak)$	$\frac{p+1-\cosh a}{p^2+2(p+1)(1-\cosh a)}$

Table 2. List of Discrete Laplace Transforms

Sequence	**d- Laplace Transforms**
$\delta_k(n)$	$(\frac{1}{p+1})^{n+1}$
$u_k(n)$	$\frac{1}{p(p+1)^n}$
kx_k	$-(p+1)X'(p) - X(p)$
$x_k * y_k$	$(p+1)X(p).Y(p)$
$\sum_{m=0}^{k} x_m$	$\frac{p+1}{p}X(p)$
$a^k x_k$	$\frac{1}{a}X(\frac{p+1-a}{a})$
x_{k+n}	$(p+1)^n L_d\{x_k\} - \sum_{m=0}^{n-1} x_m(p+1)^{n-m-1}$
$x_{k-n}x_k(n)$	$\frac{1}{(p+1)^n}X(p)$

4. CONCLUSION

The Laplace transformation is considered to be the most important one of the various integral transformations that contribute to the development of the theory of differential equations. The question of whether these transformations has a separate analogue still needs an answer. In this paper, the discrete analogue of the Laplace transform is defined using the Laplace transform in the general time scale. The basic theorems related to this transformation are mentioned and Laplace transforms of the most important functions are derived. Unlike the differential equations which can be solved by a few transformation methods, there is only one total transformation called Z-transformation which is used to solve differential equations. However, solving the difference equations using Z-transformation requires tedious and complex calculations. Although there is a strong relationship between Z-transformation and Discrete Laplace Transform, Discrete Laplace Transform can be considered as a competitor for Z-transformation, which minimizes these calculations.

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