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Demonstration for Fermat's last theorem and Beal's conjecture

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Abstract

Fermat's Last Theorem (FLT), 1637, states that if n is an integer greater than 2, then it is impossible to find three positive integer numbers x, y and z in $x^n + y^n = z^n$ where such equality is met being (x, y) coprime. Beal's Conjecture (BC), 1993, states that in equation $A^x + B^y = C^z$, where $(A, B, C, x, y, z) \in \mathbb{Z}^+$ and $(x, y, z) > 2$ are different exponents, then (A, B, C) must have a prime factor, for positive integer solutions, but if are coprime and the exponents $(x, y, z) > 2$ are different, there are no positive integer solutions. The present proof contains two theorems that finally allow us to demonstrate the Beal Conjecture, transforming the equation of Beal conjecture into the form of Fermat's Last Theorem equation. Since there are no solutions in positive integer numbers for Fermat's Last Theorem equation, then the Beal's Conjecture does not have solution in positive integer numbers for unequal exponents or with two equal exponents, but all greater than two, being two of their bases coprime.

Keywords: Pythagorean Theorem, Fermat's Last Theorem, Beal Conjecture

Classification AMS: 11D41

1. Introduction

1.1 Fermat's last theorem

Fermat's last theorem (FLT) or Fermat-Wile's theorem (1995) was one of the theorems where great mathematicians tried to solve it during the last three centuries [1–3].

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Using modern notation, Fermat's last theorem can be stated as follows:

If n is an integer number greater than 2, then it can't be found three integer numbers: x, y and z being $(x, y) > 0$ coprime and $x \neq y$ in the equation:

$$x^n + y^n = z^n$$

Some of the mathematicians who managed to find partial solutions were: Pierre de Fermat (1667) [4], for $n = 4$, other alternative tests for $n = 4$ were made later [5]; although Fermat actually showed that given two integers (x, y), it is impossible for $z^{4k} = x^{4k} + y^{4k}$ for $k \geq 1$, to have integer solutions in z, that is, (z^{2k}, x^{2k}, y^{2k}), do not belong to a primitive Pythagorean triple where $z^{4k} = x^{4k} + y^{4k}$, Leonard Euler (1735) for $n = 3$, [6–8]. Sophie Germaine, [9], Fermat's Last Theorem can be divided into two cases. Case 1 involves all powers p that do not divide any of x, y, or z. Case 2 includes all p that divide at least one of x, y, or z. Germaine proposed the following, commonly called "Sophie Germaine's theorem".

Let p be an odd prime. If there exists an auxiliary prime $P = 2Np + 1$ (N is any positive integer not divisible by 3) such that:

1. If $x^p + y^p + z^p \equiv 0 \pmod p$, then p divides xyz, and
2. p is not a p-th power residue (mod p).

Then the first case of Fermat's Last Theorem holds true for p.

Germain used this result to prove the first case of Fermat's Last Theorem for all odd primes $p < 100$ and Legendre extended it to $p < 197$. Dirichlet and Legendre, (1823–1825), [10–11], went from $n = 3$ to $n = 5$. Lame (1840), [12–14], did so for $n = 7$. Likewise the test has been extended to exponents $n = [6,10,14]$.

Andrew Wiles (1995), [15–18], finally managed to solve it using modern mathematics that did not exist in Fermat's time.

Wiles could prove Fermat's last theorem from the connection, outlined by Frey, and demonstrated by Ken Ribet in 1985 [19–20], that a demonstration of the so-called Taniyama-Shimura conjecture [21–22], would directly lead to a demonstration of Fermat's last theorem. In short, the Taniyama-Shimura conjecture states that every elliptic curve may be uniquely associated with a mathematical object called a modular. If the FLT is false, then there would be an elliptic curve such that cannot be associated with any modular form, and therefore the Taniyama-Shimura conjecture would be false. i.e., Taniyama-Shimura conjecture solution would demonstrate the FLT.

Wiles spent 8 years following the demonstration of Ribet in complete isolation working on the problem and only relying on his wife, which is a way of working unusual in mathematics, where it is common to mathematicians from around the world to share their ideas often. Wiles studied and expanded this approach in his proof and in January 1993 asked his Princeton colleague, Nick Katz, to help this reasoning. For Wiles at this time, his developments and reasoning fit right, but he wanted someone else to check it out. His conclusion at that time was that the techniques used by Wiles seemed to work properly, but had subtle errors that Wiles finally corrected and successfully completed its demonstration in 1995 [23–27].

Because Wiles used more than 100 pages and modern mathematical techniques, is in practice impossible that this demonstration is the same one that hinted at Fermat. (Fermat had a copy of the "Arithmetical of Diophantus' on whose banks scoring reflections that were emerging him. In one of these margins it enunciated the theorem and wrote in Latin: "*Cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet*", whose translation is: "I have a truly marvelous demonstration for this fact, but this margin is too narrow to contain it", [1]. Although Fermat in 1667, proved the case $n = 4$, using the method of infinite descent; it is likely that him had deceived to believe that he had a proof for the general case. It can be even that will have noticed his error further: their marginal notes were for personal use, and therefore Fermat would have not had to backtrack with his comments.

On October 7, 2013 I was invited to Paris by WASET (World Academy of Science Engineering and Technology) to give a lecture on the demonstration of the last theorem of Fermat, using mathematical tools that existed in the 17TH century. This solution also was presented at the invitation of the Department of Physics of the "Universidad del Valle Cali", Colombia on December 4, 2014, and it was published in the scientific magazine of the Military School of Cadets Colombia (2016) [28].

Here I present a different and novel proof considering that z^n is an integer of the form $(y+l)^n$, finding that $z^n = (y+l)^n \neq x^n + y^n$ being all integers, (*x*, *y*) coprime and $x \neq y$, thus proving Fermat's last theorem.

1.2 Beal's Conjecture

Andy Beal, (1993) [29–33] stated that $A^x + B^y = C^z$ (note that *x*, *y* and *z* are unique exponents) where $(A,B,C,x,y,z) \in \mathbb{Z}^+$ and $(x,y,z) > 2$ are different exponents would have no solution using (A,B,C) coprime bases. While working on Fermat's Last Theorem, Andy Beal studied equations with independent exponents. He worked on several algorithms to generate solution sets, but the nature of the algorithms he developed required a common factor in the bases. He suspected that using coprime bases might be impossible and set out to test his hypothesis. With the help of computers and a colleague, Andy Beal tested this for all variable values up to 99. Many solutions were found, and all had a common factor in the bases, but not with coprime bases.

During the period after the discovery by Andy Beal in 1993, he tried to check if there was something similar already, writing to scientific journals and teachers in number theory. One of them Dr. Harold Edwards, from the Department of Mathematics at New York University, who is well-known for being the author of "Fermat's Last Theorem, a genetic Introduction to Algebraic Numerical Theory", [34] sent the paper to Dr. Earl Taft, also a mathematician at Rutgers University who received it and sent it to his colleague Jarell Tunnell, an expert on the Fermat's last theorem. All of them confirmed that there was no known antecedent of this fact being unknown before 1993, when Beal discovered and postulated that there could be a possibly finite number of solutions between integers.[1]

The solution to the Beal conjecture necessarily requires in its bases three integers coprime, all of which must have same integer exponents, and greater than 2, that is to say if there were different exponents in their bases, can't be considered as a counter example to Beal's conjecture, as this example:

$$271^3 + 2^3 3^5 73^3 = 919^3$$
$$3^4 29^3 89^3 + 7^3 11^3 167^3 = 2^7 5^4 353^3$$

Here also is confirmed that Beal's Conjecture (BC) not only met with prime common bases, but also with composite common bases if one of the exponents is less than three and after eliminating the common bases can have solutions in $\mathbb{Z}^+$ and finally, when the common bases from BC are eliminates, that is, if all the exponents are different or two exponents are equal but all exponents are greater than two, shows that there are not solutions in $\mathbb{Z}^+$.

The proof of this conjecture is based on Fermat's Last Theorem demonstrated in this manuscript, exponent algebraic properties and *reduction ad absurdum*.

2. Lemma [1]

Let *x, y* natural numbers, *a*, *b* and *c* be positive real numbers, which satisfy the equations:

$$\begin{cases} x + y = a \\ x^2 + y^2 = b^2 \\ x^n + y^n = c^n \end{cases}$$

Then it is true that $a > b > c$ for $n \in \mathbb{Z}^+$, $n > 2$.

[1] Summarize from: http://bealconjecture.com/

Demonstration:
Let's try initially to prove that $a > b$.
Indeed:

$$a^2 = a \bullet a$$
$$a^2 = (x + y)(x + y)$$
$$a^2 = x^2 + y^2 + 2xy$$
$$a^2 > x^2 + y^2$$
$$a^2 > b^2$$

Consequently, $a > b$.
On the other hand:

$$b^n = b^2 b^{n-2}$$
$$b^n = (x^2 + y^2)\left(\sqrt{x^2 + y^2}\right)^{n-2}$$
$$b^n = x^2\left(\sqrt{x^2 + y^2}\right)^{n-2} + y^2\left(\sqrt{x^2 + y^2}\right)^{n-2}$$
$$b^n > x^2 x^{n-2} + y^2 y^{n-2}$$
$$b^n > x^n + y^n$$
$$b^n > c^n$$

Then, $b > c$, which completes the demonstration that $a > b > c$.

3. Theorems

3.1 Theorem 1: Solution of Fermat's Last Theorem

The equation $z^n = x^n + y^n$ for $n > 2$, has no solutions in positive integers $(\mathbb{Z}^+)$.

Demonstration:
It will be proved that there are no solutions in $\mathbb{Z}^+$ in the equation $z^n = x^n + y^n$ being $(x, y) = 1$ (coprime), so initially it will be assumed that there is an integer solution.

1. In fact, assume they are (x, y, z) coprime $\mathbb{Z}^+$, which satisfy the equation:

   $$z^n = x^n + y^n$$

   For some $n \in \mathbb{Z}^+$, $n > 2$.
   It can't be $x = y$, because $z = \sqrt[n]{2}x \in \mathbb{I}$.
2. Since $(x, y) = 1$, then $U^2 = x^2 + y^2$, can have $\mathbb{Z}^+$ by Pythagorean Theorem. When U is an integer the triplet (U, x, y), it is known as a primitive Pythagorean triplet. There are no two different primitive Pythagorean triples where their triangles are similar since $(x, y) = 1$; Likewise, when U is irrational, there cannot be a minor primitive Pythagorean triplet similar to the triangle (U, x, y), since (x, y)=1.
3. By Lemma [1], we have that $U > z$, from which we can say that there exists $1 < k^2 \in \mathbb{R}^+$ where:

   $$k^2 = \frac{U^2}{Z^2} = \frac{x^2 + y^2}{\left(\sqrt[n]{x^n + y^n}\right)^2}.$$

4. In accordance with the above, it can be stated that k^2 can not be a positive integer. In effect, since $k > 1$, it has the following:

$$ky < kz,$$

Then $k^2 y^2 < U^2$,

Therefore $k^2 y^2 < y^2 + y^2$,

So $(k^2 - 2)y^2 < 0$.

It follows that $1 < k^2 = \frac{U^2}{Z^2} < 2$. So things, $1 < k^2 < 2$ is rational ($\mathbb{Q}$) or irrational ($\mathbb{I}$) number.

5. On the other hand, we have:

$$\begin{aligned} z^2 &= \frac{U^2}{k^2} \\ &= \frac{x^2 + y^2}{k^2} \\ &= {x_1}^2 + {y_1}^2 \end{aligned}$$

Where:

$${x_1}^2 = \frac{x^2}{k^2}$$

$${y_1}^2 = \frac{y^2}{k^2}$$

By Pythagoras (z, x_1, y_1) form a right triangle.
In this way, the right triangles OAB and OCD of Figure 1 are constructed, which would be similar triangles since:

$$\frac{U}{z} = \frac{x}{x_1} = \frac{y}{y_1} = k$$

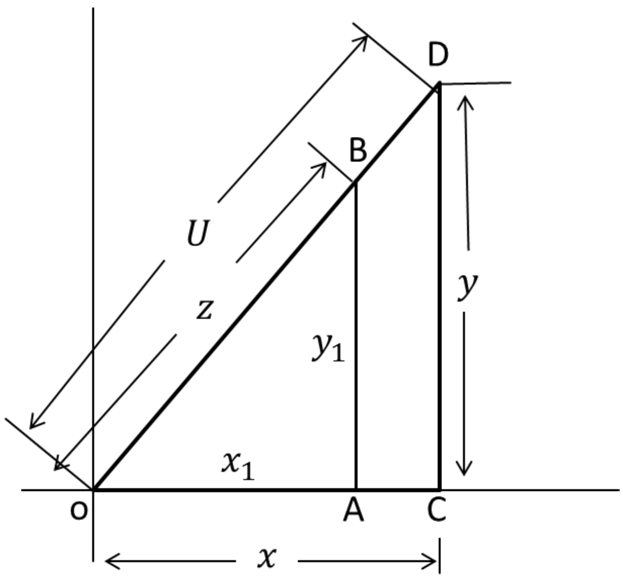

Figure 1: Similar triangles OAB and OCD

6. We already know that k^2 is rational or irrational number.
   Let $k^2 \in \mathbb{Q}$ and given that x ≠ y, in the triangle OCD it has that $U^2 = x^2 + y^2$, and in the triangle OAB:

$$z^2 = {x_1}^2 + {y_1}^2 = \frac{x^2 + y^2}{k^2} \tag{1}$$

   The fact that $(x, y) = 1$, then $(x^2{}_1, y^2{}_1) \notin \mathbb{Z}^+$, and $z^2 = {x_1}^2 + {y_1}^2$ has not solution in $\mathbb{Z}^+$ and also $z^n = z^2 z^{n-2} = x^n + y^n$ for $n > 2$ has not solution in $\mathbb{Z}^+$.

Additionally the solution of $Z^2 \in \mathbb{Z}^+$ with $(x^2{}_1, y^2{}_1) \in \mathbb{Q}$ it originates an absurdity, since there would be the same value of $Z^2 = W^2$ with the value $v \neq x$,being $Z^n = x^n + y^n$ and $W^n = v^n + y^n$, it turns out that $Z^n = W^n$ which is absurd, since $v \neq x$:

As $Z > y$ and $k^2 \in \mathbb{Q}$, there must be a $1 < k^2 = \dfrac{U^2}{\left(\sqrt[n]{x^n + y^n}\right)^2} = \dfrac{U^2}{(y+q)^2} < 2$, with $q \geq 1$, so that $Z \in \mathbb{Z}^+$:

$$Z^2 = \frac{U^2}{k^2} = \frac{x^2 + y^2}{\dfrac{x^2 + y^2}{\left(\sqrt[n]{x^n + y^n}\right)^2}} = \frac{x^2 + y^2}{\dfrac{x^2 + y^2}{(y+q)^2}} = (y+q)^2 \left(\frac{x^2 + y^2}{x^2 + y^2}\right) = (y+q)^2$$

$$Z^n = (y+q)^n = x^n + y^n$$

Since $\dfrac{x^2 + y^2}{x^2 + y^2} = 1$ with a value $v \neq x$ and $(v, y) = 1$ it has $\dfrac{v^2 + y^2}{v^2 + y^2} = 1$. with $W^n = v^n + y^n$ and $1 < K^2 = \dfrac{v^2 + y^2}{(y+q)^2} = \dfrac{v^2 + y^2}{\left(\sqrt[n]{v^n + y^n}\right)^2}$<2, we also have that $W^2 = \dfrac{v^2 + y^2}{K^2} = (y+q)^2 \dfrac{v^2 + y^2}{v^2 + y^2} = Z^2$, which is absurd in $W^n = v^n + y^n = Z^n = x^n + y^n$, since $v \neq x$, then $\left(k^2 = \dfrac{U^2}{Z^2} = \dfrac{x^2 + y^2}{\left(\sqrt[n]{x^n + y^n}\right)^2},\ K^2 = \dfrac{v^2 + y^2}{W^2} = \dfrac{v^2 + y^2}{\left(\sqrt[n]{v^n + y^n}\right)^2}\right) \notin \mathbb{Q}$.

As $k^2 \notin \mathbb{Q}$ then $k^2 \in \mathbb{I}$ and $\boldsymbol{Z}^2 = \dfrac{\boldsymbol{U}^2}{\boldsymbol{k}^2} \in \mathbb{I}$ (the integer over an irrational number is irrational), then $Z^n = x^n + y^n$ has no solution in $\mathbb{Z}^+$, and what is assumed in point 1. that they are (x, y, z) coprime positive integers, which satisfy the equation $z^n = x^n + y^n$ is false.

In this way Fermat's last theorem is proved and maybe we have followed the same path of Fermat when he said: "*Cuius rei demonstrationem mirabilem sane detexi. Hanc marginis exiguitas non caperet*" and the fact that Fermat used the infinite descent in it demonstration for $z^4 = x^4 + y^4$ has not integer solution. Also it can be concluded that in any right triangle, (U, x, y) with (x, y) coprime, any integer $z < U$ in its hypotenuse must be $z^n \neq x^n + y^n$ for $n > 2$.

Corollary 1

In equation $z^n = x^n + y^n$ for $n > 2$ if x, y or z they are irrational in the form $z = a^{(f/d)}$ or $x = b^{(h/g)}$ or $y = c^{(j/i)}$, with $(a, b, c) \in \mathbb{Z}^+$, and $[(d, f), (g, h), (i, j)] \in \mathbb{Z}^+$, where $(d, f) = 1$, $(h, g) = 1$ and $(j, i) = 1$, where $(f > d,\ h > g,\ j > i)$ and $(d, g, i) > 2$ then the equation $z^n = x^n + y^n$ for $n > 2$ has no solution in the integer numbers, since $z < U = \sqrt{x^2 + y^2}$ and $1 < k^2 < 2$, would be a similar case to what has been demonstrated in Theorem 1, and if $z \in \mathbb{I}$, by definition $z \notin \mathbb{Z}^+$.

3.2 Theorem 2: Solution of Beal's conjecture

Theorem

In Equation $A^x + B^y = C^z$, where $(A,\ B,\ C,\ x,\ y,\ z) \in \mathbb{Z}^+$ and $(x \neq y \neq z) > 2$ then $(A,\ B,\ C)$ must have a prime factor p^n where p is a prime and $n \geq 1$, to have integer solutions with the exponents (x,y,z) greater than 2.

In other words: $A^x = p^n a^r$, $B^y = p^n b^s$ and $C^z = p^n c^t$

If p^n is eliminated and $a^r + b^s = c^t$ with (a,b) coprime and the exponents (r,s,t) are different and greater than 2, there are no solutions in $\mathbb{Z}^+$.

Demonstration through Theorem 2 (FLT) and reduction ad absurdum

1. In Equation $A^x + B^y = C^z$, where $(A,\ B,\ C,\ x,\ y,\ z) \in \mathbb{Z}^+$ and $(x \neq y \neq z) > 2$ then $(A,\ B,\ C)$ must have a prime factor p^n where p is a prime and $n \geq 1$, to have integer solutions with the exponents (x,y,z) greater than 2.

   In other words: $A^x = p^n a^r$, $B^y = p^n b^s$ and $C^z = p^n c^t$

   If p^n is eliminated and $a^r + b^s = c^t$ with $(a,b) = 1$ and the exponents (r,s,t) are different and greater than 2, there are no solutions in $\mathbb{Z}^+$.
2. Let proof with a numerical example when $a^r + b^s = c^t$ with $(a,\ b) = 1$ and the exponents (r,s,t) are different and one of the exponents is less than 2, there are solutions in $\mathbb{Z}^+$.

   The equation $A^x + B^y = C^z$, where $A^x = 3^6$; $B^x = 18^3$; $C^z = 3^8$ has solution in $\mathbb{Z}^+$, but after eliminating the prime factor $p^n = 3^6$, the equation remains $a^r + b^s = c^t$, whit $(a,b) = 1$, have one of the exponent is less than 2 and has solution in $\mathbb{Z}^+$. See Table 1.

   Beal's Conjecture also complies when the common factor is a composite number, if after the elimination of the composite factor one of the exponents is less than three can have solution in $\mathbb{Z}^+$. For example, the Equation $a^2 + b^4 = c^3$ has integer solutions and can be converted to Beal's Conjecture, with generating numbers (a,b,c) [32] of the form[2]:

$$a = (3m^4 + 4n^4)(9m^8 - 408m^4n^4 + 16n^8)$$
$$b = 6mn(3m^4 - 4n^4)$$
$$c = 9m^8 + 168m^4n^4 + 16n^8$$

   This form has infinite solutions in $\mathbb{Z}^+$, (see Table 2).

   To clarify, Figure 2 shows how it is possible to go from Beal's Conjecture $A^x + B^y = C^z$ to $a^r + b^s = c^t$ equation or from $a^r + b^s = c^t$ to $A^x + B^y = C^z$ with solutions in $\mathbb{Z}^+$ if one of the exponents (r,s,t) is less than 3.
3. As we already know that there exist integer solutions in $A^x + B^y = C^z$ with common factors and after eliminating them the resulting equation $a^r + b^s = c^t$ with $(a,b,c) \in \mathbb{Z}^+$ and (a,b) coprime, has at least one of the exponents (r,s,t) less than 3, we must continue the demonstration assuming: integer solutions in the equation $a^r + b^s = c^t$ with, (a,b) coprime and$(r \neq s \neq t) > 2$.

Table 1: Example of a solution of the equation $A^x + B^y = C^z$ with $(r \neq s \neq t) > 2$, where after eliminating the prime factor p^n, the equation $a^r + b^s = c^t$ has a solution in the integers but one of the exponents r,s or t is less than 3.

A^x	B^y	C^y	Prime Factor	a^r	b^s	c^t
3^6	18^3	3^8	$p^n = 3^6$	1	2^3	3^2

[2] Rafael Parra Machío. ECUACIONES DIOFÁNTICAS, pp 22. Web: http://hojamat.es/parra/diofánticas.pdf

Table 2: Some examples with $(m=1, n=1)$ and $(m=3, n=2)$, where a^{12k} for $k \geq 1$ can be a common factor of A^x, B^y and C^z with $A^x = a^{2+12k}, B^y = (a^{3k}b)^4, C^z = (a^{4k}c)^3$ and then $(A,B,C,x,y,z) \in \mathbb{Z}^+$.

A^{14}	B^4	C^3	Composite numbers	a^2	b^4	c^3
2681^{14}	$(2681^3 \cdot 6)^4$	$(2681^4 \cdot 193)^3$	2681^{12}	2681^2	6^4	193^3
142946261^{14}	$(142946261^3 \cdot 6444)^4$	$(142946261^4 \cdot 280873)^3$	142946261^{12}	142946261^2	6444^4	280873^3

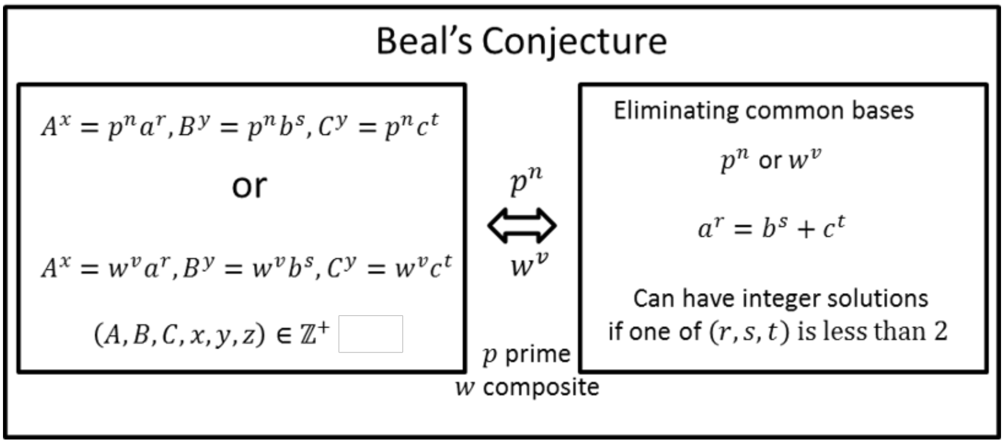

Figure 2: How is it possible to go from BC to the $a^r + b^s = c^t$ equation with solutions in $\mathbb{Z}^+ - \{0\}$ or from $a^r + b^s = c^t$ to BC with solutions in $\mathbb{Z}^+$ if one of the exponents (r,s,t) is less than 3.

4. For this part of the demonstration the method of reduction ad absurdum will be applied. In fact assume they are $(a,b,c) \in \mathbb{Z}^+$ with (a,b) coprime, which satisfy the equation:

$$a^r + b^s = c^t$$

For some $(r \neq s \neq t) > 2$, $\Rightarrow (r,s,t) \in \mathbb{Z}^+$.

5. Assuming that r is the smaller exponent and greater than 2 then:

$$(a^r + b^s = c^t) \Leftrightarrow (a^r + (b^{s/r})^r = (c^{t/r})^r) \Rightarrow \text{(exponent property)}$$

There are two cases to be analyzed for $b^{s/r}$ and $c^{t/r}$

Case 1: $r \mid s$ and $r \mid t$ or $(b = b_1^{kr} \wedge r \nmid s)$ or $(c = c_1^{kr} \wedge r \nmid t)$, $(b_1, c_1, k) \in \mathbb{Z}^+$ then:

$$b^{s/r} = u, \quad c^{t/r} = v \rightarrow (u,v) \in \mathbb{Z}^+$$

and

$$(a^r + b^s = c^t) \Leftrightarrow (a^r + u^r = v^r) \Rightarrow r > 2$$

Which leads to an absurd, since according to Theorem 2, this equation has no solution in the integer numbers.

Case 2: $r \nmid s$ or $r \nmid t$ and $(b \neq b_1^{kr} \vee c \neq c_1^{kr}, (b_1, c_1, k) \in \mathbb{Z}^+)$; therefore $b^{s/r}$ or $c^{t/r}$ or both must not be rational numbers.

Proof:
Assuming $u = b^{s/r} = \dfrac{d}{e}$ or $v = c^{t/r} = \dfrac{f}{g}$ or both are fractional numbers $(d,e,f,g) \in \mathbb{Z}^+ \Rightarrow (d,e) = 1$ and $(f,g) = 1$:

$\left[u^r = \left(\dfrac{d}{e}\right)^r\right] = b^s$ or $\left[v^r = \left(\dfrac{f}{g}\right)^r\right] = c^t$, then u or v or both can't be fractional numbers. It is impossible that an integer be a fractional number; therefore: $u = b^{s/r}$ or $v = c^{t/r}$ or both must be irrational numbers.

For this case: If any $u = b^{s/r}$ or $v = c^{t/r}$, or both bases are irrational numbers, then the equation:
$[(a^r + b^s = c^t)] \Leftrightarrow [a^r + (b^{s/r})^r = (c^{t/r})^r] \Leftrightarrow [a^r + u^r = v^r]$ is not resolving in $\mathbb{Z}^+$. (Corollary 1 of Theorem 1).

The demonstration scheme is the same for any smaller exponents and it is not necessary to expand the demonstration for each of the others exponents (s,t) if one of them is the smallest.

Corollary 1: The equation $a^r + b^s = c^t$, has no solution in $\mathbb{Z}^+$, if all the exponents $(r, s, t) > 2$ and two of them are equal.

It is obvious that if there is no solution with exponents $(r \neq s \neq t) > 2$, neither is there if two of its exponents are equal, vastly follow the same procedure from point 4 of Theorem 3, i.e. if the exponents $r = s$ or $r = t$ or $s = t$, there are no solutions in integers.

In conclusion it is ad absurdum to consider in point 4. that the equation $a^r + b^s = c^t$, $(a,b) = 1$ and $(r \neq s \neq t) > 2$ or two of the exponents are equal, can have solution in integer numbers, being demonstrated in all its forms the Beal's Conjecture.

4. Conclusion

A simple demo of FLT using 17TH-century mathematical tools such as stated by Fermat in the margin of the "Arithmetical of Diophantus' writing his notes can exist. Taking into account that Fermat was who introduced the principle of infinite descent, which was used on his show for $n = 4k$ in the FLT, it wouldn't be strange that Fermat did think that he had a general solution of his last theorem or used a similar procedure to the described here.

The theorem doesn't have a major application, but to be considered the most difficult problem in the world, for 360 years, the search for its solution, allowed the advancement of mathematical science during the last four centuries and where great mathematicians such as Euler (1707–1783), Lagrange (1736–1813), Germaine (1776–1831), Gauss (1777–1855), Cauchy (1789–1857), Lamé (1795–1870), Dirichlet (1805–1859), Liouville (1809–1882), Kumer (1810–1893), Vaudiver (1882–1973), Taniyama (1927–1958), Shimura (1930-), Wiles (1953-) and many other mathematicians who contributed to the advancement of the sciences and number theory in search of their show.

Using the demonstration of Fermat's Last Theorem as a base, mathematical transformations with exponent properties and *reduction ad absurdum*, it was possible to confirm Beal's Conjecture, i.e. when Equation $A^x + B^y = C^z$, where $(A,B,C,x,y,z) \in \mathbb{Z}^+$ and $(x,y,z) > 2$ has common bases, (prime or composite numbers) there are integer solutions if after eliminating the common bases, one of the exponents is less than three. If the resulting equation have all the exponents (r,s,t) greater than two (all different, or two exponents equal), with (a,b) coprime, confirms that the equation:
$a^r + b^s = c^t$ and $(r,s,t) > 2$ has not solutions in $\mathbb{Z}^+$, solving Beal's conjecture.

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Conflict of Interest

The authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speaker's membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent arrangements), or non (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in this manuscript. Potential conflicts of interest related to individual authors' commitments. Potential conflicts of interest related to commitments of editors, journal staff, or reviewers.

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