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A fuzzy transmission model analysis of corruption dynamics

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Abstract

In this study, we propose a corruption model that incorporates fuzzy transmission dynamics. We consider a population of individuals, each characterized by a corruption level ranging from 0 to 1, representing their degree of engagement in corrupt practices. The transmission of corruption is not strictly binary but rather influenced by fuzzy logic, where individuals with higher corruption levels are more likely to influence others towards corruption. To analyze the dynamical behavior of the corruption model, we employ mathematical techniques from fuzzy systems theory and dynamical systems theory. We investigate how the corruption levels evolve over time, considering factors such as social interactions, institutional interventions, and individual tendencies. Through simulations and mathematical analysis, we explore various scenarios and observe interesting dynamical behaviors. These include the emergence of corruption clusters, the impact of anti-corruption measures on reducing corruption levels, and the potential for corruption to spread or decline based on different initial conditions and external factors. Our findings highlight the importance of considering fuzzy transmission dynamics in corruption models, as it provides a more realistic representation of corruption in society. This research contributes to a better understanding of the complexity of corruption and provides insights for policymakers and anti-corruption agencies in designing effective strategies to combat corruption and promote ethical behavior in society.

Key words and phrases. fuzzy logic; corruption dynamics; Fuzzy control; fuzzy transmission.

Mathematics Subject Classification (2010): 93C42; 03E72; 49K45

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1. Introduction

Corruption poses a significant challenge to societies worldwide [1], hindering economic development, eroding trust in institutions, and undermining governance. Accurately measuring corruption levels [2] is crucial for effective anti-corruption efforts and policy-making. However, traditional measurement approaches often struggle to capture the inherent complexity, ambiguity, and uncertainty associated with corruption assessments [3]. In this article, we propose a novel approach measuring corruption using fuzzy logic, aiming to enhance accuracy and effectively manage uncertainty.

Conventional corruption measurement methods often rely on quantitative indicators, such as surveys and statistical data. While these approaches provide valuable insights, they may fail to fully capture the nuanced and context-dependent nature of corruption, leading to limited accuracy and potential misinterpretation.

Fuzzy logic [4], a mathematical framework that allows for reasoning and decision-making in the presence of uncertainty, provides a promising alternative for measuring corruption. By incorporating linguistic variables, fuzzy sets, and fuzzy inference systems, fuzzy logic enables a more nuanced representation of corruption levels, accommodating the inherent vagueness and ambiguity associated with corruption assessments.

In this study, we propose a corruption model that incorporates fuzzy transmission dynamics to address the limitations of deterministic models [5–7]. By introducing fuzziness into the transmission process, we aim to capture the nuanced and uncertain nature of corruption propagation. Our approach recognizes that corruption is not a binary phenomenon but exists on a spectrum, with individuals exhibiting varying degrees of engagement in corrupt practices.

The foundation of our model is a population of individuals, each characterized by a corruption level ranging from 0 to 1. This continuous scale represents the extent of an individual's involvement in corruption, with higher values indicating a higher propensity for corrupt behavior. Unlike deterministic models, where transmission is strictly binary, our fuzzy model incorporates fuzzy logic, allowing for a more realistic representation of corruption transmission. Individuals with higher corruption levels are more likely to influence others towards corruption, reflecting the complex social dynamics at play.

To analyze the dynamical behavior of our corruption model, we draw upon mathematical techniques from fuzzy systems theory and dynamical systems theory. By applying these methodologies, we investigate how corruption levels evolve over time, considering various influencing factors. Social interactions play a crucial role in the spread of corruption, but institutional interventions and individual tendencies also contribute to the dynamics. By incorporating these factors, our model aims to capture the multifaceted nature of corruption dynamics in a more comprehensive manner.

Through simulations and mathematical analysis, we explore different scenarios to uncover interesting dynamical behaviors within our corruption model. These include the emergence of corruption, the impact of anti-corruption measures on reducing corruption levels, and the potential for corruption to either spread or decline based on different initial conditions and external factors. By examining these dynamics, we gain valuable insights into the underlying mechanisms driving corruption and the potential effectiveness of interventions aimed at curbing it.

The significance of our research lies in the incorporation of fuzzy transmission dynamics into the corruption model. By recognizing the uncertainty and variability inherent in corruption dynamics, we provide a more realistic representation of corruption in society. This enables a better understanding of the complexities involved and offers insights for policymakers and anticorruption agencies in designing targeted strategies to combat corruption and foster ethical behavior.

In the following sections, we present the methodology behind our fuzzy corruption model, discuss our findings from simulations and mathematical analysis, and conclude with implications and recommendations for combating corruption based on our research. By shedding light on the intricacies of corruption dynamics through a fuzzy lens, we contribute to the advancement of corruption modeling and the development of more effective anti-corruption strategies.

2. Literature Review

2.1. Indicators of Corruption

Corruption can manifest in various forms and contexts, and its indicators may vary depending on the specific situation or environment.Indicators of corruption include both objective and subjective measures. Objective indicators, such as the Corruption Perceptions Index and the World Bank's Control of Corruption indicator, are aggregate survey-based measures that provide valid assessments of overall corruption levels in different country contexts [8]. Subjective indicators should be cross-checked with objective indicators to ensure accuracy, even though they may have a more limited scope and availability [9]. Additionally, indicators capable of alerting the system to risk factors are important for corruption prevention. Italy has developed abnormal and contrast indicators to assess the effectiveness of prevention policies and identify potential corruption risks [10]. In the field of public procurement, objective proxy measures have been developed, including single bidding in competitive markets and composite scores of tendering'red flags' [11]. These indicators are based on publicly available records and are consistent over time and across countries, providing a reliable measure of high-level corruption [12].

2.2. Determinant of Corruption

Corruption is influenced by a variety of factors. One important determinant is the public attitude towards corruption. When people perceive corruption as acceptable and have previous experience with it, they are more likely to engage in petty corruption [13]. Another determinant is the relative pecuniary gain of being corrupted compared to not being corrupted [14]. Additionally, factors such as income levels, extent of primary schooling, rule of law, foreign direct investment, trade freedom, political rights, government size, and religious fractionalization also play a role in determining corruption levels [15]. Nevertheless, elements such as the abundance of natural resources, the size of the country's population, the prevailing religious tradition, ethnic fractionalization, and political stability appear not to exert a substantial influence on corruption. [16]. Overall, understanding these determinants can help policymakers develop effective anti-corruption policies [17].

2.3. The correlation between corruption and urban population density

Corruption can have a significant impact on population density in a given area [18]. Higher levels of corruption tend to discourage investment and economic growth, resulting in limited job opportunities and lower living standards. This can lead to a decrease in population density, as individuals may choose to migrate to areas with lower levels of corruption in search of better opportunities. Additionally, corruption can also directly affect population density by influencing the distribution of resources and infrastructure development. Therefore, areas with higher levels of corruption may have inadequate access to basic services such as education, healthcare, and transportation, leading to a decline in population density. Furthermore, population density itself can also contribute to corruption [19]. As stated in [20], Urban areas with high population density offer diminished social controls and increased opportunities for corruption. This means that as population density increases, the potential for corruption and unethical behavior among government officials and institutions also increases. Thus, the relationship between corruption and population density is complex and multifaceted, with corruption influencing population density and population density potentially exacerbating corrupt practices. Additionally, the presence of a higher population density can also result in increased interactions and exchanges of ideas among individuals. This can promote technological innovation and the development of green technologies, as mentioned in [2]. To summarize, the relationship between corruption and population density is reciprocal and interdependent. Therefore, addressing corruption and promoting transparency in governance are crucial for maintaining sustainable population density and fostering social and economic development. In conclusion, both corruption and population density have a reciprocal relationship. Higher levels of corruption can negatively impact population density by discouraging investment and economic growth, leading to limited job opportunities and lower living standards.

3. Preliminaries

Definition 3.1. *A fuzzy subset* [22] *A of the universe set* Ω *is represented by the membership function* $\mu_A(x) : \Omega \to [0, 1]$, *where* $\mu_A(x)$ *indicates the degree of membership of x in the fuzzy set A.*

Definition 3.2. *Let* Ω *be a nonempty set and* $P(\Omega)$ *the set of all subsets of* Ω. *The function* $\mu : P(\Omega) \to [0, 1]$ *is called a fuzzy measure* [23] *if*

$$\mu(\emptyset) = 0 \quad and \quad \mu(\Omega) = 1$$

$$\mu(A) \le \mu(B) \quad if \quad A \subseteq B$$

Definition 3.3. *A fuzzy number* $n = (u, w, z)$ *is said to be triangular* [4] *if its membership function is given by*

$$\mu_n(x) = \begin{cases} 0, & x \le u \\ \dfrac{x-u}{w-u}, & u < x \le w \\ \dfrac{z-x}{z-w}, & w < x \le z \\ 0, & x \ge w \end{cases}$$

where $u \le w \le z$.

A fuzzy subset F is called Trapezoidal [22] if its membership function is a trapezoidal function, which is specified by four parameters as $F(x : a, b, c, d)$.

Triangular and trapezoidal functions find extensive use, especially in fuzzy control systems, owing to their straightforward expressions and computational efficiency.

Definition 3.4. *Let u be a fuzzy subset. The fuzzy integral, or fuzzy expected value of u, FEV* [*u*] [24], *is the real number, defined by the Sugeno integral* [25],

$$FEV\left[u\right] = \int_{\Omega} u d\mu = \sup_{0\le\alpha\le1} \inf\left[\alpha, F(\alpha)\right]$$

where $F(\alpha) = \mu\{u \ge \alpha\} = \mu\{x \in R : u(x) \ge \alpha\}$ *It is intriguing to note tha* $F : [0, 1] \to [0, 1]$ *exhibits a continuous decrease across almost all points within its domain* [26] .

Theorem [26]. *Consider a typical membership function* $u : \Omega \to [0, 1]$ *and* μ *a fuzzy measure on* Ω. *If the function* $F(\alpha) = \mu\{x \in \Omega : u(x) \ge \alpha\}$ *has a fixed point* α^* *then it follows that*

$$\int_{\Omega} u d\mu = \alpha^* = F(\alpha^*)$$

In simpler terms the value of the Sugeno integral of u corresponds to the fixed point of F if such a fixed point exists.

4. Mathematical formulation and description of the problem

4.1. Problem Formulation

We consider a population divided into four categories:

V_v *Very vulnerable to corruption,*
V *Vulnerable to corruption,*

C corrupts,
R Individuals to recover,

we'll assume that the whole population: $N = V_v + V + C + R$. After recovering from corruption, it is thought that those affected will have a transient immunity brought on by this act. Figure 1 depicts the transmission mode of corruption, and Table 1 provides a full breakdown of the associated factors. With the use of the transmission diagram in Figure 1 and the parameters from Table 1, The model will have the following system of differential equations:

$$\frac{\mathrm{d}V_{vt}}{\mathrm{d}t} = \delta N - \frac{\lambda C_t V_{vt}}{N} - \delta V_{vt}$$
$$\frac{\mathrm{d}V_t}{\mathrm{d}t} = \frac{\lambda C_t V_{vt}}{N} - (a+\delta)V_t$$
$$\frac{\mathrm{d}C_t}{\mathrm{d}t} = aV_t - (a+b+\delta)C_t$$
$$\frac{\mathrm{d}R_t}{\mathrm{d}t} = \mu C_t - \delta R_t$$

To simplify matters, we can redefine the occurrence as the proportions:

$$v_v = \frac{V_v}{N} \qquad v = \frac{V}{N} \qquad c = \frac{C}{N} \qquad r = \frac{R}{N}$$

We get:

$$\frac{\mathrm{d}v_v}{\mathrm{d}t} = \delta - \lambda c v_v - \delta v_v$$
$$\frac{\mathrm{d}v}{\mathrm{d}t} = \lambda c v_v - (a+\delta)v$$
$$\frac{\mathrm{d}c}{\mathrm{d}t} = av - (\mu+b+\delta)c$$
$$\frac{\mathrm{d}r}{\mathrm{d}t} = \mu c - \delta r$$

with initial conditions:

$$v_v(0) \geq 0 \quad v(0) \geq 0 \quad c(0) \geq 0 \quad r(0) \geq 0$$

Table 1. A description of the model's parameters..

Parameters	Description
λ	Transmission rate of corruption from a corrupter person in a time period
a	Infectious rate
b	Rate of imprisonment due to corruption
δ	Birth and death rate which are assumed to be equal
μ	The recovery rate

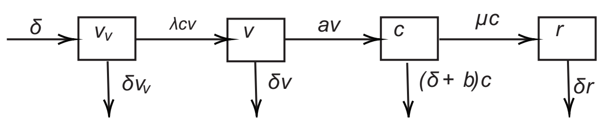

Figure 1: Transmission diagram for corruption.

Taking into account the overall population density,

we have: $v_v(t) + v(t) + c(t) + r(t) = 1 \Rightarrow r(t) = 1 - v_v(t) - v(t) - c(t)$ Thus, it is sufficient to take into account

$$\begin{aligned} \frac{dv_v}{dt} &= \delta - \lambda c v_v - \delta v_v \\ \frac{dv}{dt} &= \lambda c v_v - (a+\delta)v \\ \frac{dc}{dt} &= av - (\mu + b + \delta)c \end{aligned} \tag{1}$$

The set

$$\Omega = \{(v_v(t), v(t), c(t)) \in \mathbb{R}_+^{\nvDash} \quad v_v(t) + v(t) + c(t) \leq 1\}$$

is positively invariant for the system (1). The system (1) have two equilibrium points are given by $E^0 = (1, 0, 0)$ is the disease free equilibrium points of the system and the endemic equilibrium point $E^* = (v^*; v^*; c^*)$.

with,

$$\begin{aligned} v_v{}^* &= \frac{(a+\delta)(\delta+b+\mu)}{\lambda a} \\ v^* &= \frac{\lambda a \delta_\delta (a+\delta)(\delta+b+\mu)}{\lambda a (a+\delta)} \\ c^* &= \frac{\lambda a \delta_\delta (a+\delta)(\delta+b+\mu)}{\lambda (a+\delta)(\delta+b+\mu)} \end{aligned}$$

The endemic equilibrium point exists only when the condition $0 < v_v^* < 1$ is sufficiently necessary [27]. or $\mathcal{R}_0 > 1$, where $\mathcal{R}_0 = \dfrac{\lambda a}{(a+\delta)(\delta+b+\mu)}$ is known as reproduction number which determines the asymptotic behavior of the model.

4.2.Control problem

We will now introduce our second model, which incorporates control as an induced trait. We split whole human populations into three groups: those who are vulnerable to corruption v(t), those who are corrupts c(t), and those who quieted corruption q(t). We believe that corruption spreads among vulnerable persons as a result of their direct interaction with corrupts, where λ is the contact rate. The natural death rate is supposed to be the same for both classes. Because the model is considered to be in normalized form the birth rate δ is equal to the natural mortality rate. We evaluate natural recovery from corruption at rate μ and α is the control of corruption by the use of anti-corruption measures [28]. The proposed model is as follows:

$$\begin{aligned} \frac{dv}{dt} &= \delta(1-\alpha) - \lambda c v - \delta v \\ \frac{dc}{dt} &= \lambda c v - \mu c - \delta c \\ \frac{dr}{dt} &= \mu c - \delta r \\ \frac{dq}{dt} &= \delta \alpha - \delta q \end{aligned} \tag{2}$$

We have: $v(t) + c(t) + r(t) + q(t) = 1 \Rightarrow r(t) = 1 - s(t) - c(t) - q(t)$

Thus, it suffices to consider.

$$\frac{dv}{dt} = \delta(1-\alpha) - \lambda cv - \delta v$$
$$\frac{dc}{dt} = \lambda cv - \mu c - \delta c \tag{3}$$
$$\frac{dq}{dt} = \delta\alpha - \delta q$$

The set $\omega_1 = \{(v(t), c(t), q(t)) \in \mathbb{R}^3_+; v(t) + c(t) + q(t) \leq 1\}$ is positively invariant for the system (3).

The system (3) have two equilibrium points are given by $E_1^0 = (1 - \alpha, 0, \alpha)$ is the disease free equilibrium points of the system and the endemic equilibrium point

$$E_1^* = \left(\frac{\mu+\delta}{\lambda}, \frac{\delta(\lambda(1-\alpha)-\mu-\delta)}{\lambda(\mu+\delta)}, \alpha\right).$$

The impact of control on the disease-free equilibrium point and endemic equilibrium point can be readily observed. The susceptible population is reduced by a factor of α. Additionally, the reproduction number, which represents the number of secondary infections, is greatly affected. Following the introduction of the control in the model, the new reproduction number becomes $\mathcal{R}_\alpha = \mathcal{R}_0(1 - \alpha)$. The existence of an endemic equilibrium point is contingent on $\mathcal{R}_\alpha > 1$.

5. Measuring Corruption levvel using Fuzzy Logic

Fuzzy logic is a modeling approach that specifically addresses the prediction of a categorical variable, denoted as Y, which is considered "subjective" because it cannot be objectively defined and relies on the observer's perspective (such as categorizing an individual as "weak," "medium," or "strong"). This framework deviates from conventional statistics, where the value of variable Y can be objectively measured. Applying fuzzy logic involves striving to emulate a form of reasoning that closely resembles human thought processes.

The typical fuzzy model comprises three fundamental steps: fuzzification, fuzzy inference, and defuzzification.

Fuzzification: This involves transforming variables into fuzzy variables (also known as linguistic variables) by associating them with truth values. For instance, the anti-corruption variable is divided into categories like "weak", "medium", and "strong". This step is primarily conducted based on statistical observations (or through supervised or unsupervised learning to group variable values into homogeneous categories) or expert opinions.

Fuzzy Inference: During this stage, rules (along with corresponding results) are formulated based on linguistic variables. Each rule is assigned a truth value, and these rules are then combined to yield a singular linguistic result.

	Rule
1	if anti-corruption mesure is weak or transmission is bad then corruption level is high
2	if anti-corruption mesure is medium or transmission is average then corruption level is medium
3	if anti-corruption mesure is medium or transmission is bad then corruption level is high
4	if anti-corruption mesure is strong or transmission is good then corruption level is low

Fuzzy rules of anti-corruption measures with transmission.

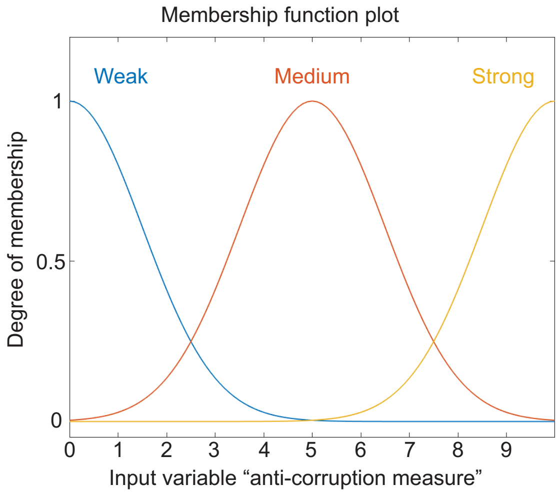

Figure 2: Membership Function of anti-corruption measure.

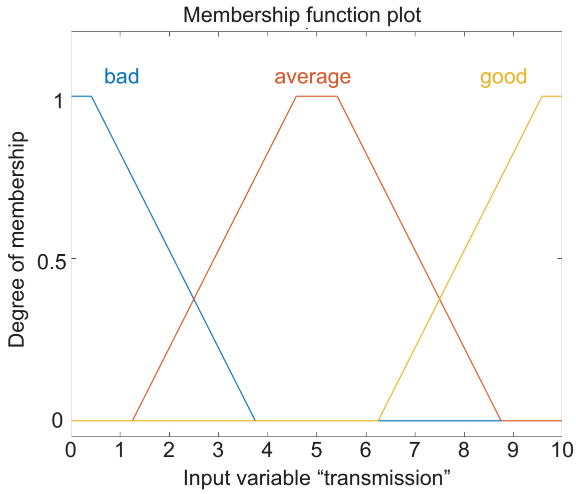

Figure 3: Membership Function of transmission of corruption.

Defuzzification: This involves transitioning from a linguistic result to a numerical result. The final step in fuzzy logic aims to transform the resulting activation curve obtained during the aggregation step into a real value. The method employed for this is the center of gravity method, which involves calculating the abscissa of the center of gravity of the result curve's surface.
Discussion: We design a system architecture for assessing levels of corruption, utilizing the methodology of a fuzzy rule-based inference system. The study incorporated two input variables: anti-corruption measures and the transmission of corruption, with one output variable being the levels of corruption. The simulation was executed using MATLAB 2023.

The findings indicated a clear correlation, showing that when a civil service exhibits a low severity level of corruption, it implies that the anti-corruption measures are strong. Additionally, the transmission factor needs to be either poor or at least average.

6. Fuzzy Control Model for Corruption Dynamic

The corruption dynamic fuzzy model is derived through modifications to the traditional model. This model accounts for the diversity in population corruption levels. Consequently, the transmission and

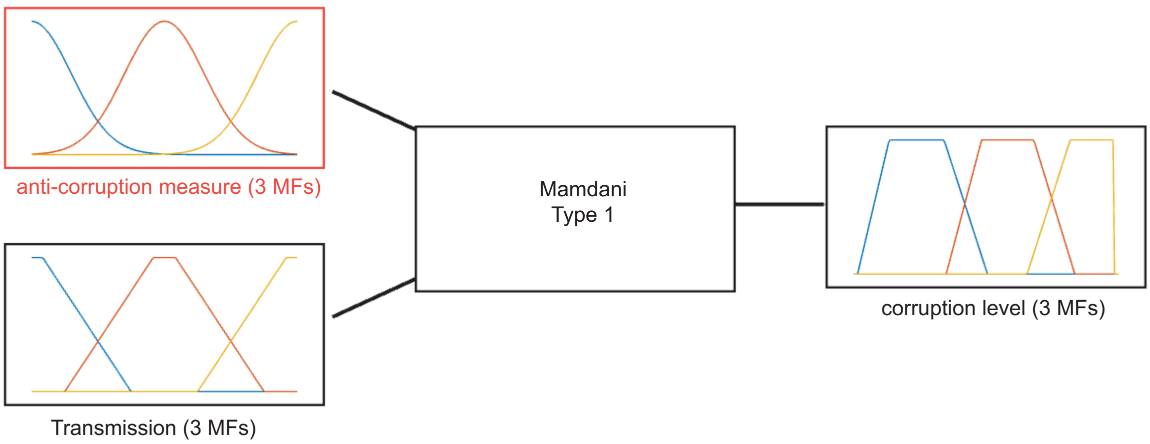

Figure 4: Membership function of output (corruption levels) based on Mamdani Type 1.

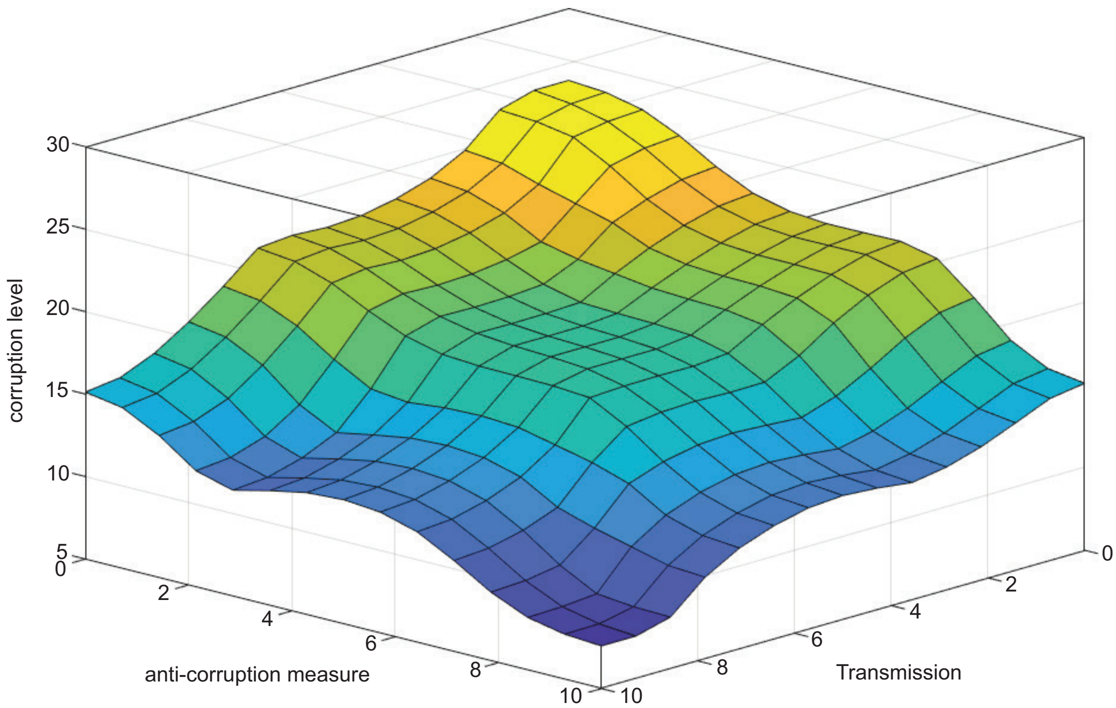

Figure 5: The surface of the function between the anti-corruption measure and transmission and corruption levels.

recovery within the population are not constant but contingent on corruption. The population is segmented into three categories based on corruption severity levels: low, medium, and high. The corruption levels β distribution within each category is represented as a triangular fuzzy number with a membership function denoted as $\pi(\beta)$.

$$\pi(\beta)=\begin{cases}1-\dfrac{|\beta-\tilde{\beta}|}{\varepsilon} & if \quad \beta\in[\tilde{\beta}-\varepsilon,\tilde{\beta}+\varepsilon],\\ 0 & if \quad \beta\notin[\tilde{\beta}+\varepsilon,\tilde{\beta}-\varepsilon];\end{cases} \tag{4}$$

The quantity $\tilde{\beta}$ is an average value and ε provides the dispersion of each of the fuzzy sets assumed by β. The graphical representation of $\pi(\beta)$ is seen in Figure 6. For a fixed β, $\pi(\beta)$ can have a linguistic meaning that is provided by a professional, such as low, medium, and high(Figure 7).

We claim that the corruption load of corrupter individuals provides population heterogeneity. As a result, the greater the corruption load, the greater the risk of corruption transmission. In other words, we suppose that $\lambda = \lambda(\beta)$ assesses the likelihood of transmission occurring in a meeting between a susceptible and a corrupter with a quantity of bribes β. As a result, some values of λ are more likely than others, which creates a membership function of a fuzzy number. The same for the recovery from corruption rate $\mu = \mu(\beta)$ and recovery rate due to anti-corruption measures $\alpha = \alpha(\beta)$.

A fuzzy model of corruption dynamic corresponding to problem (3) is:

$$\begin{aligned}\frac{\mathrm{d}\tilde{v}}{\mathrm{d}t}&=\delta(1-\alpha(\beta))-\lambda(\beta)\tilde{c}\tilde{v}-\delta\tilde{v}\\ \frac{\mathrm{d}\tilde{c}}{\mathrm{d}t}&=\lambda(\beta)\tilde{c}\tilde{v}-\mu(\beta)\tilde{c}-\delta\tilde{c}\\ \frac{\mathrm{d}\tilde{q}}{\mathrm{d}t}&=\delta\alpha(\beta)-\delta\tilde{q}\end{aligned}$$

7. Analysis of the Fuzzy Problem

In this section both the categories of susceptible and corrupts are ambiguous in the sense that various individuals in the population have varying degrees of sensitivity. Such distinctions can occur, For instance, when exploring the diverse habits and customs of the population, along with varying degrees of resistance, and the like. In this methodology, more realistic models can be considered that incorporate the individuals' diverse levels of susceptibility. The parameter λ is treated as a fuzzy number in this context.

We make the assumption that there is a minimum number of corruption acts β_m required to transmit corruption and that when the amount of corruption acts by a person is very low, the chance of transmission is insignificant. This assumption allows us to derive the membership function. Additionally, for a specific number of corruption acts β_0, the risk of corruption transmission is at its highest and equals 1. Finally, we assume that the maximum number of corruption acts that an individual may have is always set by β_M.

We have selected the following linear membership function for the fuzzy subset:

$$\lambda(\beta) = \begin{cases} 0 & if \quad \beta < \beta_m, \\ \dfrac{\beta - \beta_m}{\beta_0 - \beta_m} & if \quad \beta_m \leq \beta \leq \beta_0, \\ 1 & if \quad \beta_o \leq \beta \leq \beta_M. \end{cases} \tag{6}$$

The graphic of $\lambda(\beta)$ is shown in figure 8.

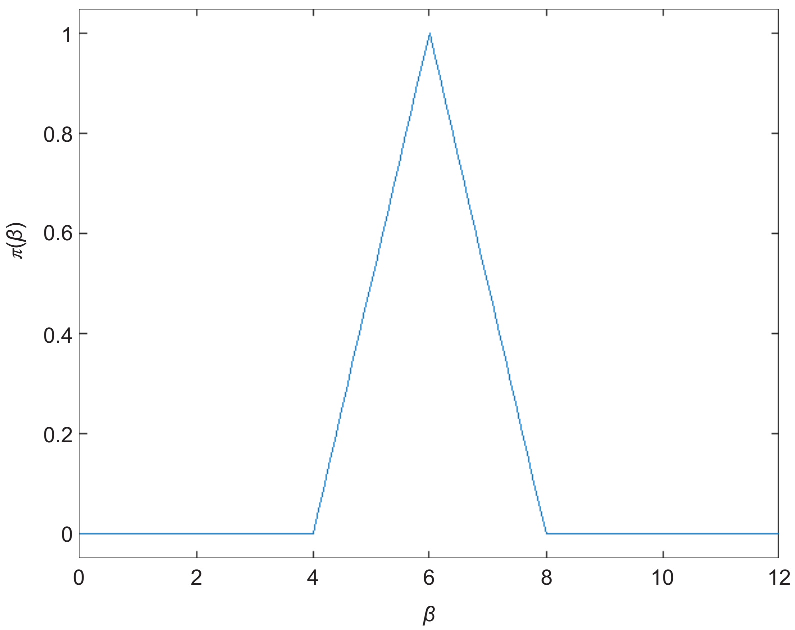

Figure 6:. Membership function of $\pi = \pi(\beta)$.

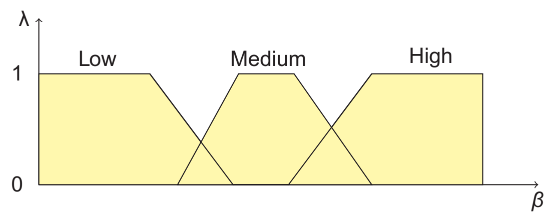

Figure 7: Low, medium, and high corruption levels.

Because anti-corruption measures control α are allocated in accordance with the level of corruption β, it is also reasonable to conclude that the recovery rate control due to anti-corruption measures is a fuzzy quantity. When the risk of fraud is minimal for low-level corruption β_m, anti-corruption measures are not required. Therefore, no anti-corruption actions are necessary if the number of corrupt activities is less than β_m.

Similar to the above, anti-corruption measures α should be used most frequently whenever the degree of corruption activities β is at its peak, or if $\beta_0 \leq \beta \leq \beta_M$. However, due to a number of factors, including high cost and human resources shortages, α might not take its maximal value as one. Therefore, let's assume that the anti-corruption measure's maximum value is $1 - \sigma, (\sigma > 0)$.

In the current case, the membership function for the anti-corruption measures α is:

$$\alpha(\beta) = \begin{cases} 0 & if \quad \beta < \beta_m, \\ \dfrac{\beta - \beta_m}{\beta_0 - \beta_m} & if \quad \beta_m \leq \beta \leq \beta_0, \\ 1 - \sigma & if \quad \beta_0 \leq \beta \leq \beta_M. \; and \; \sigma > 0; \end{cases} \tag{7}$$

Now, $\mu = \mu(\beta)$ is the recovery rate from corruption. The level of corruption affects the healing rate of a person (μ) as well. The period of time it takes to recover from corruption increases with the level of corruption activities. As a result, $\mu(\beta)$ should be a decreasing function of β:

$$\mu(\beta) = \frac{\mu_0 - 1}{\beta_M}\beta + 1 \quad \text{if} \quad 0 \leq \beta \leq \beta_M. \tag{8}$$

With $0 < \mu_0 < 1$ is the lowest recovery rate. Fig. provides a representation for the membership function of the recovery rate $\mu(\beta)$.

8. Fuzzy Basic Reproduction Number R_f (β)

Notice that $\mathcal{R}_0$ consists of λ, μ and α which are both functions of the level of corruption, hence $\mathcal{R}_0$ is also a function of β and can be denoted as $\mathcal{R}_0(\beta)$.

Following the introduction of the control in the fuzzy model, the new reproduction number becomes $\mathcal{R}(\beta) = \mathcal{R}_0(\beta)(1 - \alpha(\beta)) = \dfrac{\lambda(\beta)}{\mu(\beta) + \delta}(1 - \alpha(\beta))$.

However, $\mathcal{R}(\beta)$ is not a fuzzy set for its value can be greater than one. So we have to modify it to be less than 1.

We have : $\mu_0 < \mu(\beta)$

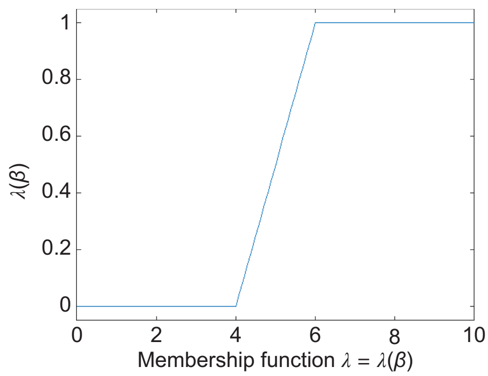

Figure 8: Membership function of $\lambda = \lambda(\beta)$.

so $\mu_0 + \delta < \mu(\beta) + \delta$

and $\dfrac{\mu_0+\delta}{\mu(\beta)+\delta} < 1$ and we know $\lambda(\beta) < 1$ and $1 - \alpha(\beta) < 1$, so we obtain :

$$\frac{(\mu_0+\delta)\lambda(\beta)}{\mu(\beta)+\delta}(1-\alpha(\beta)) < 1$$

$$(\mu_0 + \delta)\mathcal{R}(\beta) < 1$$

We define the Fuzzy Basic Reproduction Number by

$$\mathcal{R}_f(\beta) = \frac{1}{\mu_0+\delta} FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right]$$

To get the FEV$[(\mu_0 + \delta)\mathcal{R}(\beta)]$ we need to define a fuzzy measure:

$$\chi(K) = \sup_{\beta\in K} \chi(\beta), \qquad K \subset R$$

by using fuzzy measure χ

$$FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right] = \sup_{0\le\theta\le1} \inf\left[\theta, f(\theta)\right]$$

where $f(\theta) = \chi\{\beta\colon (\mu_0 + \delta)\mathcal{R}(\beta) \ge \theta\}$

As we can see FEV $[(\mu_0 + \delta)\mathcal{R}(\beta)]$ depend on $\lambda(\beta)$ and $\mu(\beta)$. As we discussed before, in this fuzzy model the population divided in 3 categories based on the severity of levels of corruption, and each category have different transmission rate, recovery rate and anti-corruption measures. Therefore, $\mathcal{R}_f$ (β) will be calculated from each category. The fuzzy sets given by the membership function $\pi(\beta)$ for different cases are:

a) low if $\bar{\beta} + \epsilon < \beta_m$
b) medium if $\bar{\beta} - \epsilon < \beta_m$ and $\bar{\beta} + \epsilon \le \beta_M$
c) high if $\bar{\beta} - \epsilon > \beta_M$

8.1. Low levels of corruption

In this category $\lambda(\beta) = 0$, $\alpha(\beta) = 0$, and $\mu(\beta) = \dfrac{(\mu_0-1)\beta}{\beta_M} + 1$

So we can calculate FEV $[(\mu_0 + \delta)\mathcal{R}(\beta)]$.

$$FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right] = \sup_{0\le\theta\le1} \min\left\{\theta, \chi(\emptyset)\right\} = 0$$

So $\mathcal{R}_f(\beta) = \dfrac{1}{\mu_0+\delta}[FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right] = 0.$

This case is the endemic case and for along time there will be no active corrupts in this category.

8.2. Medium levels of corruption

The transmission, the recovery rate and the anti-corruption measures are:

$$\lambda(\beta) = \frac{\beta-\beta_m}{\beta_0-\beta_m}, \qquad \mu(\lambda) = \frac{\mu_0-1}{\beta_M}\beta + 1 \quad \text{and} \qquad \alpha(\beta) = \frac{\beta-\beta_m}{\beta_0-\beta_m}.$$

Then we calculate FEV$[(\mu_0 + \delta)\mathcal{R}(\beta)]$

$$FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right] = \sup_{0\le\theta\le1} \min\left\{\theta, \varphi(\theta)\right\}$$

Where $\varphi(\theta) = \chi\left\{\beta : \frac{\lambda(\beta)(\mu_0+\delta)(1-\alpha(\beta))}{\mu(\beta)+\delta} \geq \theta\right\}$.

Through a direct calculation we obtain:

$$\varphi(0) = \chi(\Omega) = 1 \quad \text{and} \quad \varphi(1) = \chi(\varnothing) = 0$$

For $0 < \theta < 1$

$$\varphi(\theta) = \chi([\beta_\theta, \beta_m]) \quad \text{with} \quad \beta_\theta \text{ is when } \quad \frac{\lambda(\beta)(\mu_0+\delta)(1-\alpha(\beta))}{\mu(\beta)+\delta} = \theta$$

We divide $\varphi(\theta)$ in 3 cases:

$$\begin{aligned}
\varphi(\theta) &= 1 && \text{if} \quad 0 \leq \theta \leq \frac{\lambda(\bar\theta)(\mu_0+\delta)(1-\alpha(\bar\theta))}{\mu(\bar\theta)+\delta} \\
&= \varphi(\bar\theta) && \text{if} \quad \frac{\lambda(\bar\theta)(\mu_0+\delta)(1-\alpha(\bar\theta))}{\mu(\bar\theta)+\delta} \leq \theta \leq \frac{\lambda(\bar\theta+\epsilon)(\mu_0+\delta)(1-\alpha(\bar\theta+\epsilon))}{\mu(\bar\theta+\epsilon)+\delta} \\
&= 0 && \text{if} \quad \frac{\lambda(\bar\theta+\epsilon)(\mu_0+\delta)(1-\alpha(\bar\theta+\epsilon))}{\mu(\bar\theta+\epsilon)+\delta} \leq \theta \leq 1
\end{aligned}$$

As we see φ is a continuous and decreasing function ,so the fixed point of φ is the same as that of FEV $[(\mu_0 + \delta)\mathcal{R}(\beta)]$.

A direct calculation yields

$$\frac{\lambda(\bar\theta)(\mu_0+\delta)(1-\alpha(\bar\theta))}{\mu(\bar\theta)+\delta} \leq FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right] \leq \frac{\lambda(\bar\theta+\epsilon)(\mu_0+\delta)(1-\alpha(\bar\theta+\epsilon))}{\mu(\bar\theta+\epsilon)+\delta}$$

And we obtain

$$\mathcal{R}(\bar\beta) \leq \mathcal{R}_f(\beta) \leq \mathcal{R}(\bar\beta+\epsilon)$$

8.3. High levels of corruption

In this category

$$\lambda(\beta) = 1, \qquad \mu(\lambda) = \frac{\mu_0 - 1}{\beta_M}\beta + 1 \quad \text{and} \qquad \alpha(\beta) = 1 - \sigma.$$

Then we calculate FEV$[(\mu_0 + \delta)\mathcal{R}(\beta)]$

$$FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right] = \sup_{0\leq\theta\leq1} \min\{\theta, \varphi(\theta)\}$$

Through a direct calculation we obtain:

$$\begin{aligned}
\varphi(\theta) &= 1 && \text{if} \quad 0 \leq \theta \leq \frac{\sigma(\mu_0+\delta)}{\mu(\bar\theta)+\delta} \\
&= \varphi(\bar\theta) && \text{if} \quad \frac{\sigma(\mu_0+\delta)}{\mu(\bar\theta)+\delta} \leq \theta \leq \frac{\sigma(\mu_0+\delta)}{\mu(\bar\theta+\epsilon)+\delta} \\
&= 0 && \text{if} \quad \frac{\sigma(\mu_0+\delta)}{\mu(\bar\theta+\epsilon)+\delta} \leq \theta \leq 1
\end{aligned}$$

As we see φ is a continuous and decreasing function ,so the fixed point of φ is the same as that of FEV $[(\mu_0 + \delta)\mathcal{R}(\beta)]$.

A direct calculation yields

$$\frac{\sigma(\mu_0+\delta)}{\mu(\bar{\theta})+\delta} \leq FEV\left[(\mu_0+\delta)\mathcal{R}(\beta)\right] \leq \frac{\sigma(\mu_0+\delta)}{\mu(\bar{\theta}+\epsilon)+\delta}$$

So we obtain

$$\frac{\sigma}{\mu(\bar{\theta})+\delta} < \mathcal{R}_f(\beta) < \frac{\sigma}{\mu(\bar{\theta}+\epsilon)+\delta}$$

So we have $\mathcal{R}_f(\beta) > 1$, in this case we conclude that corruption will expand in the society.

9. Numerical simulation

This article uses an iterative approach for numerical simulations and focuses specifically on parameter values: $\lambda(\beta) = 0.3$, $\mu(\beta) = 1.2$, $\delta = 0.2$, $\alpha(\beta) = 0.2$.

The transmission rate rises together with the corruption level beta, which increases the number of corrupt people. As a result, there are less susceptible people.

The dynamics of susceptible people, corrupts, and recovered hosts are shown in Figures 10 through 12 at various levels of corruption, respectively. When corruption levels are at lowest levels (234 cases), there is hardly any corruption in the population. As a result, no transmission takes place at this specific corruption level (as seen in Figure 10).

It appears that the susceptible human population substantially declines and reaches its lowest value (as indicated in Figure 11) when the degrees of corruption reach their highest value (5.103 cases). The transmission rate has significantly increased, which is principally responsible for this

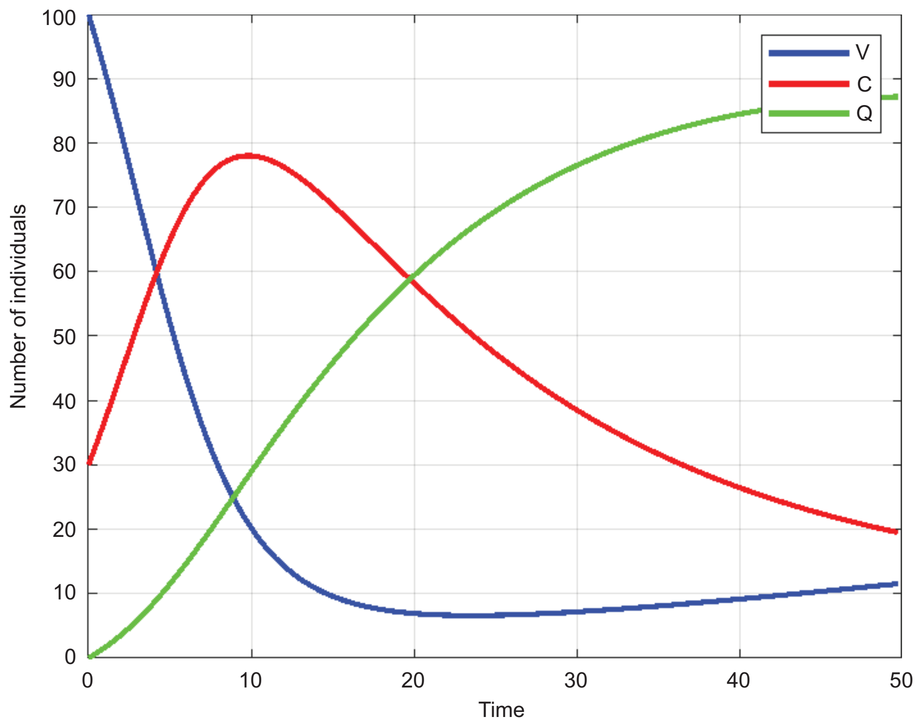

Figure 9: Dynamical behavior of the system.

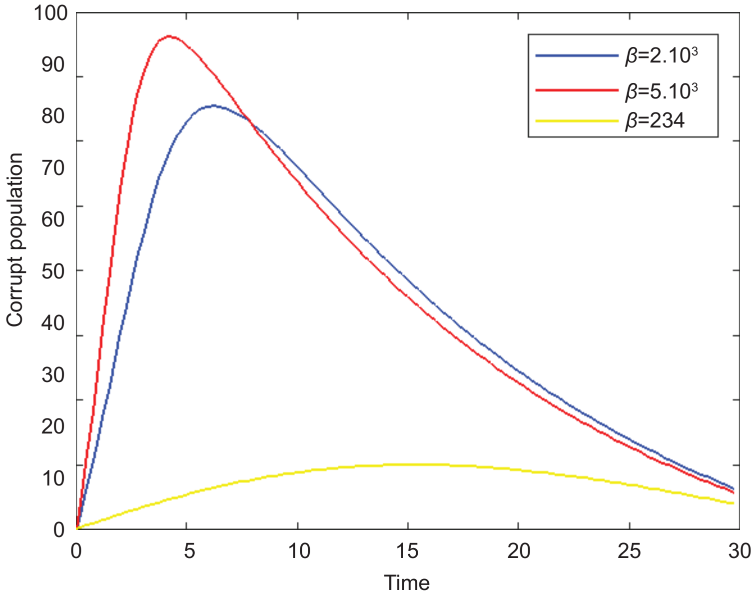

Figure 10: Dynamical behavior of corrupt individuals.

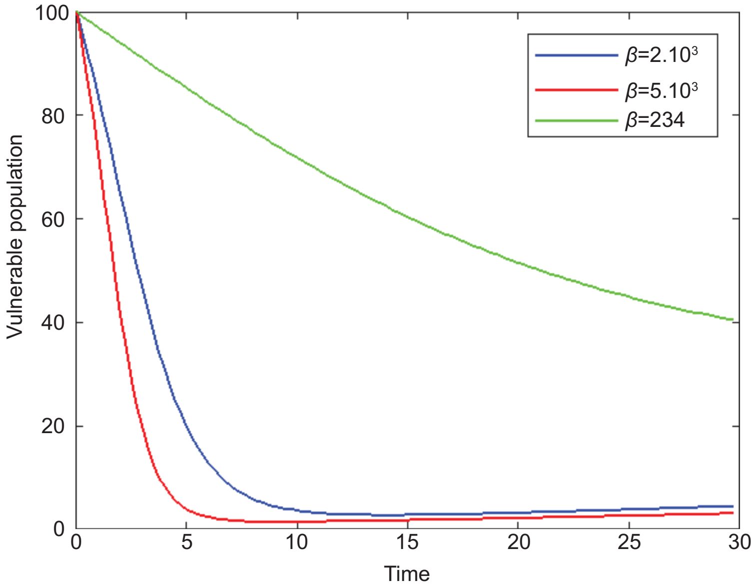

Figure 11: Dynamical behavior of vulnerable population.

decrease. Initially, the high transmission rate causes an increase in the number of corrupt people. But as the simulation goes on, the population begins to fall as a result of both corruption recovery and natural mortality (as shown in Figures 10 and 12).

An important indicator of whether corruption will eventually disappear or remain in the community is the basic reproduction number. The number of corrupt people grows along with the fuzzy transmission rate of corruption, which raises the basic reproduction number (as seen in Figure 13). A low basic reproduction number means that the corruption will eventually disappear and is unlikely to spread widely. On the other side, as the rate of transmission rises, corruption becomes endemic, a sign that it will continue to affect people over time. whereas, as the recovery rate rises, corruption is more likely to disappear, lowering its prevalence (as seen in Figure 14).

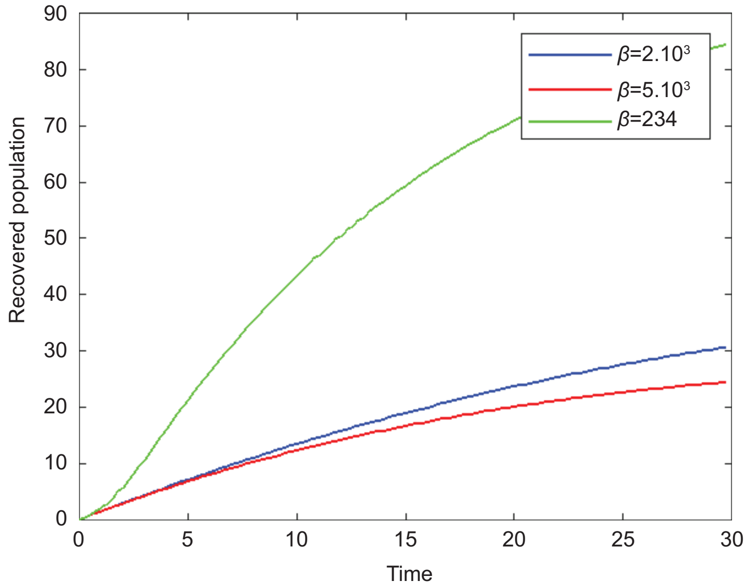

Figure 12: Dynamical behavior of recovered population.

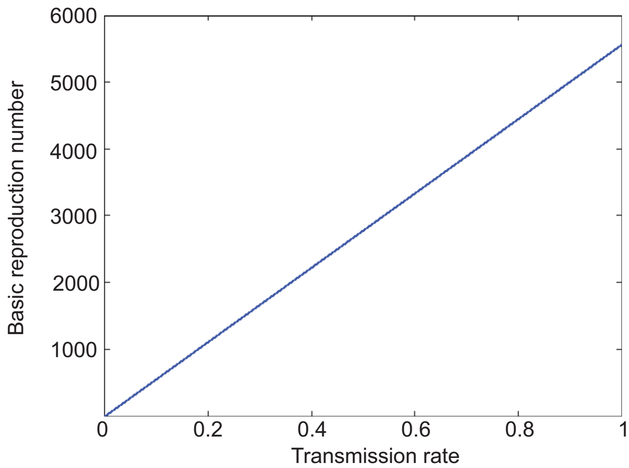

Figure 13: Basic reproduction number with transmission rate.

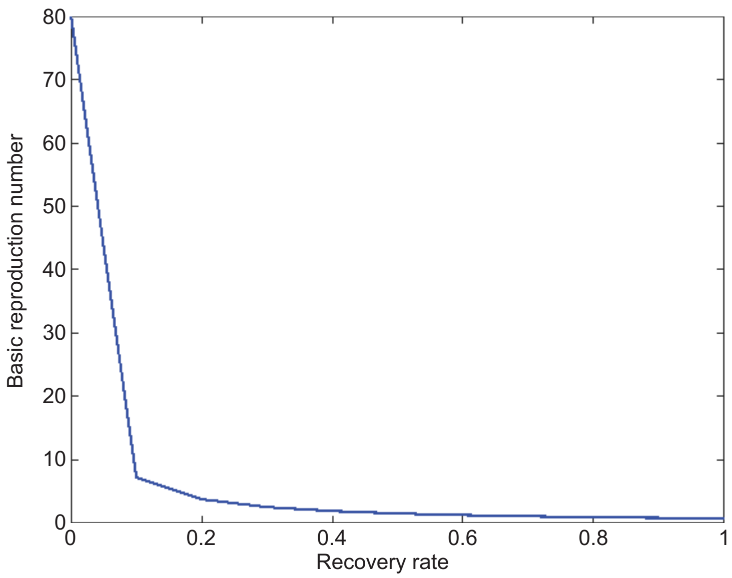

Figure 14: Basic reproduction number with recovery rate.

Different transmission dynamics phenomena in corruption can be observed as a result of the presence of fuzziness in the model parameters, which are functions of the levels of corruption. Deterministic models cannot witness these behaviors. As a result, when compared to a deterministic model of the condition, the fuzzy model offers a more accurate explanation of the dynamics of corruption's transmission. Fuzziness allows for a more precise representation of real-world events by improving the model's ability to reflect the inherent uncertainties and complexities linked to the development of corruption.

Conclusion

To provide a more accurate depiction of corruption in society, we proposed a corruption model that includes fuzzy transmission dynamics in this work. Our model accurately depicts the complex nature of corruption transmission by viewing corruption as a continuous spectrum as opposed to a binary occurrence. We showed that fuzzy logic enables a more precise representation of the transmission of corruption, where individuals with higher levels of corruption have a stronger influence on others.

We investigated the dynamics of corruption in our fuzzy model using simulations and mathematical analysis. Our research uncovered fascinating behaviors that are not visible in deterministic models. We noticed the development of corruption hotspots or clusters,the effect of anti-corruption efforts on lowering corruption levels as well as the intricate interplay between beginning circumstances and outside variables that affect how corruption spreads or declines.

Our corruption model's inclusion of fuzzy transmission dynamics facilitates a greater comprehension of the complexity of corruption. Our model offers insights into the causes causing corruption and the efficacy of intervention methods by taking uncertainties and variability into consideration. These insights can be used by policymakers and anti-corruption organizations to develop focused and efficient strategies for thwarting corruption and encouraging moral behavior in society.

This study advances the subject of corruption modeling by emphasizing the value of taking fuzzy transmission dynamics into account. It improves our comprehension of the complexities involved with corruption and offers a more thorough framework for researching and dealing with this social problem. We fill the gap between theoretical models and actual data by introducing fuzzy logic, which increases the practical relevance and applicability of corruption research.

Our fuzzy corruption model can now be expanded upon by future research by include other elements including social networks, economic situations, and cultural influences. With these improvements, corruption dynamics would be better understood in their entirety, allowing for more precise forecasts and policy suggestions.

Our research supports the importance of including fuzzy transmission dynamics in corruption modeling. We capture the nuances of corruption spread and enable policymakers to create focused strategies for battling corruption by embracing uncertainty and unpredictability. We may work to create more open, accountable, and ethical communities through continual study and pragmatic application.

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