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Dominations in bipolar picture fuzzy graphs and social networks

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Abstract

In this manuscript, we initiate the concepts of domination in bipolar picture fuzzy graphs (BPPFGs) based on the strong edges. Basically, it is the generalization of both the dominations in bipolar fuzzy graphs (BPFGs) and picture fuzzy graphs (PFGs). In the beginning, we introduce different terms related to the domination of bipolar picture fuzzy graphs (BPPFGs) like vertex cardinality, edge cardinality, strong edge, neighbors, strong neighbor of vertex, private neighborhood, independent sets, dominating sets etc. After this, we provide some important characterizations of domination in bipolar picture fuzzy graphs (BPPFGs) based on minimal dominating sets and maximal independent sets. We also investigate the lower and upper domination numbers of these graphs. Moreover, we discuss the notion of the total domination of bipolar picture fuzzy graphs (BPPFGs) and present few of its properties. In our study, we include the terms status and structurally equivalent of bipolar picture fuzzy graphs (BPPFGs). Finally, we present the application of the domination in bipolar picture fuzzy graphs towards social networking.

Keywords: domination in BPPFGs, Upper, lower and total domination numbers of BPPFGs, status of a BPPFG

2010 Mathematics Subject Classification: 03E72, 05C72

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1. Introduction

In 1965, Zadeh initiated the concepts of fuzzy sets (FSs) [1]. After this, many generalizations of the fuzzy sets (FSs) have been explored in the literature. interval-valued fuzzy set (IVFS) was the first generalization of the FSs introduced by Zadeh [2]. Subsequently, Intuitionistic fuzzy sets (IFSs), bipolar fuzzy sets (BPFs), picture fuzzy sets (PFSs) are another generalizations of FSs explored in the literature. Bipolar fuzzy sets (BPFSs) was first introduced in [3]. In BPFSs, the membership values were restricted in the interval [–1, 1]. Different types of relations were studied on BPFSs in [4]. Further extension of BPFSs named bipolar pythagorean fuzzy sets was introduced in [5]. Afterwards, the term BPFSs was shifted towards soft sets theory and the term bipolar fuzzy soft set was introduced in [6]. Atanassov [7] introduced the new type of the FSs termed intuitionistic fuzzy sets (IFSs). In IFSs, the positive and negative memberships were considered. Recently, B. C. Cuong [8] introduced the most extended form of the FSs named picture fuzzy sets (PFSs). In PFSs, three membership values were assigned to any entity which were neutral, positive and negative. C. Bo et al. [9] explored new types of operations and relations on PFSs. Different types of fuzzy logical operators towards PFSs were discussed by B. C. Cuong et al. [10]. Recently, the combination of both the BPFSs and PFS termed bipolar picture fuzzy sets (BPPFSs) and relations along with their applications have been introduced by the first and the third authors (with Faiz) [11].

On the other hand, Rosenfeld shifted the classical graphs theory towards fuzzy sets theory. He introduced the term fuzzy graphs (FGs) in [14]. FGs found more flexible and effective compared to that of the classical graphs. Due to its flexibility, numerous applications of FGs were explored in the literature. Bhattacharya introduced several new terms in the theory of FGs [15]. Several operations were defined and applied to FGs in [16]. The term complement of FGs was investigated in [17]. The term average connectivity of the classical graphs has been shifted towards fuzzy graphs by Poulik et al. [18]. An extension of the FGs named interval-valued fuzzy graphs (IVFGs) was introduced in [19]. Another generalization of the FGs termed intuitionistic fuzzy graphs (IFGs) was addressed in [20]. Different new operations on IFGs were explored in [21]. The notion of the complex intuitionistic fuzzy graphs along with their applications in cellular networking theory were explored in [22]. Similarly, another important generalization of FGs termed bipolar fuzzy graphs (BPFGs) was introduced by M. Akram in [23]. He also explored many fascinating characteristics of these graphs. However, different types of BPFGs were discussed in [24]. Regular-BPFGs was introduced in [25]. Similarly, the term m-polar fuzzy graphs was discussed in [26]. Furthermore, some applications of strong arcs in m-polar fuzzy graphs were discussed in [27]. Sequently, in 2022 Poulik and Ghorai [28] introduced the notions of perfectly regular BPFGs and perfectly edge-regular BPFGs. They also provided the worthwhile applications of these terms towards communication systems and decision making theory. The concepts of the picture fuzzy graphs (PFGs) was first introduced in [29]. Several operations were defined and applied to PFGs and applications towards social networking were also explored. The extension of PFGs termed picture fuzzy multi-graphs (PFMGs) was introduced in [30]. Subsequently, the term regular picture fuzzy graphs (RPFGs) was initiated in [31]. Recently, we (the first and third authors with Faiz) have also initiated the terms Cayley picture fuzzy graphs [32] and interval-valued picture fuzzy graphs (IVPFGs) [33]. The first and the third authors (with Ali) discussed the concepts of the bipolar picture fuzzy graphs (BPPFGs) [34] which was the extended form of both the BPFGs and PFGs. Chen et al. [12] introduced the concepts of picture fuzzy line graphs. Arif et al. [13] introduced the notion of picture (S, T)-fuzzy graphs and its applications towards MADM.

Domination in FGs based on effective edges was introduced in [35]. Afterwards, Nagoorgani et al. [36] discussed the term domination in FGs based on strong arcs. Domination in directed fuzzy graphs was investigated in [37]. Most recently, the terms broadcasts and dominating broadcasts in FGs were discussed in [38]. They also presented the application of these terms towards transportation model. Further to this, the concepts of the total efficient domination was introduced in [39]. The concepts of the domination in BPFGs were introduced in [40]. Gong et al. [41] introduced the domination in BPFGs in some different ways. Subsequently, domination in BPFGs was further investigated in [42]. They also investigated the fuzzy network connectivity in the setting of BPFGs. The domination in PFGs was discussed in [43]. More generalized form of domination termed paired domination, strong paired domination in PFGs were introduced in [44].

In this manuscript, we initiate the concepts of dominations in bipolar picture fuzzy graphs (BPPFGs) based on strong edges. This is the generalized form of both the domination in BPFGs and PFGs. In section 3, firstly we define few useful terminologies related to domination in BPPFGs such as edge cardinality, vertex cardinality, neighbors, private neighbors and strong neighbor of vertex etc of BPPFGs. Then, by considering the minimal dominating set and maximal independent set, we investigate some important characterizations of BPPFGs. During this study, we also add the terms the lower domination and upper domination numbers in BPPFGs. The concept of the status, total dominating set and total domination number of BPPFGs are also introduced in the setting of BPPFGs. Finally, we present the application of the domination in BPPFGs towards social networking.

2. Preliminaries

In this section, we review some useful terms related to FSs and FGs from the literature. However, we refer [45] for the basics of the classical graphs theory.

Definition 2.1: [1] *A fuzzy set (FS) T on a nonempty set U is described by*

$$T = \{(u, \rho(u)) : u \in U, \rho(u) \in [0,1]\}$$

Definition 2.2: [3] *A bipolar fuzzy set* (BPFS) *T on a nonempty set U is given by* $T = \{(w, \rho^+(w), \rho^-(w)) : w \in U\}$, *where* $\rho^+ : U \to [0,1]$ *and* $\rho^- : U \to [-1,0]$ *are the mappings.*

Definition 2.3: [8] *A PFS T on U is the collection* $T = \{(u, \rho_T(u), \varphi_T(u), \tau_T(u)) : u \in U\}$, *where* $\rho_T(u), \varphi_T(u)$ *and* $\tau_T(u)$ *are lying in the interval* [0, 1] *and represent the positive, neutral and negative membership degrees of u in T, respectively with* $\rho_T(u) + \varphi_T(u) + \tau_T(u) \le 1$, *for all* $u \in U$.

Definition 2.4: [14] *A fuzzy graph (FG) is a pair G* = (C, D), where* $C = \{\rho_C\}$ *and* $D = \{\rho_D\}$ *such that* $\rho_C : V \to [0,1]$ *and* $\rho_D : V \times V \to [0,1]$. *We have* $\rho_D(x,w) \le \rho_C(w) \wedge \rho_C(x)$.

Definition 2.5: [14] *A FG* $H = (C^*, D^*)$ *is said to be a fuzzy subgraph of G*, if* $\rho^*(w) \le \rho(w)$, *for all* $w \in C$, $\rho^*(w,z) \le \rho(w,z)$, *for all* $w, z \in C$.

Definition 2.6: [23] *A BPFG is a pair G* = (C, D), where* $C = \{\rho_C^+, \rho_C^-\}$ *and* $D = \{\varphi_D^+, \varphi_D^-\}$, *where* $\rho_C^+ : W \to [0,1]$, $\rho_C^- : W \to [-1,0]$, $\varphi_D^+ : W \to [0,1]$ *and* $\varphi_D^- : W \to [-1,0]$, *is said be a BPFG of underlying set W, if* $\varphi_D^+(w,z) \le \min(\rho_C^+(w), \rho_C^+(z))$ *and* $\varphi_D^-(w,z) \ge \min(\rho_C^-(w), \rho_C^-(z))$, *for all* $w, z \in D = C \times C$.

Definition 2.7: [29] *A pair G = (C, D) is called a PFG on G* = (C, D), where* $C = (\rho_C, \varphi_C, \tau_C)$ *is a PFS on V and* $D = (\rho_D, \varphi_D, \tau_D)$ *is a PFS on* $D \subseteq V \in C$ *with*

$$\rho_D(w,z) \le min(\rho_C(w), \rho_C(z))$$
$$\varphi_D(w,z) \le min(\varphi_C(w), \varphi_C(z))$$
$$\tau_D(w,z) \ge max(\tau_C(w), \tau_C(z))$$

Definition 2.8: [29] *A sequence of different vertices* $w_0, w_1, w_2, ..., w_n$ *in a PFG G* = (C, D) is a path p such that* $(\rho_D(w_{i-1}, w_i)\varphi_D(w_{i-1}, w_i)\tau_B(w_{i-1}, w_i)) > 0$, $i = 1, 2, ..., n$, *where n is the length of the path.*

Definition 2.9: [29] *If two vertices w and z of a PFG G* = (C, D) are connected through the path of length n as* $p : w_0, w_1, w_2, ...w_{n-1}, w_n$, *then* $\rho_D(w,z), \varphi_D(w,z)$ *and* $\tau_D(w,z)$ *can be described as*

$$\rho_D(w,z) = min(\rho_D(w_0,w_1), \rho(w_1,w_2), ...\rho(w_{k-1},w_n))$$
$$\varphi_D(w,z) = min(\varphi_D(w_0,w_1), \varphi(w_1,w_2), ...\varphi(w_{k-1},w_n))$$
$$\tau_D(w,z) = max(\tau_D(w_0,w_1), \tau(w_1,w_2), ...\tau(w_{k-1},w_n)).$$

Definition 2.10: [29] *Let* $(\rho_D)^\infty(w,z)),(\varphi_D)^\infty(w,z))$ *and* $(\tau_D)^\infty(w,z))$ *be the strength of connectedness between the two nodes w and z of a PFG G*. Then* $(\rho_D)^\infty(w,z)),(\varphi_D)^\infty(w,z))$ *and* $(\tau_D)^\infty(w,z))$ *are defined as follows.*

$$(\rho_D)^\infty(w,z)) = max(\rho_D^i(w,z) : i = 1,2,3....,n$$
$$(\varphi_D)^\infty(w,z)) = max(\varphi_D^i(w,z) : i = 1,2,3....,n$$
$$(\tau_D)^\infty(w,z)) = min(\tau_D^i(w,z) : i = 1,2,3....,n$$

Definition 2.11: [35] *Let G* = (C, D) be a FG of a crisp graph G and* $w,z \in C$. *We say w dominates z in G* , if* $\rho(wz) = \rho(w) \wedge \rho(z)$. *A subset* C_1 *of C is the dominating set (DS) in G*, if for each* $w \in C_1$ *there is* $z \in V - C_1$ *such that w dominates z. A dominating set* C_1 *of a FG G* is a minimal dominating set (MDS), if there is no any proper subset of* C_1 *which is a dominating set (DS) of G*. The minimum (fuzzy) cardinality of a dominating set (DS) in G* is the domination number (DN) of G*.*

Definition 2.12: [35] *If G* be any FG not containing any isolated vertex, then the subset* C_1 *of a DS C is called a total dominating set (TDS), if every vertex lying in C is dominated by a vertex in* C_1. *The minimum fuzzy cardinality of a total dominating set (TDS) is a total domination number (TDN) of G*.*

Definition 2.13: [29] *Two vertices w and z are said to be neighbors in a FG, if* $\rho(w,z) > 0$, *and N(w) represents the set of all neighbors of w. Also, if the arc (w, z) is strong, then w is a strong neighbor (SNs). The collection of all the strong neighbors (SNs) of w is called a strong neighborhood (SNbhd) of w, and is abbreviated as* $N_s(w)$. *The closed strong neighborhood (CSNHd) is* $N_s[w] = N_s(w) \cup w$.

Definition 2.14: [43] *An arc (w, s) in a PFG G = (C, D) is called a strong arc, if* $\rho_D(w,z) \ge (\rho_D)^\infty(w,z)$, $\varphi_D(w,z) \ge (\varphi_D)^\infty(w,z)$ *and* $\tau_D(w,z) \ge (\tau_D)^\infty(w,z)$.

Definition 2.15: [43] *A subset* C_1 *of C in a PFG is said to be a DS, if for each vertex* $w \in C - C_1$ *there is the vertex* $z \in C_1$ *such that z dominates w. A DS* C_1 *of a PFG G = (C, D) is a MDS, if for each vertex* $w \in C_1, C - C_1$ *is not a DS of G*. The minimum cardinality among all DSs in G* is the DN of G*, abbreviated as* ∇_G^*.

Definition 2.16: [11] *A bipolar picture fuzzy set (BPPFS) on a nonempty set V is the collection* $S = \{u, \rho^P(u), \rho^N(u), \varphi^P(u), \varphi^N(u), \tau^P(u), \tau^N(u) : u \in V\}$, *where* $\rho^P : V \to [0,1]$, $\rho^N : V \to [-1,0]$, $\varphi^P : V \to [0,1]$, $\varphi^N : V \to [-1,0]$, $\tau^P : V \to [0,1]$ *and* $\tau^N : X \to [-1,0]$ *are the mappings with* $0 \le \rho^P(u) + \varphi^P(u) + \tau^P(u) \le 1$, $-1 \le \rho^N(u) + \varphi^N(u) + \tau^N(u) \le 0$.

Following [11], for each $u \in V$, $\rho^P(u)$ represents the positive membership degree, $\varphi^P(u)$ for the positive non-membership degree and $\tau^P(u)$ for the positive neutral degree. On the other hand, $\rho^N(u)$ denotes the negative membership degree, $\varphi^N(u)$ is the negative non-membership degree and $\tau^N(u)$ is a negative neutral degree. Alternatively, if $\rho^P(u) \ne 0$ (resp., $\rho^N(u) \ne 0$, $\tau^P(u) \ne 0$) while all the rest of the mappings are mapped to zero, then u has only a positive membership property (resp., negative membership property, positive neutral property) of the BPPFS. An element u has only the negative neutral property of a BPPFS if $\tau^N(u) \ne 0$ and the other maps are mapped to zero. Similarly, if $\varphi^P(u) \ne 0$ and all of the other mapping matched to zero, then u has a positive nonmembership property of a BPPFS. Finally, if all of the other mappings matched to zero except $\varphi^N(u) \ne 0$, then we say that u has a negative nonmembership property in a BPPFS.

Definition 2.17: [34] *A pair G* = (C, D) is said to be a BPPFG on G = (V, E), where* $C = \{\rho_C^+, \rho_C^-, \varphi_C^+, \varphi_C^-, \tau_C^+, \tau_C^-\}$ *is a BPPFS on V and* $D = \{\rho_D^+, \rho_D^-, \varphi_D^+, \varphi_D^-, \tau_D^+, \tau_D^-\}$ *is a bipolar picture fuzzy set (BPPFS) on* $E \subseteq V \times V$ *such that for every edge* $uv \in E$,

$$\rho_D^+(u,v) \le min(\rho_C^+(u), \rho_C^+(v))$$
$$\rho_D^-(u,v) \ge max(\rho_C^-(u), \rho_C^-(v))$$

$$\varphi_D^+(u,v) \le min(\varphi_C^+(u),\varphi_C^+(v))$$
$$\varphi_D^-(u,v) \ge max(\varphi_C^-(u),\varphi_C^-(v))$$
$$\tau_D^+(u,v) \ge max(\tau_C^+(u),\tau_C^+(v))$$
$$\tau_D^-(u,v) \le min(,\tau_C^-(u),,\tau_C^-(v))$$

satisfying

$$0 \le \rho_D^+(uv) + \varphi_D^+(uv) + \tau_D^+(uv)(uv) \le 1$$
$$-1 \le \rho_D^-(uv) + \varphi_D^-(uv) + \tau_D^-(uv) \le 0.$$

We refer [40] for useful terminologies related to dominations in BPFGs. For more on domination in PFGs, one may consult [43, 44].

3. Dominations in Bipolar Picture Fuzzy Graphs

Recently, we (the first and the third authors (with Faiz)) have introduced the notions of the bipolar picture fuzzy sets (BPPFSs) [11]. We (the first and the third authors (with Ali)) have also been explored the concepts of the bipolar picture fuzzy graphs (BPPFGs) in [34]. In this section, we introduce the dominations and total dominations in BPPFGs. We also present the concepts of minimal dominating, total dominating sets etc along with few of their important relationships. The terms status and structurally equivalent are also discussed in the setting of BPPFGs.

Definition 3.1: *A vertex cardinality or the order of a BPPFG G* = (C, D) denoted by O (G*) = (O⁺ (G*), $O^-(G^*)$) can be defined as* $O^+(G^*) = \sum_{v\in V}(\rho_C^+(v),\varphi_C^+(v),\tau_C^+(v))$ *and* $O^-(G^*)\sum_{v\in V}(\rho_C^-(v),\varphi_C^-(v),\tau_C^-(v))$.

Definition 3.2: *The edge cardinality or the size of a BPPFG G* = (C, D) is denoted by S(G*) =* $(S^+(G^*),S^-(G^*))$ *and is defined as* $S^+(G^*) = \sum_{uv\in E}(\rho_C^+(uv),\varphi_C^+(uv),\tau_C^+(uv))$ *and* $S^-(G^*) = \sum_{uv\in E}(\rho_C^-(uv), \varphi_C^-(uv),\tau_C^-(uv))$.

Example 3.3: *The vertex cardinality or the size of the BPPFG shown in Figure 1(b) is* $O(G^*) = (O^+(G^*),O^-(G^*)) = ((1.4,1.4,1.1),(-0.9,-0.9,-1.0))$. *Similarly, the edge cardinality of BPPFG shown in Figure 1(b) is* $S(G^*) = (S^+(G^*),S^-(G^*)) = ((0.8,0.36,1.6),(-0.26,-0.75,-1.2))$.

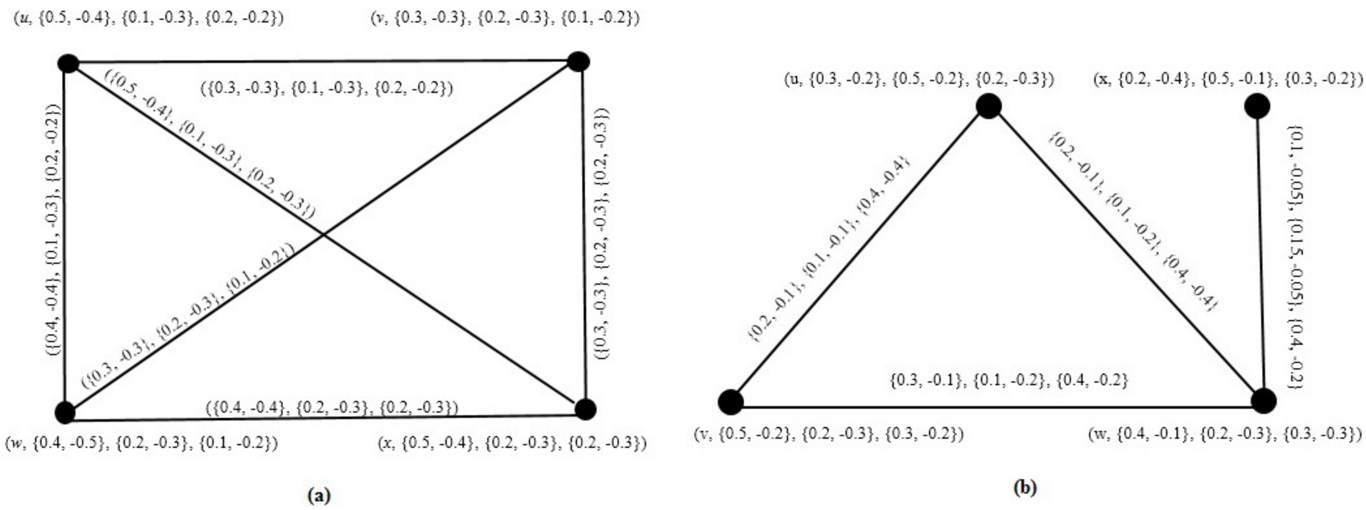

Figure 1: Bipolar Picture Fuzzy Graphs

Definition 3.4: *A path p in a BPPFG, G* = (C, D) is a sequence of different vertices* $w_0, w_1, w_2, \ldots, w_n$ *such that* $(\rho_D^+(w_{i-1}, w_i), \varphi_D^+(w_{i-1}, w_i), \tau_D^+(w_{i-1}, w_i)) > 0$ *and* $(\rho_D^-(w_{i-1}, w_{i)}, \varphi_D^-(w_{i-1}, w_i), \tau_D^-(w_{i-1}, w_i)) < 0$*,* $i = 1, 2, \ldots, n$*. Where n is the length of the path.*

Definition 3.5: *If any two nodes w and z are connected through the path of length n in a BPPFG G* = (C, D) such as* $p : w_0, w_1, w_2, \ldots w_{n-1}, w_n$*, then* $\rho_D^+(w,z), \varphi_D^+(w,z)$*,* $\tau_D^+(w,z)$ $\rho_D^-(w,z), \varphi_D^-(w,z)$ *and* $\tau_D^-(w,z)$ *can be described as*

$$\rho_D^+(w,z) = min(\rho_D^+(w_0, w_1), \rho^+(w_1, w_2), \ldots \rho^+(w_{k-1}, w_n))$$
$$\rho_D^-(w,z) = max(\rho_D^-(w_0, w_1), \rho_D^-(w_1, w_2), \ldots \rho_D^-(w_{k-1}, w_n))$$
$$\varphi_D^+(w,z) = min(\varphi_D^+(w_0, w_1), \varphi_D^+(w_1, w_2), \ldots \varphi_D^+(w_{k-1}, w_n))$$
$$\varphi_D^-(w,z) = max(\varphi_D^-(w_0, w_1), \varphi_D^-(w_1, w_2), \ldots \varphi_D^-(w_{k-1}, w_n))$$
$$\tau_D^+(w,z) = max(\tau_D^+(w_0, w_1), \tau_D^+(w_1, w_2), \ldots \tau_D^+(w_{k-1}, w_n))$$
$$\tau_D^-(w,z) = min(\tau_D^+(w_0, w_1), \tau_D^-(w_1, w_2), \ldots \tau_D^-(w_{k-1}, w_n)).$$

Definition 3.6: *Let* $(\rho_D^+)^\infty(w,z)$*,* $(\varphi_D^+)^\infty(w,z))$*,* $(\tau_D^+)^\infty(w,z))$*,* $(\rho_D^-)^\infty(w,z))$*,* $(\varphi_D^-)^\infty(w,z))$ *and* $(\tau_D^-)^\infty(w,z))$ *be the strength of connectedness between the two nodes w and z of a PFG G*. Then* $(\rho_D^+)^\infty(w,z))$*,* $(\varphi_D^+)^\infty(w,z))$*,* $(\tau_D^+)^\infty(w,z))$ $(\rho_D^-)^\infty(w,z)),(\varphi_D^-)^\infty(w,z))$ *and* $(\tau_D^-)^\infty(w,z))$ *are defined as follows.*

$$(\rho_D^+)^\infty(w,z)) = max(\rho_D^+)^i(w,z) : i = 1,2,3....,n$$
$$(\rho_D^-)^\infty(w,z)) = min(\rho_D^-)^i(w,z) : i = 1,2,3....,n$$
$$(\varphi_D^+)^\infty(w,z)) = max(\varphi_D^+)^i(w,z) : i = 1,2,3....,n$$
$$(\varphi_D^-)^\infty(w,z)) = min(\varphi_D^-)^i(w,z) : i = 1,2,3....,n$$
$$(\tau_D^+)^\infty(w,z)) = min(\tau_D^+)^i(w,z) : i = 1,2,3....,n$$
$$(\tau_D^-)^\infty(w,z)) = max(\tau_D^-)^i(w,z) : i = 1,2,3....,n.$$

Definition 3.7: *We call an edge (w, z) a strong edge in a BPPFG G* = (C, D), if*

$$\rho_D^+(w,z) \geq (\rho_D^+)^\infty(w,z); \rho_D^-(w,z) \leq (\rho_D^-)^\infty(w,z)$$
$$\varphi_D^+(w,z) \geq (\varphi_D^+)^\infty(w,z); \varphi_D^-(w,z) \leq (\varphi_D^+)^\infty(w,z)$$
$$\tau_D^+(w,z) \leq (\tau_D^+)^\infty(w,z); \tau_D^-(w,z) \geq (\tau_D^-)^\infty(w,z).$$

Definition 3.8: *Let G* = (C, D) be a BPPFG and* $u, x \in C$*. We call a vertex u dominates x, if there exists a strong edge between them.*

Remark 3.9: *Evidently, domination is a symmetric relation on C such as for* $u, x \in C$*, if u dominates x then x dominates u and vice versa. Moreover, if* $x \in C$*, then N(x) consists of all the vertices in C which are dominated by the vertex x. Similarly, if*

$$\rho^+(u,x) < (\rho_D^+)^\infty(u,x); \rho^-(u,x) > (\rho_D^+)^\infty(u,x)$$
$$\varphi^+(u,x) < (\varphi_D^+)^\infty(u,x); \varphi^+(u,x) > (\varphi_D^+)^\infty(u,x)$$
$$\tau^+(u,x) > (\tau_D^+)^\infty(u,x); \tau^+(u,x) < (\tau_D^+)^\infty(u,x)$$

for all $u, x \in C$, then the only DS of *G** is *C*.

Example 3.10: *The edges (u, v), (u, w), (w, x) and (u, x) are the strong edges in Figure 1(a) while the edges (w, v) and (x, v) are not the strong edges. Clearly, a vertex u dominates v, w and x vertices.*

Similarly, the vertex w dominates v as there is a strong edge between these two vertices. However, a vertex v does not dominate w and x as there are no strong edges between them.

Definition 3.11: *The open neighborhood of* u_1 *in a BPPFG abbreviated by* $N(u_1)$ *is defined as* $N(u_1) = \{u_2 \in V: \rho^+(u_1,u_2) > 0, \varphi^+(u_1,u_2) > 0, \tau^+(u_1,u_2) > 0$ or $\rho^-(u_1),u_2) < 0, \varphi^-(u_1),u_2) < 0, \tau^-(u_1),u_2) < 0\}$. *However, a vertex* u_2 *is said to be a strong neighbor of* u_1*, if an arc* (u_1, u_2) *is a strong arc, the set of all strong neighbors of* u_1 *is called the strong neighborhood of* u_1 *and is abbreviated by* $N_s(u_1)$*. Similarly,* $N_s[u_1] = N_s(u_1) \cup \{u_1\}$ *is the closed strong neighborhood of* u_1.

Definition 3.12: *A vertex* $u \in C$ *of a BPPFG,* $G^* = (C, D)$ *is an isolated vertex, if* $\rho_D^+(w,z) = 0$, $\rho_D^-(w,z) = 0$, $\varphi_D^+(w,z) = 0$, $\varphi_D^-(w,z) = 0$, $\tau_D^+(w,z) = 0$, $\tau_D^-(w,z) = 0$, *for all* $v \in C$, $w \neq z$*. Alternatively,* $N(w) = \varnothing$ *implies that there does not exist any neighborhood of z. Hence it is clear that the isolated vertex can never dominates any vertex in G*. If w dominates z, then z dominates w. Thus the domination possesses the symmetric relation.*

Example 3.13: *In Figure 1(a), the edges (u, v), (u, w), (w, x) and (u, x) are the strong edges while the edges (w, v) and (x, v) are not the strong edges. Here, the vertex u dominates the vertices v, w and x and hence* $N(w) = \{u,v,x\}$*. The strong neighbors of vertex w are u, x. Hence* $N_S[w] = \{u,x\} \cup w = \{u,w,x\}$.

Definition 3.14: *Let G = (C, D) be a BPPFG defined on G* = (V, E) and S be the set of vertices. Then the vertex v is said to be a (bipolar picture fuzzy) private neighbor of* $u \in S$ *with respect to S, if* $N[v] \cap S = \{u\}$*. The (bipolar picture fuzzy) neighborhood of* $u \in S$ *with respect to S is* $PN[u,S] = \{v : N[v] \cap = \{u\}\}$*. Alternatively,* $PN[u,S] = N[u] - N[S - \{u\}]$*. Also, if there is a strong edge between two vertices u and v, then we call v a private strong neighbor of* $u \in S$ *and is denoted by PNS.*

Example 3.15: *From Figure 1(b), we have* $N(u) = \{u,v\}$, $N(v) = \{u,w\}$, $N(x) = \{w\}$ *and* $N(w) = \{u,v,x\}$*. However, in Figure 1(a), the strong neighbors of w is the set* $N_s(w) = \{u,x\}$*. The set* $N(v) = \{u,x\}$ *is a neighborhood of v but not a strong neighborhood of v. Similarly, the close strong neighbors of w is the set* $N_s[w] = \{u,x,w\}$*. The private neighbor with respect to S = V is* $PN(u,S) = \{v : N_S(v) \cap S = \{u\}\}$*. The strong neighborhood of v is* $N_S(v) = \{u\}$*. Also,* $N_S(v) \cap S = \{u\} \cap \{u,v,w,x\} = \{u\}$*. Hence v is a private strong neighborhood of u.*

Definition 3.16: *A subset* C_1 *of C is said to be a dominating set (DS) in BPPFG, if for each w not in* C_1 *there exists* $z \in C_1$ *such that w dominates z. A DS* C_1 *in BPPFG is a minimal dominating set (MDS), if there doesn't exist any proper subset of* C_1 *which is a DS. By a lower domination number (LDN) of G* abbreviated as* $L_d(G^*)$*, we mean a minimum cardinality among all MDS. Similarly, an upper domination number (UDN) of G* abbreviated as* $U_d(G^*)$ *is the maximum cardinality among all MDS.*

In other words, the minimum cardinality of a dominating set (DS) in a BPPFG *G** is said to be a domination number (DN) of *G**, abbreviated as $\lambda(G^*)$. The DS which contains a minimum vertices is called a minimal dominating set (MDS).

Definition 3.17: *The DS* $\lambda(G^*)$ *of a BPPFG G* is the cardinality of a MDS in G* i.e.,* $\lambda(G^*) = |V_\rho| + |V_\varphi| + |V_\tau|$.

Example 3.18: *Let* $C = \{u,v,w,x\}$ *be the set of vertices in a graph shown in Figure 1(a). Let* $C_1 = \{u,v\}$ *be the DS lying in C. Let {w, x} be set other than* C_1 *such that each of its vertex dominates at least one vertex in C which implies that* C_1 *is a DS.*

Example 3.19: *Let* $C = \{u,v,w,x\}$ *be the set of vertices in a Figure 1(b). Here, the DSs are* $\{u,v,x\}$ *and* $\{u,w,x\}$ *while the sets* $\{u,v\}$, $\{w,x\}$, $\{u,x\}$, $\{w,v\}$, $\{u,w\}$, $\{u,v,w\}$ *and* $\{u,w,x\}$ *are not DSs. Here, MDS is* $\{u,v,x\}$.

Theorem 3.20: *A DS* C_1 *of a BPPFG G* = (C, D) is a MDS if and only if for any* $d \in C_1$ *satisfies one of the followings.*

(i) *d* is not a strong neighbor of any vertex in C_1 (ii) There exists a vertex $v \in C - C_1$ with $N(v) \cap C_1 = d$.

Proof. Let C_1 be a MDS in a BPPFG G^*. Clearly, for any $d \in C_1$, the set $C_1 - \{d\}$ is not a DS and hence $v \in \{C - \{C_1 - \{d\}\}\}$ is not dominated by any vertex of $C_1 - \{d\}$. However, if $v = d$, then v is not a strong neighbor of any other vertex lying in C_1. But, if $v \neq d, v$, then v is not dominated by $C_1 - \{d\}$, but if dominated by C_1, then there is a vertex v which is the only strong neighbor to d. It implies $N(v) \cap C_1 = d$.

Conversely, let C_1 be a DS such that for each vertex $d \in C_1$, one of the two given conditions does hold. Let C_1 is not a MDS. It means that there is a vertex $d \in C_1$ such that $C_1 - d$ is a DS. Which implies that d is a strong neighbor to a minimum of one of the vertex in $C_1 - d$ and consequently the condition (1) fails. Similarly, if $C_1 - d$ is a DS, then each vertex in $C - C_1$ become a strong neighbor to at least one of the vertex in the set $C_1 - d$. Then, the second condition violated, which contradicts our hypothesis i.e., at least one of the conditions holds true. Hence C_1 is a MDS.

Theorem 3.21: *Let $G^* = (C, D)$ be a BPPFG with no any isolated vertex and C_1 is a MDS. Then $C - C_1$ is a DS of G^*.*

Proof. Let C_1 be a MDS and $d \in C_1$. Since G^* doesn't have any isolated vertex implies there is a vertex $v \in N(d)$. Then, certainly v is dominated by at least one of the vertex in $C_1 - d$ which implies $C_1 - d$ is a DS. Following Theorem.1, $v \in C - C_1$. Hence each vertex in C_1 is dominated by at least one of the vertex in set $C - C_1$. Thus, $C - C_1$ is a DS.

Definition 3.22: *Two vertices of a BPPFG $G^* = (C, D)$ are independent, if there doesn't exist any strong edge between them. A subset F of C is an independent set in a BPPFG G^*, if it satisfies the followings.*

$$\rho_F^+(w,z) < (\rho_D^+)^\infty(w,z)$$
$$\rho_F^-(w,z) > (\rho_D^-)^\infty(w,z)$$
$$\varphi_F^+(w,z) < (\varphi_D^+)^\infty(w,z)$$
$$\varphi_F^-(w,z) > (\varphi_D^-)^\infty(w,z)$$
$$\tau_F^+(w,z) > (\tau_D^+)^\infty(w,z)$$
$$\tau_F^-(w,z) < (\tau_D^-)^\infty(w,z)$$

for all $w, z \in F$.

Definition 3.23: *An independent set $F \subseteq C$ of BPPFG $G^* = (C, D)$ is a maximal independent, if for each $w \in C - F$, the set $F \cup \{w\}$ is not an independent. An independent set $F \subseteq C$ in a BPPFG $G^* = (C, D)$ is said to be a maximal independent, if for each $z \in C - F$ the set $F \cup \{w\}$ is not an independent. The minimum (resp., maximum) cardinality among all the maximal independent sets is said to be a lower (resp., upper) independent number of a BPPFG G^*, abbreviated as $i_d(G^*)$ (resp., $I_d(G^*)$).*

Theorem 3.24: *Every maximal independent set in a BPPFG $G^* = (C, D)$ is a MDS.*

Proof. Assume that F is a maximal independent set of a BPPFG. By assumption, F is a DS and let F is not a MDS. Consequently, there must exists at least one of the vertex $v \in F$ such that $F - \{v\}$ is a DS. However, if $F - \{v\}$ dominates $C - \{F - \{v\}\}$, then at least one of the vertex in $F - \{v\}$ is necessary a strong neighbor of u. Which violates the fact that F is an independent set of G^*. Hence F is necessarily a MDS.

Remark 3.25: *An independent dominating set of a BPPFG G^* is both the minimal and maximal independent set. Alternatively, any maximal independent set F_1 in G^* is the independent dominating set of G^*.*

Corollary 3.26: *Let G^* be a BPPFG with no any isolated vertex. Then, $L_d(G^*) \leq \dfrac{O(G^*)}{2}$.*

Proof. Consider a BPPFG G^* without isolated vertex. Then, it has two disjoint DSs which implies $L_d(G^*) \leq \frac{O(G^*)}{2}$.

Theorem 3.27: *In any BPPFG G* defined on G = (V, E)* $L_d + \overline{L_d} \leq 2O(G^*)$, *where* $\overline{L_d}$ *is the lower domination number of* G^* *and equality hold iff*

$$
\begin{aligned}
&0 < \rho_D^+(w,z) < min(\rho_C^+(w), \rho_C^+(z))\\
&0 < \rho_D^-(w,z) > max(\rho_C^-(w), \rho_C^+(z))\\
&0 < \varphi_D^+(w,z) < min(\varphi_C^+(w), \varphi_C^+(z))\\
&0 < \varphi_D^-(w,z) > max(\varphi_C^-(w), \varphi_C^-(z))\\
&0 < \tau_D^+(w,z) > max(\tau_C^+(w), \tau_C^+(z))\\
&0 < \tau_D^-(w,z) < min(\tau_C^-(w), \tau_C^-(z)
\end{aligned}
$$

for all $w, z \in C$.

Proof. The inequality is trivial, further $L_d = O(G^*)$ iff

$$
\begin{aligned}
&\rho_D^+(w,z) < min(\rho_C^+(w), \rho_C^+(z))\\
&\rho_D^-(w,z) > max(\rho_C^-(w), \rho_C^-(z))\\
&\varphi_D^+(w,z) < min(\varphi_C^+(w), \varphi_C^+(z))\\
&\varphi_D^+(w,z) < max(\varphi_C^-(w), \varphi_C^-(z)\\
&\tau_D^+(w,z) > max(\tau_C^+(w), \tau_C^+(z))\\
&\tau_D^+(w,z) < min(\tau_C^-(w), \tau_C^-(z))
\end{aligned}
$$

for all $w, z \in C$ and $L_d = O(G^*)$ iff

$$
\begin{aligned}
&min(\rho_C^+(w), \rho_C^+(z)) - \rho_D^+(w,z) < min(\rho_C^+(w), \rho_C^+(z))\\
&max(\rho_C - +(w), \rho_C^-(z)) - \rho_D^-(w,z) > max(\rho_C^-(w), \rho_C^-(z))\\
&min(\varphi_C^+(w), \varphi_C^+(z)) - \varphi_D^+(w,z) < min(\varphi_C^+(w), \varphi_C^+(v))\\
&max(\varphi_C^-(w), \varphi_C^-(z)) - \varphi_D^-(w,z) > max(\varphi_C^-(w), \varphi_C^-(v))\\
&max(\tau_C^+(w), \tau_C^+(z)) - \tau_D^+(w,z) > max(\tau_C^+(w), \tau_C^+(v))\\
&min(\tau_C^-(w), \tau_C^-(z)) - \tau_D^-(w,z) < min(\tau_C^-(w), \tau_C^-(v))
\end{aligned}
$$

for all $w, z \in C$ which is equivalent to $\rho_D^+(w,z)0, \rho_D^-(w,z) < 0, \varphi_D^+(w,z) > 0, \rho_D^-(w,z) < 0, \tau_D^+(w,z) > 0$ and $\rho_D^-(w,z) < 0$, hence $C_l + L_d^- = 2O(G^*)$ iff

$$
\begin{aligned}
&0 < \rho_D^+(w,z) < min(\rho_C^+(w), \rho_C^+(z))\\
&0 < \rho_D^-(w,z) > max(\rho_C^-(w), \rho_C^+(z))\\
&0 < \varphi_D^+(w,z) < min(\varphi_C^+(w), \varphi_C^+(z))\\
&0 < \varphi_D^-(w,z) > max(\varphi_C^-(w), \varphi_C^-(z))\\
&0 < \tau_D^+(w,z) > max(\tau_C^+(w), \tau_C^+(z))\\
&0 < \tau_D^-(w,z) < min(\tau_C^-(w), \tau_C^-(z))
\end{aligned}
$$

Corollary 3.28: *Let G* be a BPPFG defined on the graph G, and both G* and* $\overline{G^*}$ *having no isolated vertex. Then,* $L_d(G^*) + \overline{L_d(G^*)} \le O(G^*)$ *and the equality holds iff* $L_d(G^*) = \overline{C_L} = \frac{O(G^*)}{2}$.

Proof. By Corollary 1, $L_d(G^*) \le \frac{O(G^*)}{2}$ and also $\overline{L_d(G^*)} \le \frac{O(G^*)}{2}$ which implies

$$L_d(G^*) + \overline{L_d(G^*)} \le \frac{O(G^*)}{2} + \frac{O(G^*)}{2} = O(G^*)$$

that is,

$$L_d(G^*) + \overline{L_d(G^*)} \le O(G^*)$$

if $L_d(G^*) = \overline{L_d(G^*)} = \frac{O(G^*)}{2}$ then $L_d(G^*) + \overline{L_d(G^*)} = O(G^*)$ conversely, let $L_d(G^*) + \overline{L_d(G^*)} = O(G^*)$. Then, by corollary 1, $L_d(G^*) \le \frac{O(G^*)}{2}$ and $\overline{L_d(G^*)} \le \frac{O(G^*)}{2}$ If either $L_d(G^*) < \frac{O(G^*)}{2}$ or $\overline{L_d(G^*)} < \frac{O(G^*)}{2}$ then both $L_d(G^*) + \overline{L_d(G^*)} < O(G^*)$, which is a contradiction. Hence,

$$L_d(G^*) = \overline{L_d(G^*)} = \frac{O(G^*)}{2}$$

Definition 3.29: *Let G*= (C, D) be a BPPFG without isolated vertex. Then the set* C_1 *is called a total dominating set (TDS), if for each vertex* $w \in C$ *there is a vertex* $z \in C_1$ *such that* $w \ne z$ *and w dominates z.*

Definition 3.30: *By a minimal total dominating set (MDS) in BPPFGs, we mean a total dominating set (TDS)* C_1 *of a BPPFG such that no proper subset of* C_1 *is a total dominating set (TDS) except* C_1*. The minimum (resp., maximum) cardinality of a MDS is said to be a lower (resp., upper) total dominating number (LTDN) of G*, and is abbreviated as* $L_{td}(G^*)$*. The maximum cardinality of a minimal total dominating set is the upper total dominating number (UTDN) of G*, denoted by* $U_{td}(G^*)$.

Example 3.31: *The edges* (u,v), (v,z), (u,w) *and* (x,z) *of a BPPFG shown in Figure 2 are the strong edges. The sets* $\{u,v,z\}$, $\{u,v,w,z\}$ *and* $\{u,v,x,z\}$ *are the TDSs. Also,* $\{u,v,z\}$ *is a MDS and the cardinalities*

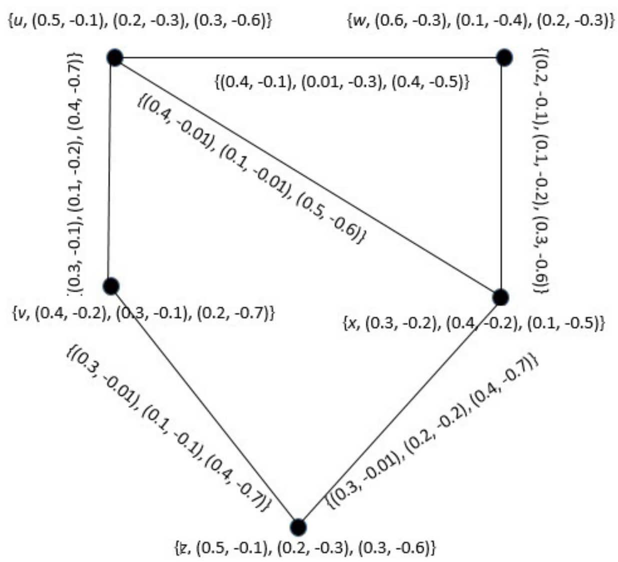

Figure 2: Bipolar Picture Fuzzy Graph

of the vertices u, v, and z are 1.55, 1.45 and 1.55, respectively. Hence the lower and upper total dominations of the BPPFG G are* $L_{td}(G^*) = 1.45$ *and* $U_{td}(G^*) = 1.55$*, respectively.*

Theorem 3.32: *In a BPPFG G*,* $L_{td}(G^*) = O(G^*)$ *if and only if each of its vertex has a unique neighbor.*

Proof. If every vertex of G^* has a unique neighbor, then the vertex set C is the only TDS in G^*. Hence $L_{td}(G^*) = O_B(G^*)$. Conversely, let $L_{td}(G^*) = O(G^*)$. Now, if there is a vertex w whose neighbors are u and x, then the set $C - \{w\}$ is the TDS of G^*. And $L_{td}(G^*) \le O(G^*)$, a contradiction. Hence each vertex in G^* has a unique neighbor.

Definition 3.33: *A subset of the vertex set C of a BPPFG G* is called a status S, if every vertex* $w, z \in S$ *has the property that the vertices in* $C - S$ *dominated by w is equal to the set of nodes in* $C - S$ *dominated by z. Hence all of the vertices in a status S dominate the same set of vertices outside of the status. It is notable that every status S must have at least two vertices.*

Since we know that the DS which contains a minimum vertices of a BPPFG is a MDS. We interrelate the terms minimum dominating set, independent dominating set and the status in BPPFGs in the below theorem.

Theorem 3.34: *If MDS S of a nontrivial connected BPPFG G* is a status of G*, then S is an independent dominating set of cardinality* 2.

Proof. Let S be a MDS of G^*, which is a status. Since G^* is connected and has no isolated vertex, there must be at least one of the vertex $w \in C - S$. Since S is a MDS, w must be adjacent to at least one vertex in S. But, as S is a status, every vertex of S must be adjacent to w. Furthermore, each vertex in S must be adjacent to every vertex in $C - S$. Since S is status, $|S| \ge 2$. Assume that $|S| \ge 3$, and let $z \in S$ and $w \in C - S$. Since S is a status it implies z is adjacent to every vertex in $C - S$ and w is adjacent to every other vertex in S. Hence $\{w, z\}$ is a dominating set, which Contradicts the minimality of S. Therefore, $|S| = 2$. However, if z is adjacent to w, then z is a dominating set of G^*, again contradicting the minimality of S. Consequently, $|S| = 2$ is an independent set.

Definition 3.35: *Two vertices w and z are said to be a structurally equivalent, if either* $N_s(w) = N_s(z)$ *or* $N_s[w] = N_s[z]$*. A set S is said to be a structurally equivalent set whenever every two vertices in S are structurally equivalent.*

Example 3.36: *Clearly, the vertices x and w are structurally equivalent in Figure 1(a), as* $N_s[x] = N_s[w] = \{u, w, x\}$.

Corollary 3.37: *Let G* be a connected BPPFG and S be a minimum dominating set which is also structurally equivalent. Then the set S consists of two independent vertices each of which has degree* $(O(G^*) - 1)$.

Definition 3.38: *A total domination number (TDN) of G* abbreviated as* $\lambda_t(G^*)$*, is the minimum cardinality of a TDS of G*.*

Theorem 3.39: *If G* is connected BPPFG with* $O(G^*) \ge 3$*, then* $\lambda_t(G^*) \le \dfrac{2O(G^*)}{3}$.

Proof. Let S be a total minimum DS of G^*. By minimality, each $w \in S$ either has a private neighborhood or induced subgraph $\{S - \{w\}\}$ contains an isolated vertex. Let $P = \{w \in S : Pn[w, S] \ne 0\}$. Let B be the set of isolated vertices in P and $A = P - B$. Further to this, let T be a minimal set of vertices of $S - P$ such that each vertex of B is adjacent to some vertex of T. We note that $|T| \le |B|$. Finally, let $B = S - \{P \cup T\}$. Then, by definition of I, $\lambda_t(G^*)$ is an induced bipolar picture fuzzy subgraph $< I > = |I|$ and hence $< I > = kK_2$, $k \ge 0$.

Let $a_i b_i$, $1 \le i \le k$ be the distinct edges of $< I >$. The connectivity of G^* implies that each a_i is adjacent to some other vertex x_i. If $x_i \in P \cup T$, then $S - \{B_i\}$ would be a smaller TDS than the set S.

Hence $x_i \in C-S$. If $x_i = x_j$ for $i \neq j$, then $S-\{b_i,b_j\}\cup\{x_i\}$ which predicts the same contradiction. By the definition of I each x_i is adjacent to at least two vertices of S, and by the definition of P there are at least $|P|$ vertices of $C-S$ that are adjacent to exactly one vertex of S. Thus $|P|+k \leq |C-S|$, that is $|A|+|B|+k \leq O(G^*)-\lambda_t(G^*)$. So, $\lambda_t(G^*)=|A|+|B|+|T|+|I| = (|A|+|B|+k)+(|T|+k)$. Since $|T| \leq |A|+|B|$, we have $\lambda_t(G^*) \leq 2(|A|+|B|+k) \leq 2(O(G^*)-\lambda_t(G^*))$. Hence $\lambda_t(G^*) \leq 2(O(G^*)-\lambda_t(G^*)$. Hence $\lambda_t(G^*) \leq \frac{2O(G^*)}{3}$.

4. Application of Domination in BPPFGs towards Social Networks

Modeling by using graphs has diverse applications towards different fields of sciences like computer science, physics, mathematics, chemistry, biology, social sciences etc. In the study of social networks, it has been observed that there is the relation between two people in a group, and it is important to conclude that who is more social, influential or dominating in a group. We can depict this scenario through the graph. We can construct a graph in which the vertex u stands for each person in a particular group. The undirected edges in the graph represent the relationship between the two persons at vertices u and v. In such types of graphs there are no need of multiple edges and loops. The edge between any two vertices shows that there is a relationship between the two persons. Since each vertex is of equal importance in the classical graphs theory, so it is not possible to graph the social networks model, accurately. Moreover, all of social units (individual or organization) in social groups must be given an equal importance in the classical graph theory. However, in the real life the situation is very different. Similarly, every edge (relationship) in the classical graphs has an equal strength. Moreover, in the classical graphs theory the relationships between any two social units have equal strengths. However, in real life it is not realistic. Thus, the acquaintance or influence of the person has fuzzy boundaries and thus it would be better to represent such situations through the fuzzy graphs. In the fuzzy acquaintanceship or influential graph, each vertex stands for the person and its membership value is the strength of his influence within the social network. No doubt, since picture fuzzy set is the most developed form of the fuzzy sets, domination in BPPFGs would demonstrate the best results as compared to the other form of the fuzzy graphs such as intuitionistic fuzzy graphs, bipolar fuzzy graphs etc.

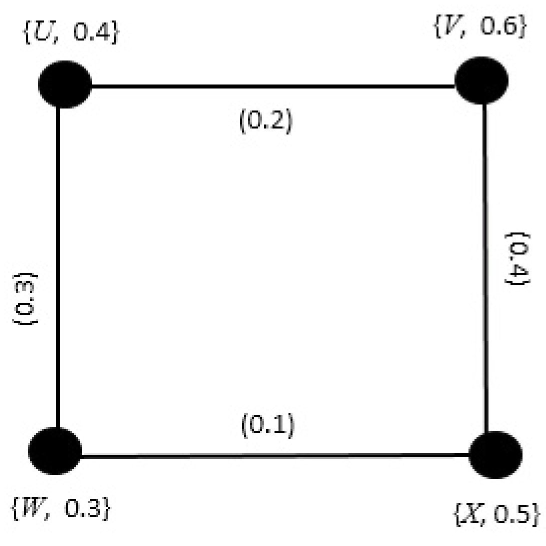

Figure 3: Fuzzy influence graph

4.1 Fuzzy influence graph

We take a fuzzy influential graph of some social network which is shown in Figure 3. In which the nodes represent the degree of the level of influence of a person within the social group. The degree of the level of influence is expressed in its membership value. Degree of membership states that how much a person is influential e.g., X has 60% level of influence within the group. The edges of a graph describe the influential level of one person on the other person. The membership degree of edges can be considered in terms of positive percentage e.g., Y has 40% influential level with X and so. However, U dominates the other that is why it is most busy and hence influential than the others.

4.2 Bipolar fuzzy influence graph

As well as concerned about FGs we are unable to find the negative or positive levels of influence. But practically, the influence of a person may be positive or negative. Suppose if a persona A and B belong to a social network but having not a good relationships between them then the influence of one person between them is negative. We can depict such circumstances through the bipolar fuzzy influential graph. Hence we can only describe such scenario through bipolar fuzzy influential graph. Let us consider a bipolar fuzzy influential graph of some social group shown in Figure 4. In which the nodes are reflecting the degree of the level of influence of a person belongs to a social group and the edges represent the degree of influential levels among the persons. Degree of positive membership can be interpreted as how much a person influential while a negative membership tells us that how much a person losses the level of influence, X has 50% level of influence within the group but he losses 20% level in the same group. Edges of the graph reflect the influence of one person with the other persons in the group. The positive and negative memberships degrees of edges describes the percentage of positive and negative influences for instance e.g., X is acquainted 10% with W and W is not acquainted 10% with X. Again, since there exist strong edges between all the vertices, but U dominates the others and hence most influential and busy in the group.

4.3 Bipolar picture fuzzy influence graph

In FG and BPFGs the neutral term is not involved. In practical, neutrality has its own importance which can be described through PFGs. Here, we analyze the influence of the person in a group based on the domination of a BPPFG model shown in Figure 1(a).

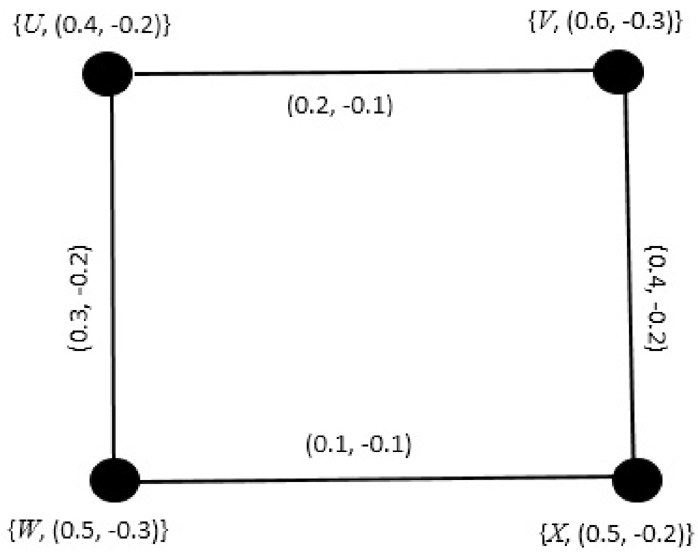

Figure 4: Bipolar fuzzy influence graph

The level of the influence of any person is referred to its membership (positive, negative), non-membership (positive, negative) and neutral membership (positive, negative) values. The degree of the membership (positive, negative) can be interpreted as a power of a person to influence or true influence (gaining, loosing). The degree of non-membership (positive, negative) is referred to how much power he losses to influence or false influence (gaining, loosing). Also, the degree of neutral membership (positive, negative) reflects the person's with neutral influence (gaining, loosing). In neutral influence, the person does not know well about someone, but he finds him around infrequently.

Let us assume that Figure 1(a) represents the group of four people in some social network. Then, x has a 50% (resp., looses influence 40%) true influence. Similarly, he has 20% (resp., looses 30%) false influence but he gains (resp., looses) 20% (resp., looses 30%) neutral influence within the social network. However, the edges of a graph in Figure 1(a) represent the influence between any two persons in the group. The degrees of a membership (positive and negative), non-membership (positive and negative) and neutral membership (positive and negative) of the edges show the percentage of the power of influence (gaining, loosing) or true influence, how much he has a false influence (gaining, loosing) and neutral influence or non-influence (gaining, loosing). Clearly, the edges (u,v), (u,w), (w,x) and (u,x) are the strong edges. One can easily deduce that the vertex u dominates v, w and x and hence a person u is the most influential than the persons at vertices v, w and x in a group.

5. Discussion and Conclusion

Graph theory has many useful applications in different branches of science specifically in computer science, physics, chemistry, operation research, economics etc. Mainly, the problems related to the graphs contain uncertainties, it is most appropriate to deal these problems through the fuzzy graphs. From last few decades, the fuzzy graphs have shown an enormous applications towards modern sciences and technologies, particularly in the fields of computer sciences. In this work, we have introduced the concepts of dominations of the most generalized form of the fuzzy graphs named bipolar picture fuzzy graphs (BPPFGs). Initially, we have introduced different terms related to the dominations of BPPFGs such as vertex cardinality, edge cardinality, strong edge, neighbors, strong neighbor of vertex, private neighborhood, independent sets, dominating sets etc. Then, we have presented some important characterizations of the dominations in BPPFGs which are based on minimal dominating sets and maximal independent sets. During this study, we have also investigated the lower and upper domination numbers of BPPFGs. In addition, we have discussed the concepts of total domination along with few of its characteristics in BPPFGs. We have also described the terms status and structurally equivalent in the setting of BPPFGs. At the end, we have presented an application of the domination in a BPPFG. We expect that dominations in BPPFGs would be most beneficial for solving many problems related to computer sciences such as networks, social media etc. One may extend this study to introduce more generalized forms of the domination in BPPFGs such as double domination, distance 2 domination etc. This study can be extended towards spherical and T-spherical fuzzy sets.

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