Results in Nonlinear Analysis 6 (2023) No. 2, 114–121
https://doi.org/10.31838/rna/2023.06.02.011
Available online at www.nonlinear-analysis.com

Results in Nonlinear Analysis
ISSN 2636-7556

Peer Reviewed Scientific Journal

A comparative review of solution methods and techniques for analyzing reliability in the transient state

Zeenat Zaidi

Department of Mathematics, Deanship of Educational Services, Qassim University, Buraidah 51452, Kingdom of Saudi Arabia

Abstract

The primary goal of this comparative study is to address various solution methods and techniques for transient analysis of a three-component system's reliability. To analyse the system's transient reliability, a Markov model is used. To determine the outcome of the differential equations of a three-component system in the transient state, three methods are used: the Laplace Transform method, the Matrix approach, and simple integration. These procedures are executed that has been reviewed using MATLAB 7.8.0 (R2009a). These tools and numerical methods offer a more reliable mathematical framework and methodology for evaluating the transient reliability of the system. The introduction of these techniques is useful for researchers to assess reliability and availability in large scale systems.

Keywords: Comparative study 1, Differential Equations 2, Markov model 3, Transient state 4, Reliabiity 5, Numerical methods 6, large scale systems 7

1. Introduction

The estimation of component reliability using information of element reliabilities and their configuration is a fundamental process in reliability technology. Markov analysis is used when there is some sort of dependency between component breakdowns. The fundamental premise of a Markov

Email address: z.hasain@qu.edu.sa (Zeenat Zaidi)

Received April 4, 2023; Accepted May 20, 2023; Online June 13, 2023

process is that, irrespective of any potential previous states the system may have experienced, the likelihood of the system changing from one state to another entirely relies on its present state. When it comes to failure, the exponential distribution satisfies this Markovian characteristic. The two approaches that are most frequently used by many investigators to ascertain the availability of complex systems are the Laplace transform technique and regeneration methodology. The method yields precise findings. The criteria are not satisfied by the numerical approach since it approximates the solution of differential difference equations. Singh [1] examined a multiple channel system using the matrix technique for solving differential difference equations. The idea and associated theory of the multi-state system were progressively suggested by Barton [2] and Murchland [3]. In his PhD thesis, Mahajan [4] examined the availability of multiple repairable systems. The technique necessitates computing the coefficients matrix's eigenvalues. Lisnianski and Levitin [5] investigated the theory of multi-state system reliability and the practical use of analytical techniques. The overall generation function was used by Levitin [6] to assess the reliability of an irreversible MSS with common cause failure. Comparing different systems with dependent elements was done by Novarro et al. [7]. Dembinska [8], Navarro et al. [9], and Parsa et al. [10] have all conducted additional study on this subject. A k-out-of-n method with independent exponential components was introduced by Li et al. [11]. Their research revealed that some operating elements would cease to function as soon as the system failed; repairs start as soon as a component breaks, and repair times are independent and exponentially distributed. Analysis was done on the mean time between failures, mean working time and the mean down time during a failure repair cycle. Azaron et al. [12] proposed a new framework for the reliability analysis of an L-dissimilar-unit redundant system using the minimal path approach and continuous time Markov methods. Amiri and Ghassemi [13, 14] suggested an approach for transient evaluation of availability with repairable elements using Markov method and eigenvectors. They gave a technique for calculating the system's availability, survivability, MTTF, and MTSF. Systems that only rely on the number of failed elements are included in the large family of dependability systems, and they are essential to the system's reliability. K-out-of-n: F system is a configuration that fails as soon as k out of n units fail while K-out-of-n: G system is a configuration that works if and only if at least k of its n units is functioning. These systems are frequently used in reliability theory and were formalised (Esary and Proschan [15]; Birnbaum et al. [16]. Eryilmaz evaluates its usefulness for systematic study and practical applications [17–19]. Gurler and Bairamov [20], Freixas and Puente [21], Petchrompo et al. [22] also studied k-out-of-n system. Ram [23] investigated reliability strategies in several engineering and scientific disciplines. The major areas, or past, present, and future trends in reliability technique, have been attempted to be identified by the authors. Franko and Tutunco [24] studied reliability of weighted k-out-of-n: G systems using system signature. Eryilmaz [25] examined failed components in a failed and operating k-out-of-n system. In a random environment, Zhang et al. [26] examined the reliability-based measures and prognostic problems of a k-out-of-n system. Reliability analysis of multi-state systems under common cause failure conditions is investigated by Jia et al. [27]. Zhang [28] examined the reliability analysis of k-out-of-n systems with heterogeneous components. By Cerqueti [29], who also introduced and studied the usual k-out-of-n systems, it is predicted that each component's part in anticipating the system's failure is distinct. In a typical system reliability analysis, the state of a system and unit are usually taken to be normal or entirely failed. Due to changes in productivity, working circumstances, environmental effects, and system complexities, reliability analysis using a two-state correlation theory system is no longer used in real-world engineering. Complex systems and units can live in a variety of states, each of which is delineated by a unique efficiency stage, in a multi-state system. This research analyses a three-component system using three alternative solution techniques to evaluate reliability in the transient state. The possibility of extending the matrix method to estimate the likelihood of different complex system states without calculating the matrix's eigenvalues is discussed. MATLAB is used to implement and test these techniques.

2. Assumptions and Notations

This research aims to evaluate the transient state reliability of a three-component system.

- Consider the instantaneous (failure) rate as the transition between states.
- Because the method is stable and the probabilities of transformation do not shift over time, the transition rates will remain constant. This is equal to assuming exponential failure rates.
- No units fail when in standby, and the failure rates for all three systems are constant when they are operational.

λ: denote the instantaneous failure rate of three component standby system.
$P_n(t), (n = 1, 2, 3)$: denote the probability is in nth state at time t.

3. The Proposed Methodology

The probabilities of different system states are represented by a Markov model as a function of time. This method can be used to analyse the component's time-dependent reliability. When the failure and repair rates are constant, the approach is successful. The system state behavior is defined by a process known as a Markov process if the probability rules of its future state of existence depends only on the state it is in and not on how the system arrived at that state. State and time are its two variables. The overall condition of a method can be defined in a variety of ways. Because of this, there are three potential states for a two-component system with components of 1 and 2: no failure, one failure (either 1 or 2), and two failures. Another set of states include no component failed, component 1, component 2, and both components failed. Consider about a system that has two standby (backup) components and one active (operational) component. It is possible to create differential equations by using the mnemonic rule. According to the mnemonic rule, the sum of all probability flows coming into a particular state from other states minus the sum of all probability flows leaving that state and going to other states equals the derivative of probability at that state. These equations can be solved using the matrix technique, direct integration, or Laplace transforms. The following definitions describe the various system states:

State	Component 1	Component 2	Component 3
1	Operational	Backup	Backup
2	Failed	Operational	Backup
3	Failed	Failed	Operational
4	Failed	Failed	Failed

The resulting differential difference equations are as follows.

$$\frac{dP_1(t)}{dt} = -\lambda P_1(t) \tag{3.1}$$

$$\frac{dP_2(t)}{dt} = \lambda P_1(t) - \lambda P_2(t) \tag{3.2}$$

$$\frac{dP_3(t)}{dt} = \lambda P_2(t) - \lambda P_3(t) \tag{3.3}$$

With initial conditions $P_1(0) = 1, P_2(0) = 0$ and $P_3(0) = 0$

Solution of governing differential difference equations (3.1), (3.2), and (3.3) for $P_1(t), P_2(t)$, and $P_3(t)$ helps to study the system for various parameters like MTBF, availability, maintainability etc. The analysis is carried out in detail, with the help of simple integration method, Laplace transform and Matrix method. The precision of all the strategies is tested using MATLAB 7.8.0 (R2009a).

3.1 Simple Integration Method

From equation (3.1),

$$\frac{dP_1(t)}{dt} = -\lambda P_1(t)$$
$$\frac{dP_1(t)}{P_1(t)} = -\lambda dt$$

Integrating both sides

$$\ln P_1(t) = -\lambda t$$
$$P_1(t) = e^{-\lambda t}$$

From equation (3.2),

$$\frac{dP_2(t)}{dt} = \lambda P_1(t) - \lambda P_2(t)$$
$$\frac{dP_2(t)}{dt} = \lambda e^{-\lambda t} - \lambda P_2(t)$$

with $e^{-\lambda t}$ as an integrating factor

$$P_2(t)e^{\lambda t} = \lambda \int e^{-\lambda t} e^{\lambda t} dt + c$$

or $P_2(t) = \lambda t e^{-\lambda t} + c e^{-\lambda t}$

The initial conditions are $P_1(0) = 1, P_2(0) = 0$ and $P_3(0) = 0$.

Therefore, $c = 0$

we get $P_2(t) = \lambda t e^{-\lambda t}$

$P_3(t)$ is also derived in a similar way,

From equation (3.3),

$$\frac{dP_3(t)}{dt} = \lambda P_2(t) - \lambda P_3(t)$$
$$\frac{dP_3(t)}{dt} = \lambda^2 t e^{-\lambda t} - \lambda P_3(t)$$

with $e^{\lambda t}$ as an integrating factor

$$P_3(t)e^{\lambda t} = \int \lambda^2 t e^{-\lambda t} e^{\lambda t} dt + c$$
$$P_3(t)e^{\lambda t} = \lambda^2 \int t dt + c$$
$$P_3(t) = \frac{\lambda^2 t^2}{2} e^{-\lambda t} + c e^{-\lambda t}$$

using initial conditions, $P_3(0) = 0,$ we get $c = 0$

Hence, $P_3(t) = \dfrac{\lambda^2 t^2}{2} e^{-\lambda t}$

The system's Reliability is;

$$R(t) = P_1(t) + P_2(t) + P_3(t);$$
$$R(t) = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2}\right]$$

3.2 Laplace Transform Method

Let $F(t)$ be a function described for all $t \geq 0$. The function $f(s) = \int e^{-st} F(t) dt$, where $s > 0$ is known as the Laplace transform of the function $F(t)$.

$$L(F(t)) = f(s) = \int e^{-st} F(t) dt, \text{ where } s > 0$$

3.2.1 Inverse Laplace Transform

If $f(s)$ is the Laplace transform of a function $F(t)$, then the function $F(t)$ is known as the inverse of the function $f(s)$.

Consider the equations (3.1)–(3.3)

The Laplace Transforms, using initial conditions are:

$$(s+\lambda)\bar{p}_1(s) = -1,$$
$$(s+\lambda)\bar{p}_2(s) = \frac{\lambda}{s+\lambda},$$
$$(s+\lambda)\bar{p}_3(s) = \frac{\lambda^2}{(s+\lambda)^2},$$

Solving these equations, we get

$$\bar{p}_1(s) = \frac{-1}{[s+\lambda]},$$
$$\bar{p}_2(s) = \frac{\lambda}{(s+\lambda)^2},$$
$$\bar{p}_3(s) = \frac{\lambda^2}{(s+\lambda)^3},$$

Taking Laplace Transforms,

$$P_1(t) = L^{-1}\left[\frac{-1}{(s+\lambda)}\right]$$
$$P_2(t) = L^{-1}\left[\frac{\lambda}{(s+\lambda)^2}\right]$$
$$P_3(t) = L^{-1}\left[\frac{\lambda^2}{(s+\lambda)^3}\right]$$
$$P_1(t) = e^{-\lambda t}$$
$$P_2(t) = \lambda t e^{-\lambda t}$$
$$P_3(t) = \frac{\lambda^2 t^2}{2} e^{-\lambda t}$$

The system's Reliability is;

$$R(t) = P_1(t) + P_2(t) + P_3(t);$$
$$R(t) = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2}\right]$$

3.3 Matrix Method

Notation: We write $P_i(t) = p(i,t), i = 0,1,2.$

The coefficient matrix of equations (3.1)–(3.3) is

$$A = \begin{bmatrix} -\lambda & 0 & 0 \\ \lambda & -\lambda & 0 \\ 0 & \lambda & -\lambda \end{bmatrix}$$

$$A\bar{p}(k,0) = \begin{bmatrix} -\lambda & 0 & 0 \\ \lambda & -\lambda & 0 \\ 0 & \lambda & -\lambda \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A\bar{p}(k,0) = \begin{bmatrix} -\lambda \\ \lambda \\ 0 \end{bmatrix}$$

$$A^2\bar{p}(k,0) = \begin{bmatrix} \lambda^2 \\ -2\lambda^2 \\ \lambda^2 \end{bmatrix}$$

$$A.A^2\bar{p}(k,0) = \begin{bmatrix} -\lambda & 0 & 0 \\ \lambda & -\lambda & 0 \\ 0 & \lambda & -\lambda \end{bmatrix}\begin{bmatrix} \lambda^2 \\ -2\lambda^2 \\ \lambda^2 \end{bmatrix}$$

$$A^3\bar{p}(k,0) = \begin{bmatrix} -\lambda^3 \\ 3\lambda^3 \\ -3\lambda^3 \end{bmatrix}$$

$$\begin{aligned} P(1,t) &= 1 + (-\lambda)t + \frac{\lambda^2 t^2}{2!} + (-\lambda)\lambda^2\frac{t^3}{3!} + \ldots\ldots \\ &= e^{-\lambda t} \\ P(2,t) &= \lambda t - \lambda^2 t^2 + \frac{3\lambda^3 t^3}{3!}\ldots\ldots \\ &= \lambda t(1 - \lambda t + \frac{\lambda^2 t^2}{2!} \\ &= \lambda t e^{-\lambda t} \\ P(3,t) &= \frac{\lambda^2 t^2}{2!} - \frac{3\lambda^3 t^3}{3!} + \ldots\ldots \\ &= \frac{\lambda^2 t^2}{2}(1 - \lambda t + \frac{\lambda^2 t^2}{2!}\ldots\ldots \\ &= \frac{\lambda^2 t^2}{2} e^{-\lambda t} \end{aligned}$$

The system's Reliability is;

$$R(t) = P_1(t) + P_2(t) + P_3(t);$$

$$R(t) = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2}\right]$$

3.4 Matlab 7.8.0 (R009a)

It is possible to complete computationally intensive tasks more quickly with MATLAB than with traditional programming because it was created specifically for mathematical calculations. The acronym for matrix laboratory is MATLAB. It is an interactive high-performance software package for computations in the fields of science and engineering that includes mathematical operations.

Using Matlab, equations (3.1) through (3.2) are solved. $P(1,t) = x, P(2,t) = y, P(3,t) = z, \lambda = x1$. In Matlab, the differential equations with initial conditions are computed using the dsolve syntax.

syms $x1$
$[x, y, z] = dsolve('Dx = -x1 * x',' Dy = x1 * x - x1 * y',' Dz = x1 * y - x1 * z',' x(0) = 1',' y(0) = 0',' z(0) = 0')$
$x = 1/exp(t * x1)$
$y = (t * x1)/exp(t * x1)$
$z = (t \wedge 2 * x1 \wedge 2)/(2 * exp(t * x1))$
>>

Hence,

$$P(1,t) = e^{-\lambda t}$$
$$P(2,t) = \lambda t e^{-\lambda t}$$
$$P(3,t) = \frac{\lambda^2 t^2}{2} e^{-\lambda t}$$

The system's Reliability is:

$$R(t) = P_1(t) + P_2(t) + P_3(t);$$
$$R(t) = e^{-\lambda t}\left[1 + \lambda t + \frac{\lambda^2 t^2}{2}\right]$$

4. Conclusion and Discussion

This study's primary goal is to present various problem-solving approaches and techniques for figuring out a three-component system's transient reliability. The Markov Method is implemented to generate the model for the time dependent reliability of such system. Mnemonic rule is used for formulating difference equations. It has been noticed that the outcomes are consistent when we use the aforementioned techniques on a three component standby system. Although the Laplace transforms technique is useful for simple systems, it is not well suited for complex systems due to the difficulty of inverting Laplace transforms. Matrix method is more suitable for complex systems. However, MATLAB makes technical computation issues easier to tackle than with more conventional programming languages like C, C++, and FORTRAN. These techniques can be used in a variety of industries, giving management the greatest possible advantage. The management tries to improve the maintenance facilities it offers or, in some other way, to reduce the likelihood that any subsystem will fail. As a consequence, reliability engineers and managers can benefit from the findings.

Acknowledgement

I would like to thank the Deanship of Scientific Research, Qassim University for funding publication of this research.

References

[1] Singh, J., *On multiple parallel channels with a finite source*, Jnanabha Sect. A, 5, (1975), 1–7.
[2] Barton, R., and Damon, W., Reliability in a multi-state system, Proceedings of the Sixth Annual South eastern Symposium on Systems theory, Baton Rouge, LA, USA (1974).
[3] Murchland, J., *Fundamental concepts and relations for 0 reliability analysis of multi-state systems and fault tree analysis*, Theoretical and Applied Aspects of System Reliability and Fault Tree Analysis, (1975), 581–618 SIAM, Philadelphia, PA, USA.
[4] Mahajan, P., *Availability analysis of redundant repairable systems*, Ph.D. thesis, Kurukshetra University, (1999).
[5] Lisnianski, A., and Levitin, G., Multi-State System Reliability: Assessment, Optimization and Applications, World Scientific Publishing Co Pte Ltd, New York, NY, USA, (2003).
[6] Levitin, G., *Common supply failures in linear multi-state sliding window systems*, Reliability Engineering and System Safety, 82 (1), (2003), 55–62.
[7] Navarro, J., Ruiz, J. M., and Sandoval, C. J., *A note on comparisons among coherent systems with dependent components using signatures*, Statistics and Probability Letters, 27, (2005), 179–185.
[8] Dembinska, A., *On reliability analysis of k-out-of-n systems consisting of heterogeneous components with discrete lifetimes*, IEEE Transactions on Reliability, 67 (3), (2018), 1071–1083.
[9] Navarro, J., del Aguila, Y., Sordo, M. A., Suarez Llorens, A., *Stochastic ordering properties for systems with dependent identically distributed components*, Applied Stochastic Models in Business and Industry, 29, (2013), 264–278.
[10] Parsa, M., Di Crescenzo, A., and Jabbari, H., *Analysis of reliability systems via Gini-type index*, European Journal of Operational Research, 264, (2018), 340–353.
[11] Li, X., Zuo, M. J., Yam, R. C. M., *Reliability analysis of a repairable k-out-of-n system with some components being suspended when the system is down*, Reliability Engineering and System Safety, 91, (2006), 305–310.
[12] Azaron, A., Katagiri, H., Kato, K., Sakawa, M., *Reliability evaluation of multicomponent cold standby redundant systems*, Applied Mathematics and Computation, 173, (2006), 137–149.
[13] Amiri, M., and Ghassemi, F., *A methodology for analysing the transient availability and survivability of a system with repairable components*, Applied Mathematics and Computation, 184, (2007), 300–307.
[14] Amiri, M., and Ghassemi, F., *A methodology for analysing the transient reliability of systems with identical components and identical repairmen*, International Journal of Science and Technology, Iranica, 14, (2007), 72–77.
[15] Esary, J. D., and Proschan, F., *Coherent structures of non-identical components*, Technometrics, 5 (2), (1963), 191–209.
[16] Birnbaum, Z. W., Esary, J. D., and Saunders, S. C., *Multi-component systems and structures and their reliability*, Technometrics, 3 (1), (1961), 55–77.
[17] Eryilmaz, S., *Dynamic behavior of k-out-of-n: G systems*, Operations Research Letters, 39, (2011), 155–159.
[18] Eryilmaz, S., *On the mean residual life of a k-out-of-n: G system with a single cold standby component*, European Journal of Operational Research, 222, (2012), 273–277.
[19] Eryilmaz, S., *On reliability of a k-out-of-n system equipped with a single warm standby component*, IEEE Transactions on Reliability, 62, (2013), 499–503.
[20] Gurler, S., and Bairamov, I., *Parallel and k-out-of-n: G systems with nonidentical components and their mean residual life functions*, Applied Mathematical Modeling, 33, (2009), 1116–1125.
[21] Freixas, J., and Puente, M. A., *Consecutive expansions of k-out-of-n systems*, Operations Research Letters, 37, (2009), 438–442
[22] Petchrompo, S., Li, H., Erguido, A., Riches, C., and Parlikad, A. K., *A value-based approach to optimising long-term maintenance plans for a multi-asset k-out-of-N system*, Reliability Engineering and System Safety, (2020), 106924.
[23] Ram, M., *On system reliability approaches: A brief survey*, International Journal of systems assurance engineering and management, 4 (2), (2013), 101–117.
[24] Franko, C., and Tutuncu, G. Y., *Signature based reliability analysis of repairable weighted k-out-of-n: G systems*, IEEE Transactions on Reliability, 65 (2), (2016), 843–850.
[25] Eryilmaz, S. *The number of failed components in a k-out-of-n system consisting of multiple types of components*, Reliability Engineering and System Safety, 175, (2018), 246–250.
[26] Zhang, N., Fouladirad, M., and Barros, A., *Reliability-based measures and prognostic analysis of a K-outof-N system in a random environment*, European Journal of Operational Research, 272 (3), (2019), 1120–1131.
[27] Jia, J., Li, Z., Jia, P., and Yang, Z., *Reliability analysis of common cause failure multistate system based on CUGF*, Journal Mathematical problems in Eingineering, Article ID 4608124, (2020), 14 pages.
[28] Zhang, Y., *Reliability analysis of randomly weighted k-out-of-n systems with heterogeneous components*, Reliability Engineering and System Safety, 205, (2021), 101784.
[29] Cerqueti, R., *A new concept of reliability system and applications in finance*, Statistical reliability modelling and optimization, Annals of operations research, 312, (2022), 45–64.