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Fractional electromagnetic fields in DPS and DNG regions with standard fractional vector cross product

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Abstract

In this research we have obtained fractional electromagnetic fields in DPS and DNG regions using standard fractional vector cross product(SFVCP). We have also calculated fractional electromagnetic fields of a travelling electromagnetic wave in DPS and DNG region with oblique incidence using SFVCP. This approach is new as the definition of SFVCP is not yet explored for calculating fractional electromagnetic fields in DPS and DNG regions. This study will add a new dimension to the study of electromagnetic fields that have application in mobile communication, transmission from antenna, in lens design etc.

Mathematics Subject Classification (2010): 78A02, 78A50, 78A97, 78M05

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1. Introduction

For last few years the electromagnetic wave interaction in meta-material media is getting considerable attention. Electromagnetic wave interaction with any conventional medium depends on permittivity and permeability of the medium. These parameters are positive in double conventional (DPS) medium where electric and magnetic fields follow right handed rule of physics. In 1968, Veselago [1]

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studied plane wave propagation in a medium where permittivity and permeability were both negative. This was proved experimentally by Smith et al. [2] in which they created artificial structures for the first time called as Left handed material (LHM), Double negative material (DNG) or meta-material.

Fractional fields via fractional cross product developed in these regions by [3], [4] has many applications in physics. Fractional fields using SFVCP are developed by Kankarej and Singh in [5]. In this research, fractional electromagnetic fields are obtained for DNG and DPS medium using definition of SFVCP given by Kankarej and Singh in [6], [7].

2. Standard Fractional Vector Cross Product

Crowe [8] laid the foundation of vector analysis. Further to the study of SFVCP in [6], [7], [5] where we defined:

Definition 2.1: *Let R^3 be the Euclidean 3-space equipped with standard inner product $\langle .,. \rangle$. Let (e_1, e_2, e_3) be standard orthonormal basis of R^3 and $\gamma \in [0,1]$ a real number. Then, for vectors $a = a_1e_1 + a_2e_2 + a_3e_3, b = b_1e_1 + b_2e_2 + b_3e_3$ in R^3, the Standard Fractional Vector Cross Product is defined by*

$$\begin{aligned} a \times^{\gamma} b = &\left\{ (a_2b_3 - a_3b_2)\sin\left(\frac{\gamma\pi}{2}\right) + (a_2 + a_3)b_1\cos\left(\frac{\gamma\pi}{2}\right) - (b_2 + b_3)a_1\cos\left(\frac{\gamma\pi}{2}\right) \right\} e_1 \\ &+ \left\{ (a_3b_1 - a_1b_3)\sin\left(\frac{\gamma\pi}{2}\right) + (a_3 + a_1)b_2\cos\left(\frac{\gamma\pi}{2}\right) - (b_3 + b_1)a_2\cos\left(\frac{\gamma\pi}{2}\right) \right\} e_2 \\ &+ \left\{ (a_1b_2 - a_2b_1)\sin\left(\frac{\gamma\pi}{2}\right) + (a_1 + a_2)b_3\cos\left(\frac{\gamma\pi}{2}\right) - (b_1 + b_2)a_3\cos\left(\frac{\gamma\pi}{2}\right) \right\} e_3 \end{aligned} \tag{1}$$

From (1) we have,

$$e_i \times^{\gamma} e_j = \cos\left(\frac{\gamma\pi}{2}\right)e_j + \sin\left(\frac{\gamma\pi}{2}\right)e_k - \cos\left(\frac{\gamma\pi}{2}\right)e_i \tag{2}$$

$$e_j \times^{\gamma} e_i = \cos\left(\frac{\gamma\pi}{2}\right)e_i - \sin\left(\frac{\gamma\pi}{2}\right)e_k - \cos\left(\frac{\gamma\pi}{2}\right)e_j \tag{3}$$

$$e_l \times^{\gamma} e_l = 0 \textit{ for } l = \{1,2,3\} \tag{4}$$

where (i, j, k) is a cyclic permutation of (1, 2, 3). The equations (2), (3) and (4) are similar to that in [6], [7].

3. Fractional Electromagnetic Fields in DPS and DNG region

Let us consider a x - y plane which is a source of time harmonic current at $z = 0$ directed in x direction with unit current density $\bar{J} = \hat{x}\delta(z)$ as in [3]. In this plane half space $z > 0$ is media with $\varepsilon > 0$ and $\mu > 0$ i.e. with DPS properties and the other half space $z < 0$ is media with $\varepsilon < 0$ and $\mu < 0$ i.e. with DNG properties. The DPS region $z > 0$ has the propagation wave vector directed away from the source while the DNG region $z < 0$ has the propagation wave vector directed towards the source. Thus the DPS region has the fields

$$\begin{aligned} \bar{E} &= \hat{x}E_0e^{ikz} \\ \eta\bar{H} &= \hat{y}E_0e^{ikz} \end{aligned} \tag{5}$$

Thus the DNG region has the fields

$$\begin{aligned} \bar{E} &= \hat{x}E_0 e^{ikz} \\ \eta\bar{H} &= -\hat{y}E_0 e^{ikz} \end{aligned} \tag{6}$$

Using the definition of field vectors in [3], matrix representation from [9] and fractional curl and fractional fields in [6], [7] for $\bar{F} = \bar{E} = \hat{x}E_0 e^{ikz}$ and $z > 0$, we have

$$\nabla \times^{\gamma} \bar{F} = \cos\left(\frac{\gamma\pi}{2}\right)\begin{bmatrix} \nabla^{\gamma}_{yz} & -\partial^{\gamma}_{x} & -\partial^{\gamma}_{x} \\ -\partial^{\gamma}_{y} & \nabla^{\gamma}_{zx} & -\partial^{\gamma}_{y} \\ -\partial^{\gamma}_{z} & -\partial^{\gamma}_{z} & \nabla^{\gamma}_{xy} \end{bmatrix}\begin{bmatrix} \hat{x}E_0 e^{ikz} \\ 0 \\ 0 \end{bmatrix} + \sin\left(\frac{\gamma\pi}{2}\right)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial^{\gamma}_{x} & \partial^{\gamma}_{y} & \partial^{\gamma}_{z} \\ E_0 e^{ikz} & 0 & 0 \end{vmatrix} \tag{7}$$

where $\nabla^{\gamma}_{yz} = \frac{\partial^{\gamma}}{\partial y^{\gamma}} + \frac{\partial^{\gamma}}{\partial z^{\gamma}}, \nabla^{\gamma}_{zx} = \frac{\partial^{\gamma}}{\partial z^{\gamma}} + \frac{\partial^{\gamma}}{\partial x^{\gamma}}, \nabla^{\gamma}_{xy} = \frac{\partial^{\gamma}}{\partial x^{\gamma}} + \frac{\partial^{\gamma}}{\partial y^{\gamma}}, \frac{\partial^{\gamma}}{\partial x^{\gamma}} = \partial^{\gamma}_{x}, \frac{\partial^{\gamma}}{\partial y^{\gamma}} = \partial^{\gamma}_{y}, \frac{\partial^{\gamma}}{\partial z^{\gamma}} = \partial^{\gamma}_{z}$

$$\begin{aligned} \nabla \times^{\gamma} \bar{E} &= \sin\left(\frac{\gamma\pi}{2}\right)\left\{\hat{y}\frac{\partial^{\gamma}}{\partial z^{\gamma}}E_0 e^{ikz} - \hat{z}\frac{\partial^{\gamma}}{\partial y^{\gamma}}E_0 e^{ikz}\right\} \\ &= \sin\left(\frac{\gamma\pi}{2}\right)E_0 (ik)^{\gamma}(\hat{y} - \hat{z})e^{ikz} \end{aligned} \tag{8}$$

Thus the fractional field $\bar{E} = (ik)^{-\gamma}\nabla^{\gamma}\bar{E}$ is given by

$$\bar{E}_f = \sin\left(\frac{\gamma\pi}{2}\right)E_0 e^{ikz}(\hat{y} - \hat{z}) \tag{9}$$

Similarly, for $\bar{F} = \eta\bar{H} = \hat{y}E_0 e^{ikz}$ for $z > 0$, we have

$$\nabla \times^{\gamma} \bar{F} = \cos\left(\frac{\gamma\pi}{2}\right)\begin{bmatrix} \nabla^{\gamma}_{yz} & -\partial^{\gamma}_{x} & -\partial^{\gamma}_{x} \\ -\partial^{\gamma}_{y} & \nabla^{\gamma}_{zx} & -\partial^{\gamma}_{y} \\ -\partial^{\gamma}_{z} & -\partial^{\gamma}_{z} & \nabla^{\gamma}_{xy} \end{bmatrix}\begin{bmatrix} 0 \\ \hat{y}E_0 e^{ikz} \\ 0 \end{bmatrix} + \sin\left(\frac{\gamma\pi}{2}\right)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial^{\gamma}_{x} & \partial^{\gamma}_{y} & \partial^{\gamma}_{z} \\ 0 & E_0 e^{ikz} & 0 \end{vmatrix} \tag{10}$$

$$\begin{aligned} \nabla \times^{\gamma} \eta\bar{H} &= \sin\left(\frac{\gamma\pi}{2}\right)\left\{-\hat{x}\frac{\partial^{\gamma}}{\partial z^{\gamma}}E_0 e^{ikz} + \hat{z}\frac{\partial^{\gamma}}{\partial z^{\gamma}}E_0 e^{ikz}\right\} \\ &= \sin\left(\frac{\gamma\pi}{2}\right)E_0 (ik)^{\gamma}(-\hat{x} + \hat{z})e^{ikz} \end{aligned} \tag{11}$$

Thus the fractional field for $\eta\bar{H} = (ik)^{-\gamma}\nabla^{\gamma}\eta\bar{H}$ is given by

$$\eta\bar{H}_f = \sin\left(\frac{\gamma\pi}{2}\right)E_0 e^{ikz}(-\hat{x} + \hat{z}) \tag{12}$$

Similarly, for $\bar{F} = \eta\bar{H} = -\hat{y}E_0 e^{ikz}$ and $z < 0$, we have

$$\nabla \times^{\gamma} \bar{F} = \cos\left(\frac{\gamma\pi}{2}\right)\begin{bmatrix} \nabla^{\gamma}_{yz} & -\partial^{\gamma}_{x} & -\partial^{\gamma}_{x} \\ -\partial^{\gamma}_{y} & \nabla^{\gamma}_{zx} & -\partial^{\gamma}_{y} \\ -\partial^{\gamma}_{z} & -\partial^{\gamma}_{z} & \nabla^{\gamma}_{xy} \end{bmatrix}\begin{bmatrix} 0 \\ -\hat{y}E_0 e^{ikz} \\ 0 \end{bmatrix} + \sin\left(\frac{\gamma\pi}{2}\right)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial^{\gamma}_{x} & \partial^{\gamma}_{y} & \partial^{\gamma}_{z} \\ 0 & -E_0 e^{ikz} & 0 \end{vmatrix} \tag{13}$$

$$\nabla\times^{\gamma}\eta\bar{H}=\sin\left(\frac{\gamma\pi}{2}\right)\left\{\hat{x}\frac{\partial^{\gamma}}{\partial z^{\gamma}}E_0e^{ikz}-\hat{z}\frac{\partial^{\gamma}}{\partial z^{\gamma}}E_0e^{ikz}\right\}$$
$$=\sin\left(\frac{\gamma\pi}{2}\right)E_0(ik)^{\gamma}(-\hat{x}+\hat{z})e^{ikz} \tag{14}$$

Thus the fractional field $\eta\bar{H}=(ik)^{-\gamma}\nabla^{\gamma}\eta\bar{H}$ is given by

$$\eta\bar{H}_f=\sin\left(\frac{\gamma\pi}{2}\right)E_0e^{ikz}(\hat{x}-\hat{z}) \tag{15}$$

For $\gamma=1, z>0$ we have electric and magnetic fields as,

$$\begin{aligned}\bar{E}_f&=E_0e^{ikz}(\hat{y}-\hat{z})\\ \eta\bar{H}_f&=E_0e^{ikz}(-\hat{x}+\hat{z})\end{aligned} \tag{16}$$

For $\gamma=1, z<0$ we have electric and magnetic fields as,

$$\begin{aligned}\bar{E}_f&=E_0e^{ikz}(\hat{y}-\hat{z})\\ \eta\bar{H}_f&=E_0e^{ikz}(\hat{x}-\hat{z})\end{aligned} \tag{17}$$

4. Fractional Electromagnetic Fields of a travelling electromagnetic wave in DPS and DNG region with oblique incidence

Let us take an example of travelling electromagnetic wave where

$$\begin{aligned}\bar{E}&=\hat{y}e^{ik_xx+ik_zz}\\ \eta\bar{H}&=[-\hat{x}+\hat{z}]e^{ik_xx+ik_zz}\end{aligned} \tag{18}$$

let us take the exponent $i\bar{k}\bar{r}=ik_xx+ik_zz=(i\bar{k}).(\bar{r})$. The wave is propagating in $z<0$ region, in the direction of $\bar{k}$ and in increasing x and z or in the direction of increasing $\bar{r}$ and hence the gradient $\frac{d}{dr}$ is positive. We take it's projection on x and z coordinates as $\frac{k_x}{k}$ and $\frac{k_z}{k}$ respectively. This will give the values for $\frac{\partial^{\gamma}}{\partial x^{\gamma}}$ and $\frac{\partial^{\gamma}}{\partial z^{\gamma}}$ in the calculations respectively.

Using the definition of fractional curl from [7], we have

$$\nabla\times^{\gamma}\bar{F}=\cos\left(\frac{\gamma\pi}{2}\right)\begin{bmatrix}\nabla^{\gamma}_{yz} & -\partial^{\gamma}_{x} & -\partial^{\gamma}_{x}\\ -\partial^{\gamma}_{y} & \nabla^{\gamma}_{zx} & -\partial^{\gamma}_{y}\\ -\partial^{\gamma}_{z} & -\partial^{\gamma}_{z} & \nabla^{\gamma}_{xy}\end{bmatrix}\begin{bmatrix}0\\ \hat{y}e^{i\bar{k}\bar{r}}\\ 0\end{bmatrix}+\sin\left(\frac{\gamma\pi}{2}\right)\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \partial^{\gamma}_{x} & \partial^{\gamma}_{y} & \partial^{\gamma}_{z}\\ 0 & e^{i\bar{k}\bar{r}} & 0\end{vmatrix} \tag{19}$$

$$\nabla\times^{\gamma}\bar{E}=\sin\left(\frac{\gamma\pi}{2}\right)\left\{\hat{y}\frac{\partial^{\gamma}}{\partial z^{\gamma}}e^{i\bar{k}\bar{r}}-\hat{z}\frac{\partial^{\gamma}}{\partial y^{\gamma}}e^{i\bar{k}\bar{r}}\right\}$$
$$=\sin\left(\frac{\gamma\pi}{2}\right)(ik)^{\gamma}e^{i\bar{k}\bar{r}}\hat{y}\frac{k_z}{k} \tag{20}$$

Thus the fractional field $\bar{E}=(ik)^{-\gamma}\nabla^{\gamma}\bar{E}$ is given by

$$\bar{E}_f=\sin\left(\frac{\gamma\pi}{2}\right)e^{i\bar{k}\bar{r}}\hat{y}\frac{k_z}{k}=\sin\left(\frac{\gamma\pi}{2}\right)e^{ik_xx+ik_zz}\hat{y}\frac{k_z}{k} \tag{21}$$

Similarly the fractional magnetic field is calculated as below:

$$\nabla \times^{\gamma} \bar{F} = \cos\left(\frac{\gamma\pi}{2}\right)\begin{bmatrix} \nabla^{\gamma}_{yz} & -\partial^{\gamma}_{x} & -\partial^{\gamma}_{x} \\ -\partial^{\gamma}_{y} & \nabla^{\gamma}_{zx} & -\partial^{\gamma}_{y} \\ -\partial^{\gamma}_{z} & -\partial^{\gamma}_{z} & \nabla^{\gamma}_{xy} \end{bmatrix}\begin{bmatrix} -\hat{x}e^{i\bar{k}\bar{r}} \\ 0 \\ \hat{z}e^{i\bar{k}\bar{r}} \end{bmatrix} + \sin\left(\frac{\gamma\pi}{2}\right)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial^{\gamma}_{x} & \partial^{\gamma}_{y} & \partial^{\gamma}_{z} \\ -e^{i\bar{k}\bar{r}} & 0 & e^{i\bar{k}\bar{r}} \end{vmatrix} \tag{22}$$

$$\begin{aligned} \nabla \times^{\gamma} \eta\bar{H} &= \sin\left(\frac{\gamma\pi}{2}\right)\left\{\hat{x}\frac{\partial^{\gamma}}{\partial y^{\gamma}}e^{i\bar{k}\bar{r}} + \hat{y}\left[\frac{\partial^{\gamma}}{\partial z^{\gamma}}(-e^{i\bar{k}\bar{r}}) - \frac{\partial^{\gamma}}{\partial x^{\gamma}}(e^{i\bar{k}\bar{r}})\right] + \hat{z}\frac{\partial^{\gamma}}{\partial y^{\gamma}}e^{i\bar{k}\bar{r}}\right\} \\ &= \sin\left(\frac{\gamma\pi}{2}\right)(ik)^{\gamma}e^{i\bar{k}\bar{r}}\hat{y}\left(-\frac{k_z}{k} - \frac{k_x}{k}\right) \end{aligned} \tag{23}$$

Thus the fractional field $\eta\bar{H} = (ik)^{-\gamma}\nabla^{\gamma}\eta\bar{H}$ is given by

$$\begin{aligned} \eta\bar{H}_f &= \sin\left(\frac{\gamma\pi}{2}\right)(ik)^{\gamma}e^{i\bar{k}\bar{r}}\hat{y}\left(-\frac{k_z}{k} - \frac{k_x}{k}\right) \\ &= \sin\left(\frac{\gamma\pi}{2}\right)(ik)^{\gamma}e^{ik_x x + ik_z z}\hat{y}\left(-\frac{k_z}{k} - \frac{k_x}{k}\right) \end{aligned} \tag{24}$$

For the region $z > 0$, the LHM region with DPS medium have the fields as

$$\begin{aligned} \bar{E} &= \hat{y}e^{ik_x x - ik_z z} = \hat{y}e^{-ik_x x + ik_z z} \\ \eta\bar{H} &= [-\hat{x} - \hat{z}]e^{ik_x x - ik_z z} = [-\hat{x} - \hat{z}]e^{-ik_x x + ik_z z} \end{aligned} \tag{25}$$

where $k^2 = k_x^2 + k_z^2, \bar{k} = -k_x\hat{x} + k_z\hat{z}, e^{ik_x x - ik_z z} = e^{-i\bar{k}\bar{r}}$

$$\nabla \times^{\gamma} \bar{F} = \cos\left(\frac{\gamma\pi}{2}\right)\begin{bmatrix} \nabla^{\gamma}_{yz} & -\partial^{\gamma}_{x} & -\partial^{\gamma}_{x} \\ -\partial^{\gamma}_{y} & \nabla^{\gamma}_{zx} & -\partial^{\gamma}_{y} \\ -\partial^{\gamma}_{z} & -\partial^{\gamma}_{z} & \nabla^{\gamma}_{xy} \end{bmatrix}\begin{bmatrix} 0 \\ \hat{y}e^{-i\bar{k}\bar{r}} \\ 0 \end{bmatrix} + \sin\left(\frac{\gamma\pi}{2}\right)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial^{\gamma}_{x} & \partial^{\gamma}_{y} & \partial^{\gamma}_{z} \\ 0 & e^{-i\bar{k}\bar{r}} & 0 \end{vmatrix} \tag{26}$$

$$\begin{aligned} \nabla \times^{\gamma} \bar{E} &= \sin\left(\frac{\gamma\pi}{2}\right)\left\{\hat{y}\frac{\partial^{\gamma}}{\partial z^{\gamma}}e^{-i\bar{k}\bar{r}} - \hat{z}\frac{\partial^{\gamma}}{\partial y^{\gamma}}e^{-i\bar{k}\bar{r}}\right\} \\ &= \sin\left(\frac{\gamma\pi}{2}\right)(-ik)^{\gamma}e^{-i\bar{k}\bar{r}}\hat{y}\frac{k_z}{k} \end{aligned} \tag{27}$$

Thus the fractional field $\bar{E} = (-ik)^{-\gamma}\nabla^{\gamma}\bar{E}$ is given by

$$\bar{E}_f = \sin\left(\frac{\gamma\pi}{2}\right)e^{-i\bar{k}\bar{r}}\hat{y}\frac{k_z}{k} = \sin\left(\frac{\gamma\pi}{2}\right)e^{ik_x x - ik_z z}\hat{y}\frac{k_z}{k} \tag{28}$$

Similarly the fractional magnetic field is given as

$$\nabla \times^{\gamma} \bar{F} = \cos\left(\frac{\gamma\pi}{2}\right)\begin{bmatrix} \nabla^{\gamma}_{yz} & -\partial^{\gamma}_{x} & -\partial^{\gamma}_{x} \\ -\partial^{\gamma}_{y} & \nabla^{\gamma}_{zx} & -\partial^{\gamma}_{y} \\ -\partial^{\gamma}_{z} & -\partial^{\gamma}_{z} & \nabla^{\gamma}_{xy} \end{bmatrix}\begin{bmatrix} -\hat{x}e^{-i\bar{k}\bar{r}} \\ 0 \\ -\hat{z}e^{-i\bar{k}\bar{r}} \end{bmatrix} + \sin\left(\frac{\gamma\pi}{2}\right)\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial^{\gamma}_{x} & \partial^{\gamma}_{y} & \partial^{\gamma}_{z} \\ -e^{-i\bar{k}\bar{r}} & 0 & -e^{-i\bar{k}\bar{r}} \end{vmatrix} \tag{29}$$

$$\nabla \times^{\gamma} \eta \bar{H} = \sin\left(\frac{\gamma\pi}{2}\right)\left\{-\hat{x}\frac{\partial^{\gamma}}{\partial y^{\gamma}} e^{-i\bar{k}\bar{r}} + \hat{y}\left[\frac{\partial^{\gamma}}{\partial z^{\gamma}}(-e^{-i\bar{k}\bar{r}}) - \frac{\partial^{\gamma}}{\partial x^{\gamma}}(-e^{-i\bar{k}\bar{r}})\right] + \hat{z}\frac{\partial^{\gamma}}{\partial y^{\gamma}}(-e^{-i\bar{k}\bar{r}})\right\}$$
$$= \sin\left(\frac{\gamma\pi}{2}\right)(-ik)^{\gamma} e^{-i\bar{k}\bar{r}}\hat{y}\left(-\frac{k_z}{k} + \frac{k_x}{k}\right) \tag{30}$$

Thus the fractional field $\eta\bar{H} = (-ik)^{-\gamma}\nabla^{\gamma}\eta\bar{H}$ is given by

$$\eta\bar{H}_f = \sin\left(\frac{\gamma\pi}{2}\right)e^{-i\bar{k}\bar{r}}\hat{y}\left(-\frac{k_z}{k} - \frac{k_x}{k}\right) = \sin\left(\frac{\gamma\pi}{2}\right)e^{ik_x x - ik_z z}\hat{y}\left(-\frac{k_z}{k} + \frac{k_x}{k}\right) \tag{31}$$

For $\gamma = 1 \; for z < 0$ the electromagnetic field is given as,

$$\bar{E}_f = e^{ik_x x + ik_z z}\hat{y}\frac{k_z}{k}$$
$$\eta\bar{H}_f = e^{ik_x x + ik_z z}\hat{y}\left(-\frac{k_z}{k} - \frac{k_x}{k}\right) \tag{32}$$

For $\gamma = 1 \; for z > 0$ the electromagnetic field is given as,

$$\bar{E}_f = e^{ik_x x - ik_z z}\hat{y}\frac{k_z}{k}$$
$$\eta\bar{H}_f = e^{ik_x x - ik_z z}\hat{y}\left(-\frac{k_z}{k} + \frac{k_x}{k}\right) \tag{33}$$

5. Conclusion

The study of electromagnetic fields in DNG and DPS regions gave a new flame to electromagnetism. In this research we have analysed fractional electromagnetic fields for DNG and DPS medium using SFVCP. The results obtained through this research show that the propagation of an electric field of x direction occurs in y - z plane and the propagation of a magnetic field of y direction occurs in x - z plane in DPS and DNG region. Whereas for a travelling wave with oblique incidence there is no change in the direction. This approach can lead to the construction of new types of microwave structures that can be used in mobile communications, military applications, other communication devices like antenna and lens.

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