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Weighted pretopological approach for decision accuracy in information system

M. Kamel EL-Sayed

Department of Basic Science, Higher Institute of Engineering and Technology, Kafrelsheikh, Egypt

Abstract

Computing decision accuracy is an important step in making and choosing decision in information system. Most works in this direction does not use the concepts of topology. This work is to use pretopological structures generated from weighted similarity classes to find accuracy of decision sets. Example is given to indicate the approach and comparison between weighted accuracy and other types of accuracy.

Keywords and Phrases: Pretopology, Information system, Decision accuracy

1. Introduction and Preliminaries

In this era which data overlap the issues of uncertainty increase, this affects decision-making in a particular field, Pawlak developed what is called the information system which used to analyze them the topics of induction rough set [1, 2, 3] that depend on equivalence relations in which the equivalence classes are represented to find the decision set that are described by the upper approximations and its lower. The most important concepts of this theory [4, 5], although many generalizations of this theory appeared were used, but there are some cases that do not lead to an important distinction in decision-making. In the first part of this research, we use pretopological [6–8] concepts to calculate approximations for the decision sets and its accuracy applies in the reduction of attributes. We know the symmetry matrix for the elements of the information system [9, 10] that are effective pre-closure using the weighted similarity classes, we know the effects of pre-closure and pre-interior with weights, we get from them the accuracy of the decision for the pretopological resulting from all the attributes as a basis, we compare it with the result using the subsets of attributes [4, 11–14], the second part we

Email address: dr.mkamel123@gmail.com (M. Kamel EL-Sayed)

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present the concepts of pretopology and examples of it, the third part explains the method that was used, and the fourth part presents the illustrative example.

2. Preliminaries

Let $P(E)$ contains all subsets of a non-empty set of E.

Definition 1: [15] *A map $d(\cdot)$ from $P(E)$ is known as a Pseudoclosure satisfy*

1. $d(\varphi) = \varphi$,
2. $\forall D \in P(E)$, $D \subset d(D)$.
   A pair (E,d) is defined as a pretopological space where E has been granted with a pseudoclosure $d(\cdot)$ [15].
   The subset $d(D)$ is named as pseudoclosure of D, $d(\cdot)$ can be applied as set D [9].
3. $\forall D \in P(E)$, $\forall Y \in P(E)$.
   In [15], named as D-pretopological space.

Definition 2: [15] (E,d) *is pretopological space satisfied:*

1. $\forall D \in P(E)$, $\forall Y \in P(E)$, $D \cap Y \Rightarrow d(D) \subset d(Y)$.

3. Data Mining

Huge amounts of data are collected daily [12]. Analyzing of it is important. We are living in the data age [16]. Classification of it is an important part. Scientists must specified data based on various attributes to reduce their complexity.

3.1 The Type of Information

Data have been collected, from measurements of complex information.

- **Medical data:** Large amounts of information are constantly being collected about individuals and groups.
- **Surveillance video and pictures:** there is a trend today to store tapes and this is overcome the lost of contents.

4. Data Selection in Data Mining

Data selection means data type and source that . It precedes the actual data collection. This definition explains the difference between data selection and selective data

The process of identifying relevant information is called attribute selection [15] and remove as much duplicate information as possible [17] from the set of features native to a given dataset.

The steps of information system analysis:

1. Construction of stationary matrix

$$m_{ij} = [\mu_{ij}]_{i,j=1}^{n}, \mu(ij) = \sum_{1}^{n} \delta_{ij},$$

$$\delta_{ig} = \begin{cases} 1 & v(i) = v(j), \\ 0 & v(i) \neq v(j). \end{cases}$$

2. Construction neighborhood
   Choosing a value λ for classes limits arrounding expert opinion

$$[p_i] = \{p_j : \mu(i,j) > \lambda\}.$$

3. Weights of every objects

$$w(p_i) = \sum \delta(p_i){:}p_i \in [p_i]{:}\ \ j = (1,\cdots,n).$$

4. Computing pre-closure $(P \cdot cl(A))$, pre-interior $(P \cdot \mathrm{int}(A))$

$$p \cdot cl(A) = \cup\{[p_i]{:}p_i \in A\},$$
$$p \cdot \mathrm{int}(A) = (p \cdot clA^c)^c.$$

5. Accuracy of A

$$\mathrm{acc}(A) = \frac{|\, p{\cdot}\mathrm{int}(A)\,|}{|\, p{\cdot}cl(A)\,|}.$$

These concepts are defined without using weights.

$$[p_i]_w = \{p_\mu^{(i)}{:}\mu \succ \lambda\},$$
$$p{\cdot}cl(A) = \cup\{p_\mu^{(i)}\ \forall\ \mu \succ \lambda\},$$

The following are the corresponding definition with weight.

Example 4.1: *The weather problem* [12] *is a small data set frequently. It's completely fictional, presumably about the right conditions to play some unspecified game. Generally, cases in data set are described by four attributes: (Temperature), (Windy), (Humidity) and (Outlook). Play (Yes (Y) or Not (N)) are the outcomes. In Table 1, Outlook (sunny (S) overcast (O) rainy (R)); Temperature (hot (H) – mild (M) – cool (C)); Humidity (high (G) normal (L)); and Windy (weak (W) strong (T)), of which* 13 *objects are present in the following example.*

Form a relation

$$m_{ij} = \mu(\lambda_i, \lambda_j) = \frac{\text{numberofsimilarities}}{\text{numberofattributes}}.$$

Table 1:

	Out look	Temperature	Humidity	Windy	Play
λ_1	*S*	*H*	*L*	*W*	*N*
λ_2	*S*	*H*	*G*	*T*	*N*
λ_3	*O*	*H*	*G*	*W*	*Y*
λ_4	R	M	G	*W*	*Y*
λ_5	R	C	L	*W*	*Y*
λ_6	*R*	*C*	*L*	*T*	*N*
λ_7	*O*	*C*	*L*	*T*	*Y*
λ_8	*S*	*M*	*G*	*W*	*N*
λ_9	*S*	*C*	*L*	*W*	*Y*
λ_{10}	*R*	*M*	*L*	*W*	*Y*
λ_{11}	*R*	*M*	*L*	*T*	*Y*
λ_{12}	*O*	*M*	*G*	*T*	*Y*
λ_{13}	*O*	*H*	*L*	*W*	*Y*

Table 2:

	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	λ_9	λ_{10}	λ_{11}	λ_{12}	λ_{13}
λ_1	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{3}{4}$
λ_2	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{3}{4}$
λ_3	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{3}{4}$
λ_4	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$
λ_5	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	0	$\frac{1}{2}$
λ_6	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{3}{4}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
λ_7	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{3}{4}$	1	0	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
λ_8	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{4}$	0	0	1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$
λ_9	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{4}$	0	$\frac{1}{2}$
λ_{10}	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$
λ_{11}	$\frac{1}{4}$	$\frac{1}{4}$	0	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{3}{4}$	1	$\frac{1}{2}$	$\frac{1}{4}$
λ_{12}	0	0	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{1}{2}$	1	$\frac{1}{4}$
λ_{13}	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	1

We choose the classes for each element, and it is written with the degree of similarity
Let $\lambda \geq \frac{1}{2}$. We find the classes for each element grater than or equal $\frac{1}{2}$

$$[\lambda_1] = \{\lambda_1^{(1)}, \lambda_{\frac{1}{2}}^{(2)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{3}{4}}^{(13)}\},$$

$$[\lambda_2] = \{\lambda_{\frac{1}{2}}^{(1)}, \lambda_1^{(2)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{3}{4}}^{(13)}\},$$

$$[\lambda_3] = \{\lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(2)}, \lambda_1^{(3)}, \lambda_{\frac{1}{2}}^{(4)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{3}{4}}^{(13)}\},$$

$$[\lambda_4] = \{\lambda_{\frac{1}{2}}^{(3)}, \lambda_1^{(4)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{3}{4}}^{(8)}, \lambda_{\frac{3}{4}}^{(10)}, \lambda_{\frac{1}{2}}^{(11)}, \lambda_{\frac{1}{2}}^{(12)}\},$$

$$[\lambda_5] = \{\lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(4)}, \lambda_1^{(5)}, \lambda_{\frac{3}{4}}^{(6)}, \lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{3}{4}}^{(10)}, \lambda_{\frac{1}{2}}^{(11)}, \lambda_{\frac{1}{2}}^{(13)}\},$$

$$[\lambda_6] = \{\lambda_{\frac{3}{4}}^{(5)}, \lambda_1^{(6)}, \lambda_{\frac{3}{4}}^{(7)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{3}{4}}^{(11)}\},$$

$$[\lambda_7] = \{\lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{3}{4}}^{(6)}, \lambda_1^{(7)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(11)}, \lambda_{\frac{1}{2}}^{12)}, \lambda_{\frac{1}{2}}^{(13)}\},$$

$$
\begin{aligned}
[\lambda_8] &= \{\lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(2)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{1}^{(8)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(12)}\},\\
[\lambda_9] &= \{\lambda_{\frac{3}{4}}^{(1)}, \lambda_{\frac{3}{4}}^{(2)}, \lambda_{\frac{3}{4}}^{(5)}, \lambda_{\frac{1}{2}}^{(6)}, \lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(13)}\},\\
[\lambda_{10}] &= \{\lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(2)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{\frac{3}{4}}^{(5)}, \lambda_{\frac{1}{2}}^{(6)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{1}^{(10)}, \lambda_{\frac{3}{4}}^{(11)}, \lambda_{\frac{1}{2}}^{(13)}\},\\
[\lambda_{11}] &= \{\lambda_{\frac{1}{2}}^{(4)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{3}{4}}^{(6)}, \lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{3}{4}}^{(10)}, \lambda_{1}^{(11)}, \lambda_{\frac{1}{2}}^{(12)}\},\\
[\lambda_{12}] &= \{\lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(4)}, \lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(11)}, \lambda_{\frac{1}{2}}^{(12)}\},\\
[\lambda_{13}] &= \{\lambda_{\frac{3}{4}}^{(1)}, \lambda_{\frac{3}{4}}^{(2)}, \lambda_{\frac{3}{4}}^{(3)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{1}^{(13)}\}.
\end{aligned}
$$

These classes we consider to be effective preclosure each symmetry element upper approximation. These weights for each element are attributed to the entire information upper weight

$$
\begin{aligned}
\lambda(1) &= \{\lambda_{1}^{(1)}, \lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{3}{4}}^{(1)}, \lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{3}{4}}^{(1)}\} = \lambda_{\frac{5}{8}}^{(1)},\\
\lambda(2) &= \{\lambda_{\frac{1}{2}}^{(2)}, \lambda_{1}^{(2)}, \lambda_{\frac{1}{2}}^{(2)}, \lambda_{\frac{1}{2}}^{(2)}, \lambda_{\frac{3}{4}}^{(2)}, \lambda_{\frac{1}{2}}^{(2)}, \lambda_{\frac{3}{4}}^{(2)}\} = \lambda_{\frac{18}{25}}^{(2)},\\
\lambda(3) &= \{\lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{1}^{(3)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{3}{4}}^{(3)}\} = \lambda_{\frac{15}{24}}^{(3)},\\
\lambda(4) &= \{\lambda_{\frac{1}{2}}^{(4)}, \lambda_{1}^{(4)}, \lambda_{\frac{1}{2}}^{(4)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{\frac{1}{2}}^{(4)}, \lambda_{\frac{1}{2}}^{(4)}\} = \lambda_{\frac{1}{2}}^{(4)},\\
\lambda(5) &= \{\lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{1}^{(5)}, \lambda_{\frac{3}{4}}^{(5)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{3}{4}}^{(5)}, \lambda_{\frac{3}{4}}^{(5)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{1}{2}}^{(5)}\} = \lambda_{\frac{23}{36}}^{(5)},\\
\lambda(6) &= \{\lambda_{\frac{3}{4}}^{(6)}, \lambda_{1}^{(6)}, \lambda_{\frac{3}{4}}^{(6)}, \lambda_{\frac{1}{2}}^{(6)}, \lambda_{\frac{1}{2}}^{(6)}, \lambda_{\frac{3}{4}}^{(6)}\} = \lambda_{\frac{17}{24}}^{(6)},\\
\lambda(7) &= \{\lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{3}{4}}^{(7)}, \lambda_{1}^{(7)}, \lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{1}{2}}^{(7)}, \lambda_{\frac{1}{2}}^{7}, \lambda_{\frac{1}{2}}^{(7)}\} = \lambda_{\frac{17}{28}}^{(7)},\\
\lambda(8) &= \{\lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{3}{4}}^{(8)}, \lambda_{1}^{(8)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(8)}\} = \lambda_{\frac{19}{32}}^{(8)},\\
\lambda(9) &= \{\lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}\} = \lambda_{\frac{36}{21}}^{(9)},\\
\lambda(10) &= \{\lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{3}{4}}^{(10)}, \lambda_{\frac{3}{4}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{1}^{(10)}, \lambda_{\frac{3}{4}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}\} = \lambda_{\frac{25}{40}}^{(10)},\\
\lambda(11) &= \{\lambda_{\frac{1}{2}}^{(11)}, \lambda_{\frac{1}{2}}^{(11)}, \lambda_{\frac{3}{4}}^{(11)}, \lambda_{\frac{1}{2}}^{(11)}, \lambda_{\frac{3}{4}}^{(11)}, \lambda_{1}^{(11)}, \lambda_{\frac{1}{2}}^{(11)}\} = \lambda_{\frac{18}{28}}^{(11)},\\
\lambda(12) &= \{\lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{1}{2}}^{(12)}\} = \lambda_{\frac{7}{12}}^{(12)},\\
\lambda(13) &= \{\lambda_{\frac{3}{4}}^{(13)}, \lambda_{\frac{3}{4}}^{(13)}, \lambda_{\frac{3}{4}}^{(13)}, \lambda_{\frac{1}{2}}^{(13)}, \lambda_{\frac{1}{2}}^{(13)}, \lambda_{\frac{1}{2}}^{(13)}, \lambda_{\frac{1}{2}}^{(13)}, \lambda_{1}^{(13)}\} = \lambda_{\frac{21}{32}}^{(13)}.\\
\bar{A} &= \{\lambda_{\frac{5}{8}}^{(1)}, \lambda_{\frac{18}{28}}^{(2)}, \lambda_{\frac{15}{24}}^{(3)}, \lambda_{\frac{1}{2}}^{(4)}, \lambda_{\frac{23}{36}}^{(5)}, \lambda_{\frac{17}{24}}^{(6)}, \lambda_{\frac{17}{28}}^{(7)}, \lambda_{\frac{19}{32}}^{(8)}, \lambda_{\frac{21}{36}}^{(9)}, \lambda_{\frac{25}{40}}^{(10)}, \lambda_{\frac{18}{28}}^{(11)}, \lambda_{\frac{7}{12}}^{(12)}, \lambda_{\frac{21}{32}}^{(13)}\}.
\end{aligned}
$$

These classes we consider to be effective preclouser each symmetry element lower approximation. These weights for each element are attributed to the entire information lower weight and take the complements of it

$$
\begin{aligned}
A &= \{\lambda_3, \lambda_4, \lambda_5, \lambda_7, \lambda_9, \lambda_{11}, \lambda_{12}, \lambda_{13}\},\\
A^c &= \{\lambda_1, \lambda_2, \lambda_6, \lambda_8\},
\end{aligned}
$$

$$\bar{A}^c$$

$$\lambda(1) = \{\lambda_{1}^{(1)}, \lambda_{\frac{1}{2}}^{(1)}, \lambda_{\frac{1}{2}}^{(1)}\} = \lambda_{\frac{2}{3}}^{(1)},$$

$$\lambda(2) = \{\lambda_{\frac{1}{2}}^{(2)}, \lambda_{1}^{(2)}, \lambda_{\frac{1}{2}}^{(2)}\} = \lambda_{\frac{2}{3}}^{(2)},$$

$$\lambda(3) = \{\lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{2}}^{(3)}\} = \lambda_{\frac{1}{2}}^{(3)},$$

$$\lambda(4) = \{\lambda_{\frac{1}{4}}^{(4)}\} = \lambda_{\frac{3}{4}}^{(4)},$$

$$\lambda(5) = \{\lambda_{\frac{1}{2}}^{(5)}, \lambda_{\frac{3}{4}}^{(5)}\} = \lambda_{\frac{5}{8}}^{(5)},$$

$$\lambda(6) = \{\lambda_{1}^{(6)}\} = \lambda_{1}^{(6)},$$

$$\lambda(8) = \{\lambda_{\frac{1}{2}}^{(8)}, \lambda_{\frac{1}{2}}^{(8)}, \lambda_{1}^{(8)}\} = \lambda_{\frac{2}{3}}^{(8)},$$

$$\lambda(9) = \{\lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{3}{4}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}, \lambda_{\frac{1}{2}}^{(9)}\} = \lambda_{\frac{5}{8}}^{(9)},$$

$$\lambda(10) = \{\lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{1}{2}}^{(10)}\} = \lambda_{\frac{1}{2}}^{(10)},$$

$$\lambda(11) = \{\lambda_{\frac{3}{4}}^{(11)}\} = \lambda_{\frac{3}{4}}^{(11)},$$

$$\lambda(12) = \{\lambda_{\frac{1}{2}}^{(12)}\} = \lambda_{\frac{1}{2}}^{(12)},$$

$$\lambda(13) = \{\lambda_{\frac{3}{4}}^{(13)}, \lambda_{\frac{3}{4}}^{(13)}\} = \lambda_{\frac{3}{4}}^{(13)},$$

$$\bar{A} = \{\lambda_{\frac{2}{3}}^{(1)}, \lambda_{\frac{2}{3}}^{(2)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{1}^{(6)}, \lambda_{\frac{3}{4}}^{(7)}, \lambda_{\frac{2}{3}}^{(8)}, \lambda_{\frac{5}{8}}^{(9)}, \lambda_{\frac{1}{2}}^{(10)}, \lambda_{\frac{3}{4}}^{(11)}, \lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{3}{4}}^{(13)}\}.$$

Lower weight

$$\underline{A} = \{\lambda_{\frac{1}{3}}^{(1)}, \lambda_{\frac{1}{3}}^{(2)}, \lambda_{\frac{1}{2}}^{(3)}, \lambda_{\frac{1}{4}}^{(4)}, \lambda_{\frac{1}{2}}^{(5)}, \lambda_{0}^{(6)}, \lambda_{\frac{1}{4}}^{(7)}, \lambda_{\frac{1}{3}}^{(8)}, \lambda_{\frac{3}{8}}^{(9)}, \lambda_{\frac{2}{4}}^{(10)}, \lambda_{\frac{1}{4}}^{(11)}, \lambda_{\frac{1}{2}}^{(12)}, \lambda_{\frac{1}{4}}^{(13)}\}.$$

Similarly, when making a reduction for each attribute, we find the following data

1. **Outlook**
   Upper weight

$$\bar{A} = \{\lambda_{\frac{11}{15}}^{(1)}, \lambda_{\frac{4}{6}}^{(2)}, \lambda_{\frac{7}{9}}^{(3)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{\frac{8}{12}}^{(5)}, \lambda_{\frac{3}{4}}^{(6)}, \lambda_{\frac{3}{4}}^{(7)}, \lambda_{\frac{3}{4}}^{(8)}, \lambda_{\frac{11}{15}}^{(9)}, \lambda_{\frac{13}{18}}^{(10)}, \lambda_{\frac{3}{4}}^{(11)}, \lambda_{\frac{7}{9}}^{(12)}, \lambda_{\frac{11}{15}}^{(13)}\}.$$

   Lower weight

$$\underline{A} = \{\lambda_{0}^{(1)}, \lambda_{0}^{(2)}, \lambda_{\frac{1}{3}}^{(3)}, \lambda_{0}^{(4)}, \lambda_{\frac{2}{6}}^{(5)}, \lambda_{0}^{(6)}, \lambda_{0}^{(7)}, \lambda_{0}^{(8)}, \lambda_{\frac{1}{3}}^{(9)}, \lambda_{\frac{2}{6}}^{(10)}, \lambda_{\frac{1}{3}}^{(11)}, \lambda_{\frac{2}{6}}^{(12)}, \}.$$

2. **Temperature**
   Upper weight

$$\bar{A} = \{\lambda_{\frac{2}{3}}^{(1)}, \lambda_{\frac{2}{3}}^{(2)}, \lambda_{\frac{3}{4}}^{(3)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{\frac{10}{12}}^{(5)}, \lambda_{\frac{3}{4}}^{(6)}, \lambda_{\frac{3}{4}}^{(7)}, \lambda_{\frac{4}{6}}^{(8)}, \lambda_{\frac{2}{3}}^{(9)}, \lambda_{\frac{4}{5}}^{(10)}, \lambda_{\frac{7}{9}}^{(11)}, \lambda_{\frac{7}{9}}^{(12)}, \lambda_{\frac{11}{15}}^{(13)}\}.$$

Lower weight

$$\underline{A} = \{\lambda_{\frac{1}{6}}^{(1)}, \lambda_{\frac{1}{6}}^{(2)}, \lambda_{\frac{1}{3}}^{(3)}, \lambda_{\frac{1}{3}}^{(4)}, \lambda_{\frac{2}{6}}^{(5)}, \lambda_{0}^{(6)}, \lambda_{\frac{1}{3}}^{(7)}, \lambda_{\frac{2}{9}}^{(8)}, \lambda_{\frac{1}{6}}^{(9)}, \lambda_{\frac{2}{6}}^{(10)}, \lambda_{0}^{(11)}, \lambda_{\frac{1}{3}}^{(12)}, \}.$$

3. **Humidity**
   Upper weight

$$\bar{A} = \{\lambda_{\frac{2}{3}}^{(1)}, \lambda_{0}^{(2)}, \lambda_{\frac{5}{6}}^{(3)}, \lambda_{\frac{7}{9}}^{(4)}, \lambda_{\frac{3}{4}}^{(5)}, \lambda_{\frac{2}{3}}^{(6)}, \lambda_{\frac{5}{6}}^{(7)}, \lambda_{\frac{4}{6}}^{(8)}, \lambda_{\frac{5}{6}}^{(9)}, \lambda_{\frac{7}{9}}^{(10)}, \lambda_{\frac{7}{9}}^{(11)}, \lambda_{\frac{5}{6}}^{(12)}, \lambda_{1}^{(13)}\}.$$

   Lower weight

$$\underline{A} = \{\lambda_{\frac{2}{9}}^{(1)}, \lambda_{\frac{1}{6}}^{(2)}, \lambda_{\frac{1}{3}}^{(3)}, \lambda_{\frac{1}{3}}^{(4)}, \lambda_{\frac{1}{3}}^{(5)}, \lambda_{0}^{(6)}, \lambda_{\frac{1}{3}}^{(7)}, \lambda_{\frac{1}{6}}^{(8)}, \lambda_{\frac{2}{6}}^{(9)}, \lambda_{1}^{(10)}, \lambda_{\frac{1}{3}}^{(11)}, \lambda_{1}^{(12)}, \lambda_{\frac{1}{3}}^{(13)}\}.$$

4. **Windy**
   Upper weight

$$\bar{A} = \{\lambda_{\frac{4}{6}}^{(1)}, \lambda_{\frac{2}{3}}^{(2)}, \lambda_{\frac{7}{9}}^{(3)}, \lambda_{\frac{3}{4}}^{(4)}, \lambda_{\frac{11}{15}}^{(5)}, \lambda_{\frac{11}{15}}^{(6)}, \lambda_{\frac{3}{4}}^{(7)}, \lambda_{\frac{4}{6}}^{(8)}, \lambda_{\frac{4}{9}}^{(9)}, \lambda_{\frac{10}{12}}^{(10)}, \lambda_{\frac{10}{12}}^{(11)}, \lambda_{\frac{5}{6}}^{(12)}, \lambda_{\frac{7}{9}}^{(13)}\}.$$

   Lower weight

$$\underline{A} = \{\lambda_{\frac{1}{6}}^{(1)}, \lambda_{\frac{2}{9}}^{(2)}, \lambda_{\frac{1}{3}}^{(3)}, \lambda_{\frac{1}{3}}^{(4)}, \lambda_{0}^{(5)}, \lambda_{0}^{(6)}, \lambda_{\frac{1}{3}}^{(7)}, \lambda_{\frac{1}{6}}^{(8)}, \lambda_{\frac{2}{6}}^{(9)}, \lambda_{\frac{1}{3}}^{(10)}, \lambda_{\frac{1}{3}}^{(11)}, \lambda_{\frac{1}{3}}^{(12)}, \lambda_{\frac{1}{3}}^{(13)}\}.$$

5. Results

1. We compute the accuracy of decision without elimination of any attribute.

$$\text{Accuracy} = \frac{\text{Lower}A}{\text{Upper}A} = \frac{\underline{A}}{\bar{A}}$$

$$\text{Accuracy} = \frac{4.374}{7.918} = 0.552.$$

2. We compute the accuracy of decision after elimination of each attribute.

Reduction	Outlook	Temperature	Humidity	Windy
Accuracy	0.209	0.284	0.511	0.340

6. Conclusion

Topology can help us solve all kinds of mathematical problems where distances and size don't matter, only the structure of a shape. There are many problems in daily life and by using topological modeling we can overcome these problems. In this paper we did the following:

1. Changing the upper by using the relation between $\bar{A} = (A^{c^0})^c$.
2. Finding equivalence classes using weights.
3. Calculating the accuracy in this way gave different results and improved the accuracy of the decision.

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